EXPERIMENTAL RESEARCHES

ON THE

SPECIFIC GRAVITY AND THE DISPLACEMENT OF SOME SALINE SOLUTIONS

J. Y. BUCHANAN, M.A., F.R.S.

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* From the Greek ἐννεάς, which signifies a body of nine.

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$$\frac{d\Delta}{dn} = \text{Const.}$$

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$$\frac{d \log \Delta}{dn} = \text{Const.}$$

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SECTION XV.

On a Remarkable State of Unrest in a Supersaturated Solution of Calcium Chloride before Crystallising.

104. The supersaturated solution was 7·225 $CaCl_2$ + 1000 grams of water. It was expected that this solution would crystallise easily and furnish a truly saturated solution. As it showed no inclination to crystallise although every opportunity was offered it to do so, it was adopted as an example of a supersaturated solution peculiarly adapted to closer study. Table I. contains the constants of the open hydrometers A and B, as loaded for the experiments of this section, (*a*) when floating in distilled water, and (*b*) when floating in the supersaturated solution of calcium chloride. 185

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Time in minutes:	−2·0	−1·5	−1·0	−0·5	0·0	+0·5	1·0	1·5	2·0
Temperature:	23·23°	23·21°	23·19°	23·17°	23·16°	23·14°	23·12°	23·09°	23·07°

During the 4 minutes the temperature fell 0·16°, whence 0·04° per minute represents the mean rate of fall of temperature when the system has the temperature 23·16° and is cooling in air of constant temperature 19·30° C.

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SECTION XVI.

THE DETERMINATION OF THE SPECIFIC GRAVITY OF THE CRYSTALS OF A SOLUBLE SALT BY DISPLACEMENT IN ITS OWN MOTHER-LIQUOR, AND THE VOLUMETRIC RELATIONS BETWEEN THE CRYSTALS AND THE MOTHER-LIQUOR WHICH ARE ESTABLISHED BY THE EXPERIMENT.

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120. This work was undertaken owing to the arrival of the great anticyclone or heat-wave of the summer of 1904, which made observations of specific gravity at 19·5° impossible. The liquid in which every soluble salt is quite insoluble is its own mother-liquor at the temperature at which the one parted from the other. It was in this liquid that the specific gravity of the crystals of the salts of the two enneads MR and MRO_3 was determined. It is obvious that this method is applicable only to salts which *have* a mother-liquor, such as KCl, RbBr, $CaCl_26H_2O$, $BaCl_22H_2O$. It is inapplicable to salts such as $CaCl_2$, $BaCl_2$, and the like, which have no legitimate mother-liquor. The anti-cyclone prevailed throughout the greater part of July and August 1904, during which time the determinations of the specific gravity of the crystals and the mother-liquors of the salts of the ennead MR were determined. 202

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Section I.—Introduction.

§ 1. *The Principles of Archimedes.**—It is well known that the mechanics of floating bodies, and the laws which govern their equilibrium, were established and enunciated by Archimedes, the Sicilian, in the third century before our era. The following propositions, demonstrated in the first book of his treatise on this subject, embody the fundamental principles of the hydrometer :—

(*a*) The surface of any fluid at rest is the surface of a sphere whose centre is the same as that of the earth.

(*b*) Of solids, those which, size for size, are of equal weight with a fluid will, if let down into the fluid, be immersed so that they do not project above the surface, but do not sink lower.

(*c*) A solid lighter than a fluid will, if immersed in it, not be completely submerged, but part of it will project above the surface.

(*d*) A solid lighter than a fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of the fluid displaced.

(*e*) If a solid lighter than a fluid be forcibly immersed in it, the solid will be driven upwards by a force equal to the difference between its weight and the weight of the fluid displaced.

(*f*) A solid heavier than a fluid will, if placed in it, descend to the bottom of the fluid, and the solid will, when weighed in the fluid, be lighter than its true weight by the weight of the fluid displaced.

Archimedes considered only one solid and one fluid, and his laws regulate exactly what takes place in such a system when the solid is totally immersed in the fluid; or, if only partially immersed in it, when the non-immersed portion of the solid is immersed in no other fluid—in other words, when the experiment is being made in a vacuum, or in a medium the density of which is insensible. When, however, the experiment is being made in air, it is not necessary to postulate that its density is insensible; Archimedes' laws still hold good, only the solid falls to be considered as divided into two, one of which is completely immersed in the one fluid (the liquid), and the other is completely immersed in the other fluid (the air). If the solid was immersed in three fluids, as, for instance, water, oil, and air, and floated at rest when part of it was immersed in each of these fluids, it would fall to be divided into three portions, each of which is totally immersed in one of the three fluids, Archimedes' laws would still be applicable, and the final total effect would be the sum of the partial effects.

§ 2. *Hydrometer suitable for Demonstrations on the Lecture Table.*—I constructed an instrument of this kind for use in lectures which I gave as assistant in the

* *The Works of Archimedes, edited in Modern Notation*, by T. L. Heath, Sc.D., Cambridge University Press, 1897, pp. 253-268.

University Laboratory in Edinburgh, under Professor CRUM BROWN, in the years 1869 to 1872. A description of it was published in the *Berichte der Deutschen Chemische Gesellschaft* (1871), iv. 338. Fig. 1 is a sketch of it. The stem of this instrument was made of glass tube having an external diameter of about 1 centimetre, and a truly circular section of uniform diameter. A slip of paper is attached inside the stem. It is graduated on any convenient scale of equal lengths, and the divisions are numbered upwards and downwards from the zero point in the middle. The numerals from 0 upwards have the positive sign, and those running from the 0 downwards have the negative sign.

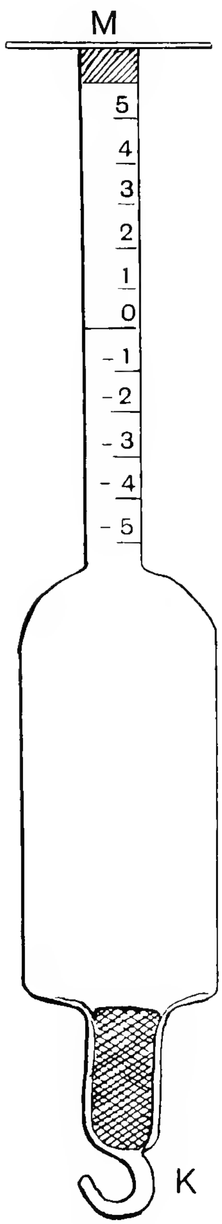


FIG. 1.

The hydrometer is ballasted with mercury or shot, so that, in its completed state, it sinks in the liquid used, at the atmospheric temperature, exactly to the zero division in the middle of the stem.

The lower extremity of the instrument takes the form shown in the figure, terminating in a hook K. The upper extremity of the stem is closed with a cork, to which a suitable disc of cardboard, M, is attached by sealing-wax.

The hydrometer was originally constructed in order to illustrate the determination of the specific gravity of solid bodies. The liquid in which it is to be immersed may be distilled water, but other liquids, for instance sea-water, may also be used. It is contained in a suitable cylinder, and should have the temperature of the room in which the experiment is being made.

When it is proposed to exhibit the determination of the specific gravity of any particular solid body, the hydrometer is immersed in the liquid, in which it sinks until the zero division on the scale is exactly in the plane of the surface of the liquid. A suitable fragment or piece of the solid body is then placed on the platform M, and the extent to which the immersion of the stem in the water is increased is noted. The solid body is then removed from the platform M and attached to the hook K, and the hydrometer is again immersed in the water. When equilibrium of flotation has been established, the immersion of the stem is again read on the scale.

Let the former of these two numbers be expressed by a and the latter by b. a and b are lengths of a cylinder of uniform diameter and of circular section; therefore the volumes of these cylinders are proportional to their lengths; and, as the same liquid is displaced in each case, the weights of the liquids so displaced are also proportional to the lengths a and b. It follows, therefore, that the expression $(a-b)$ represents the weight of a volume of the liquid equal to that of the solid body, and that the specific gravity of the solid body, referred to that of the liquid as unity, is

$$D=\frac{a}{a-b}.$$

When a solid body is placed on the platform M, the hydrometer always sinks deeper in the liquid; therefore a is always positive. When a solid body consists of a substance which is denser than the liquid, then, when it is attached to the hook K and is immersed with the instrument in the liquid, it causes the hydrometer to sink deeper in it, and b is also positive in this case. When the substance of the solid body is less dense than the liquid, a is positive as before; but when the body is attached to the hook, and is immersed with the hydrometer in the liquid, it exerts a pressure *upwards*, which causes the hydrometer to emerge and expose a part of the stem below the point 0. On this part of the scale the numerals have the negative sign, and the weight of the volume of liquid displaced by the solid body is, as before, $(a-b)$; and its specific gravity is, also as before,

$$D=\frac{a}{a-b}.$$

The identity of the expressions of the experimental data in determining the specific gravity of substances so dissimilar as, for instance, a stone and a cork never failed to arrest the attention of the students.

It will be noticed that, by using this method, *the specific gravity of a solid body is obtained without any determination of weight having been made, either in the production of the instrument or in its use, and that the only measurements made are those of length.*

§ 3. *Usefulness of the Hydrometer in the Study of Mineral Waters.*—But the hydrometer and its uses had always had a fascination for me. I began to pay particular attention to the subject in Wiesbaden, when working as a student with FRESENIUS, and afterwards (1866–67) as an assistant in his private analytical laboratory. During this period I became much interested in the mineral waters which abound in the (then) Duchy of Nassau and the neighbouring parts of the Rhineland, and especially in the Kochbrunnen of Wiesbaden, perhaps the most celebrated of them all. I had great curiosity to investigate the variations, if any, in its concentration at different times and seasons; but, as a student, I had to follow the plan of instruction laid down, and, in the private laboratory of the final referee in Germany regarding all matters of dispute or arrangement which could be decided by chemical analysis, the important and responsible work entrusted to me made it impossible for me to occupy myself with anything else at the same time. During the voyage of the *Challenger*, I many times made up my mind, on my return to Europe, to visit Wiesbaden and use the hydrometer in carrying out a systematic investigation in this sense; but my intention has not been realised.

§ 4. *The Hydrometer in the "Challenger" Expedition.*—The dispatch of the *Challenger* Expedition was decided before the end of the year 1871, and Sir WYVILLE THOMSON, who was then Professor of Natural History in the University of Edinburgh, was chosen for its leader. He did me a great honour and a very substantial service in selecting me for the post of chemist and physicist of the expedition quite a year before the date fixed for its departure. I cannot adequately express the gratitude which I feel for the confidence which he thus showed in me, and for the privilege which it gave me of taking an active part in this memorable expedition. The expedition lasted less than four years, yet these years are fuller of recollections than all the rest of my life.

During the year which elapsed between my selection and my official appointment, I occupied myself almost exclusively in preparing for my work at sea, and I considered that the specific gravity of the water of the ocean, and its variations, would be one of the most important matters for continuous observation. Here I had in view the variations of specific gravity which occur in the open ocean and far from all influence of the land. These were only imperfectly known, but there was reason to conclude that they were confined within narrow limits.

I chose the hydrometer, or "aræometer," as it is called abroad, because it appeared to me to be the only type of instrument which furnished *directly* the information demanded, namely, the specific gravity of the water, and that with the exactness required when the variations of specific gravity are so small.

Even at that early date indirect methods of all kinds were recommended to me. In theory, any physical constant of a saline solution, the expression of which includes a term depending on its specific gravity, can be used for this purpose. *But indirect methods are, in the nature of things, affected with at least a double quantity of errors. There are the errors with which the datum directly supplied by the vicarious method used is affected, and there are those which affect the operation of comparison by which that datum obtains its densimetric interpretation.*

I had then, and I have still, an instinctive dislike of all indirect methods in science; I therefore adhered to my own purpose, believing that, if nothing but manipulative difficulties stood in the way, they could be overcome by perseverance and a determination not to accept defeat too readily; and, as is so often the case, the difficulties apprehended turned out to be in no way formidable.

§ 5. In designing the hydrometer I decided that, in the values of the specific gravity obtained with it, units in the fourth place of decimals must be exact, and that the exactness should be pushed as far as possible into the fifth place. As a knowledge of the physical constants of the instrument is of the first importance, I rejected the plan of having a series of hydrometers, each to be used in the waters the specific gravity of which corresponded to the limits of its scale. I decided to have one hydrometer, made of glass, in which the dimensions of the stem and of the body should be in such proportion as to ensure the degree of accuracy above indicated, and provision for extending its range of usefulness to sea-waters of all specific gravities should be made by suitable alterations of its weight. Considerations of stability suggested attaching the accessory weights necessary for this purpose to the lower extremity of the hydrometer. But this would involve their being immersed in the liquid the specific gravity of which was to be determined, and was therefore inadmissible. The only alternative was to attach them to the upper extremity of the stem. A length of 10 centimetres of the stem was graduated into millimetres, and the external diameter of the stem was such that its graduated portion displaced rather less than 1 gram of distilled water. The body of the instrument was constructed so as to have a volume of approximately 160 cubic centimetres. The hydrometer was ballasted with mercury, so as to float in distilled water of ordinary temperature with the whole of the graduated part of the stem exposed. The system of accessory weights designed for increasing the range of the hydrometer included, as first weight, a small brass table which fitted on to the top of the stem. Its weight was designed so that, if the hydrometer alone floated at the lowest division of the scale in a particular water, and the table was then affixed to the top of the stem, the hydrometer would sink until it floated at a division near the top of the scale in the same water. Of the further weights, the first of the series was a mass of brass of about the same weight as the table. When the hydrometer carrying the table on the stem floated at the lowest division of the scale in a particular water, then, by placing the further weight on the table, the hydrometer sank until it floated at a division near the top of the scale. The weight of the next weight of the series was made approximately double that of the table, so that when the hydrometer, loaded with the table and the previous weight, floated at the lowest division on the scale in a particular water, and the previous weight was replaced on the table by the present one, the hydrometer floated in the same water at a division near the top of the scale; and so on.

§ 6. The series of weights was carried so far that waters of all densities from that of distilled water to that of water more dense than that of the Red Sea could be determined with the same hydrometer.

The accessory weights form roughly an arithmetical series, the common difference of which is equal to the first term, namely, that of the little table to be placed on the top of the stem and to carry the other weights, when required. As produced, the weights fulfilled all the conditions demanded of them, and all that it was necessary to know was the exact weight of each, and this was determined.

From the design of the system of accessory weights, it will be seen that provision was made for *single* observations of specific gravity. Duplicate observations were possible only in cases where the salinity and temperature of the water combined to produce such a specific gravity that it could be observed with one of the sets of weights near the lowest division of the scale. In that case its specific gravity could be obtained also with the next higher weight at a division near the top of the scale, because the difference between the successive weights was rather less than that required to immerse the divided portion of the stem.

On rare occasions multiple observations were made on a single water, using ordinary decigram weights, but this was found to be very inconvenient. Nevertheless, the advantage of multiple observations was clearly perceived, and provision for their being made was included in the specification of all later instruments. Also, the system of numbering the centimetres on the stem was altered. In the *Challenger* instrument the number 10 marks the lowest division on the stem, and 0 marks the highest. In all later instruments the lowest division is 0, and the centimetres are numbered 1, 2, 3, . . . 10 upwards.

In every determination of the specific gravity of a sample of water, the weight of the volume of it which was displaced by the hydrometer floating in it at an observed division on the scale was represented by the sum of the weights of the hydrometer, the table, and the accessory weight used.

The volume of the water so displaced by the hydrometer was arrived at as the result of an extensive series of observations made with it in distilled water at different temperatures.

The relation between the weight and the volume of a mass of distilled water at all ordinary temperatures, as determined by KOPP, was accepted as correct, and was used in reducing the observations made with the hydrometer in distilled water so as to arrive at the volume of its body, that is, the whole of the hydrometer below the lowest division on the stem at all ordinary temperatures. Its rate of thermal dilatability was taken to be constant within the limits of temperature considered, and its probable value was obtained by taking the mean of all those observed.

The final result was stated by giving the volume of the body of the instrument up to the lowest division on the stem at 0° C. as V, and the rate of its dilatability, dV/dt, as e.

Thus the full specification of the hydrometer, that is, the glass instrument alone, is furnished by four data.

For the hydrometer used in the *Challenger* they are :—

Weight *in vacuo* of the hydrometer . .	W	160·2128	grams.
Volume of body of hydrometer up to lowest division on stem at 0° C. . .	V	160·277	c.c.
Rate of expansion of body per ° C. . .	e	0·00455	c.c.
Total volume of divided stem (100 mm.) .	v	0·8650	c.c.

The specification of the set of accessory weights which were used with this hydrometer is as follows :—

No. . . .	0.	I.	II.	III.	IV.	V.	VI.
Weight in grams .	0·8360	0·8560	1·6010	2·4225	3·2145	4·0710	4·8245

Weight No. 0 is the small brass table which can be affixed to the top of the stem, and on it any further weight that might be required was placed. The distinctive number of the hydrometer which was used for all the determinations made in the *Challenger* was 0. In the tabulated results the combination used is indicated in the column headed "Number of Hydrometer." Thus, 00V means that Hydrometer No. 0, table No. 0, and weight No. V. were used. The combinations almost exclusively used were 00IV and 00V, which weighed 164·2633 and 165·1198 grams respectively.

In this memoir we make no use of the *volume* of the hydrometer, because in all the experiments the temperature is a constant, and we obtain directly the *displacement*, that is, the weight of distilled water displaced by the same volume of saline solution, both being at the same temperature, from which we obtain directly the specific gravity of the solution at that temperature, referred to that of distilled water at the same temperature as unity. This result is arrived at from the two observations alone, and is independent of the work of others.

As it was certain that during the voyage of the *Challenger* the specific gravity of the sea-water would have to be observed at many different temperatures, it was convenient, after having determined the displacement of the hydrometer in distilled water at different temperatures, to express the result in terms of the volume of the displacing hydrometer in standard cubic centimetres, but the difference from the later practice is only in the form of expression.

§ 7. In order to obtain all the precision of which the hydrometric method is capable, the temperature of the water must remain perfectly constant while the hydrometer is floating in it, and the temperature of the hydrometer must be the same as that of the water before it is immersed in it. In ordinary work on shore and in our latitudes this is the condition which it is most difficult to realise. In the *Challenger* it provided itself. Nearly three out of the three and a half years that the voyage lasted were spent between latitudes 40° N. and 40° S. Here the temperature of the air is relatively high, but its diurnal variation is very slight. Moreover, the *Challenger* was a wooden ship, and the laboratory was lighted and ventilated by a large main-deck gun-port, the result of which was that, especially in the tropics, the temperature of the air was almost constant, day and night.

The temperature of the surface water was usually slightly higher than that of the air, but only by a fraction of a degree, so that its specific gravity could be determined immediately after collection. Samples of water brought up from the bottom and the inferior depths arrived on board having a temperature much lower than that of the air, and it was impossible, even if it had been convenient, to proceed at once to the determinations of their specific gravity. A case containing eight large stoppered bottles was kept in the laboratory for the purpose of receiving these samples as they arrived, and they were kept in the laboratory until the next day, and their specific gravities were then determined one after the other. The twenty-four hours' sojourn in the laboratory equalised their temperature and brought it to agree sensibly with that of the air of the laboratory and that of the hydrometer, which was always kept in the laboratory. By working according to this system, the specific gravities of the waters obtained from different depths at the same station were determined at the same time and at the same temperature, and their relative specific gravities at a common temperature were thus given directly by experiment. This is an important advantage, and it is often overlooked.

A subjective precaution, but one of great importance for assuring accuracy of observation, was adopted at the beginning of the voyage and was never departed from. Before beginning to make hydrometric observations on the samples of water, or to carry out any other operation, such as the boiling out of the gases or the determination of the carbonic acid, *I locked the door of the laboratory, and it was not unlocked until the operation was finished.* Consequently none of my colleagues, or anyone else in the ship, ever witnessed the determination of the specific gravity of the water, or any other of the operations carried on in the laboratory, at any time from the beginning to the end of the voyage. I found that exactness of observation was promoted by freedom from disturbance.

TABLE VIII.*

Giving Duplicate Observations of the same Sample of Water with the same Hydrometer differently weighted.

No. of Sample.	Density observed with		Difference. 00IV – 00V.	No. of Sample.	Density observed with		Difference. 00IV – 00V.
	00IV.	00V.			00IV.	00V.	
120	1·02412	1·02411	+1	274	1·02416	1·02412	+4
127	1·02414	1·02409	+5	826	1·02411	1·02411	0
135	1·02406	1·02413	–7	829	1·02411	1·02408	+3
139	1·02407	1·02414	–7	830	1·02400	1·02405	–5
181	1·02428	1·02427	+1	831	1·02421	1·02418	+3

After the *Challenger* Expedition I used in all my deep-sea work hydrometers with sets of weights designed for making multiple observations in each water. The principal stepping-stone between the *Challenger* hydrometer and that used in the investigations of this memoir was one in which the observations were made in triplicate, the difference between successive added weights being 0·25 or 0·3 gram. One reading was made near the middle of the stem, and the other two were made near the middle of the lower and the upper halves of the stem respectively.

A very complete and important set of observations on this scheme was made in 1885, on a voyage from Southampton to Buenos Ayres, and then from Valparaiso following the west coast of South America to Panama, and thence along the west coast

* "Report on the Specific Gravity of Ocean Water," *Physics and Chemistry*, vol. i., Part II., Table VIII.

of North America to San Francisco. This pattern of hydrometer was also used a good deal in cable ships.

In all work at sea three observations, or perhaps four, are quite sufficient. When four observations are made, their arithmetical mean has theoretically two-thirds of the value of the arithmetical mean of a series of nine.

§ 8. As much misunderstanding seemed to exist not only regarding the qualifications required for the successful practice of the hydrometric method, but also, to some extent, of the principles on which the legitimacy of the method depends, I took occasion at the meeting of the Sixth International Geographical Congress, held in London in 1895, at which I read a paper entitled "A Retrospect of Oceanography during the last Twenty Years," to deal with some of these misapprehensions. The following passage may be quoted here with advantage :—

"Many writers, in passing judgment on the hydrometer as an instrument for the determination of the density of liquids, have only in their minds the hydrometer whose indications are determined by comparison with another or standard instrument; or by immersion in solutions the densities of which have been otherwise ascertained. These instruments have no greater value than that of more or less carefully constructed copies of a standard, the method and the principle of the construction of which is not always given. Rightly, therefore, they prefer the density as determined by weighing a vessel filled with the liquid and comparing it with the weight of distilled water of the same temperature filling the same vessel. The hydrometer which I constructed for the *Challenger* Expedition, and used during the whole of it, is not a hydrometer in the above sense: it does not give comparative results; it gives absolute ones. By its means, the weights of equal volumes of the solution and of the distilled water of the same temperature are determined directly. It is neither more nor less than a pyknometer, where the volume of liquid *excluded* up to a certain mark is weighed, instead of that *included* up to a similar mark. In the pyknometer, the internal surface per unit of length of the stem can be made smaller than the external surface per unit of length of stem of the hydrometer. On the other hand, the volume of the hydrometer can safely be made many times larger than that of the pyknometer, the dimensions of which must always be kept small on account of the difficulty of ascertaining the true temperature of its contents, which must be guessed, because it cannot be measured directly. The temperature of another mass of liquid is measured, and the two are assumed to be identical. With the hydrometer, the liquid being in large quantity and outside of the instrument, its temperature can be immediately ascertained with every required accuracy.

"Again, for every determination with the ordinary pyknometer, the weight of the liquid contained in it has to be determined by a separate operation of weighing. With the hydrometer, the weight of the liquid displaced, being always equal to its own, is determined once for all by repeated series of weighings, where every refinement is used to secure the true weight of the instrument. This weight can be increased at will by placing suitable small weights on the upper extremity of the stem. Their weight is also most carefully determined once for all, so that at any moment the total weight of the displacing instrument is accurately known." *

Section II.—The Principle and Construction of the Closed Hydrometer.

§ 9. It will be convenient to follow in detail the preparation of the hydrometer for use.

The instrument being closed, its true weight is constant.

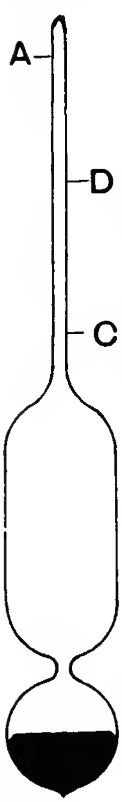


Fig. 2.

Let it be assumed that our experiments are actually made *in vacuo*, at the sea-level in lat. 45°. In these conditions the standard gram exerts a vertical pressure of 1 gram (true).

We weigh the hydrometer and find its weight to be W grams. We now float it in distilled water contained in a suitable cylinder. In the construction of the hydrometer the internal load has been so adjusted that, when immersed in distilled water of the standard temperature T, which is to remain unaltered during the whole of the experiments, the surface of the water shall cut the stem in some line C, near its junction with the body of the instrument. Then the weight of the water displaced by the hydrometer is exactly W grams.

Let pressure be now applied to the top of the stem, A, until it is completely immersed. Let the measure of this pressure be w grams. Then the weight of water displaced by the instrument when totally immersed at temperature T is $(W + w)$ grams.

We assume that the stem is a uniform cylinder of circular section and terminated by a plane surface. If we apply pressure so as to immerse the stem to the line D, which is midway between A and C, the pressure required will be $\frac{w}{2}$ grams; and, if the portion of stem so immersed, CD, stands in any other ratio to the total length CA, the pressure required to produce the immersion will stand in the same ratio to w.

Let the experiments be made in air of temperature T, and of pressure and humidity such that 1 cubic centimetre of it weighs 1·2 milligram. When the experiment was made in the vacuum and the surface of the water cut the stem in C, the weight of the water so displaced was exactly equal to that of the hydrometer, namely, W grams.

After air has been admitted, the surface of the water no longer cuts the stem exactly in C, but at a point a little lower. This difference between the lines of flotation is due to the fact that, while experimenting *in vacuo*, the portion of the stem which is not immersed in the water is immersed in a medium the density of which is insensible, whereas, after the air has been admitted, it is immersed in a medium of which 1 cubic centimetre weighs 1·2 milligram, and this exerts an upward pressure, in opposition to

* *Report of the Sixth International Geographical Congress, held in London*, 1895, p. 412.

gravity, at the rate of 1·2 milligram per cubic centimetre of stem so immersed in air. This upward pressure lifts the hydrometer until it displaces a weight of water less than it did *in vacuo* by the weight of air which the exposed stem displaces after air has been admitted.

Therefore, in ordinary laboratory practice, when the hydrometer floats in the liquid at any line C on the stem, the true weight of the liquid so displaced is equal to the true weight *in vacuo* of the hydrometer *less* the weight of the air displaced by the exposed portion of the stem.

If s be the weight of the air displaced by the exposed portion of the stem, and W, as before, be the weight *in vacuo* of the instrument, the effective vertical pressure exercised by the hydrometer when floating in equilibrium on the water is

$$H = (W - s),$$

and this is the measure of its displacement in distilled water of temperature T under existing atmospheric conditions.

§ 10. In instruments of the pattern, fig. 3, § 80, which I construct for use in dilute saline solutions I aim at a displacement of 180 grams distilled water. The stem is made from tubing selected with the greatest care so as to secure uniformity of calibre. Its total length is about 130 millimetres, and its external diameter is such that a length of 10 centimetres displaces something less than a cubic centimetre. This condition is satisfied if the glass-blower selects a suitable piece of tube having an external diameter of 3 to 3·5 millimetres by the callipers. If the diameter of the tube is exactly 3·56825 millimetres and its section circular, 10 centimetres of it will displace at 4° C. 1 cubic centimetre. The graduated portion of the stem occupies a length of 10 centimetres, which is divided into millimetres numbered at every centimetre from below upwards: 0, 1, 2, . . . 10. The zero is about 1 centimetre above the junction of the stem with the body, and the highest division, numbered 10, is found at a distance of about 2 centimetres below the top of the stem. The total length of the instrument should not exceed 33 centimetres.

If the hydrometer floats in distilled water of temperature T so that the surface of the water cuts the stem at 5 millimetres above the zero of the scale (I express this shortly by saying, the hydrometer floats at 5), and the weight of air so displaced by the exposed stem is s_5, then the true weight of water so displaced is

$$H_5 = W - s_5.$$

The other conditions remaining the same, let the distilled water in the cylinder be replaced by a saline solution at temperature T. Let the hydrometer be floated in it; the surface of the liquid will cut the stem or the body of the instrument at a lower level than the 5th millimetre on the scale. In order to immerse the instrument exactly to the 5th millimetre, we have to place a certain weight on the top of the stem. Let its weight *in vacuo* be w_5 grams. Then the weight of liquid displaced by the system is

$$H'_5 = W - s_5 + w_5 - dw_5,$$

where dw_5 is the weight of the air displaced by the small added weight w_5.

We have then two independent observations, namely, those of the weights of the distilled water and of the saline solution respectively, which occupy the same volume under identical conditions. The ratio of these two weights is the specific gravity of the heavier liquid referred to that of the lighter at the same temperature as unity. It is :—

$$S_5 = \frac{H'_5}{H_5} = \frac{W - s_5 + w_5 - dw_5}{W - s_5}.$$

Let us now repeat the double experiment, all the conditions remaining the same, except that, when the hydrometer has been immersed in the distilled water and floats at 5, a small weight v_{15} is added which immerses the hydrometer until it floats exactly at 15. Let s_{15} be the weight of air displaced by the exposed stem above the 15th division, and let dv_{15} be the weight of air displaced by the small weight v_{15}. Then the weight *in vacuo* of the distilled water displaced by the hydrometer below line 15 is

$$H_{15} = W - s_{15} + v_{15} - dv_{15}.$$

Let the hydrometer be now immersed in the heavier liquid, and let weights be placed on the top of the stem until it floats exactly at 15. As before, the weight of this liquid so displaced is

$$H'_{15} = W - s_{15} + w_{15} - dw_{15}$$

and the specific gravity of the liquid must be

$$S_{15} = \frac{H'_{15}}{H_{15}} = \frac{W - s_{15} + w_{15} - dw_{15}}{W - s_{15} + v_{15} - dv_{15}}.$$

Now H_5 and H'_5 are the weights of equal volumes of distilled water and of a heavier liquid respectively, and H_{15} and H'_{15} are also weights of equal volumes of distilled water and of the same heavier liquid respectively: therefore in the two ratios $\frac{H'_5}{H_5}$ and $\frac{H'_{15}}{H_{15}}$ we have two independent values of the specific gravity of the heavier liquid under identical conditions, namely,

$$S_5 = \frac{H'_5}{H_5} \text{ and } S_{15} = \frac{H'_{15}}{H_{15}}$$

As the specific gravity of each liquid has remained the same, these two independent determinations ought to give identical values for S: that is, $S_5 = S_{15}$.

It is evident that we can increase at will the number of independent determinations of the specific gravity of the heavier liquid as referred to that of distilled water under constant conditions, and obtain from them a mean value of continually increasing exactness.

It will be observed that the values of the specific gravity so obtained depend on our own observations alone. We have therefore the means of appraising their value exactly. Moreover, their value depends almost exclusively on determinations of weight: and this is the physical constant of a body which can be directly determined with perhaps greater precision than any other.

§ 11. *Preparation of Accessory Weights.*—We have now to consider the preparation or manufacture of the small weights to be placed on the top of the stem in order to produce small increments of the immersion of the hydrometer. They are made of wire. This is wound into spiral cones for the heavier and into rings for the lighter weights. The lighter weights are made of aluminium and the heavier ones of brass.

Generally a set of weights consists of aluminium spirals weighing 0·2, 0·5, and 1·0 gram, and rings of the same metal weighing 0·2 and 0·1 gram, also rings 0·05 and 0·02 gram. The brass weights are rings of 0·5 and 1·0 gram and spirals of 1, 3, 5, and 7 grams. At every operation I aim at making a series of nine independent observations of the displacement. In the first observation the lightest added weight is used and the reading (R_1) is near the zero of the scale. The succeeding observations are made while the added weight is increased by 0·1 gram between each observation of the series. The observations thus obtained are spread over the whole of the scale on the stem.

The weights may be made so that their nominal weight is their true weight *in vacuo*, but, as they are always used in air, it is preferable to adjust them by balancing them against standard weights in air. The standard weights exert their nominal vertical pressure only *in vacuo*, at the sea-level in latitude 45°; but we have assumed that we are in fact working at the sea-level in latitude 45°, therefore the nominal pressure of the standard weight is affected only by the density of the medium in which it is immersed. When we are actually working in a vacuum the density of the medium is insensible; when we are working in air its density is ascertained by observation. Our standard weights, which have been verified at Kew, are made of brass (gilt) for weights of 1 gram and upwards, and of platinum for weights under 1 gram. The weights destined for use on the stem of the hydrometer are also made of brass for those of 1 gram and upwards, and for those of 1 gram and under they are made of aluminium. There are gram weights and half-gram weights of both brass and aluminium.

We will consider (*a*) the preparation of a gram weight of brass as balanced against a standard gram of brass; and (*b*) the preparation of a gram weight of aluminium as balanced against a standard gram of platinum.

(*a*) As we are dealing with only one kind of material, it is sufficient to equilibrate our weight of brass wire against the brass standard gram in air of known density to obtain a weight which *in vacuo* exerts a vertical pressure equal to that of the standard gram, and it must exert the same vertical pressure as does the standard gram in air of the same density. Taking the specific gravity of brass wire at 8·38, 1 gram of it displaces 0·119 cubic centimetre of air, which, at 1·2 milligram per cubic centimetre, weighs 0·1428 milligram. Therefore, when reckoning the effective pressure exerted by the brass weights placed on the top of the hydrometer in air of the density above specified, we make a deduction from their nominal weight *in vacuo* in the proportion of 0·1428 milligram per gram used.

(*b*) Let us now consider the preparation of a weight of 1 gram in aluminium for the hydrometer, against a standard gram weight in platinum. We take the specific gravity of aluminium at 2·5 and that of platinum at 21.

The volume of a gram of platinum is therefore 1/21 cubic centimetre, and it displaces this volume of air, which weighs 0·057 milligram. Therefore the standard platinum gram weighs in air 0·999943 gram or 0·057 milligram less than *in vacuo*. If we are working actually in the vacuum and we equilibrate the platinum gram with a mass of aluminium, both masses exert the same vertical pressure. But when we admit the air the platinum gram loses only 0·057 milligram of apparent weight, whereas the aluminium gram loses $\frac{1 \cdot 2}{2 \cdot 5} = 0 \cdot 48$ milligram of weight, and its vertical pressure in air is only 0·99952 gram.

A more useful result is obtained by equilibrating the platinum and aluminium in air.

Let the standard gram of platinum be placed on the one pan of the balance, and let a mass of aluminium which *in vacuo* weighs 1 standard gram be placed on the other pan. The two masses which, *in vacuo*, would exactly balance each other, now appear to have different weights. By immersion in the air the platinum gram has lost 0·057 milligram and the aluminium gram has lost 0·48 milligram. Let aluminium be added to the aluminium weight until the balance shows equilibrium. The amount so added weighs in air 0·423 milligram. The vertical pressures exerted in the air by the masses of platinum and aluminium respectively are then equal. But this pressure is still short of the standard pressure of 1 gram by 0·057 milligram. Let this weight of aluminium be added to the mass of aluminium already on the pan. When this addition has been made, the total mass of aluminium will exert in air, weighing 1·2 milligram per cubic centimetre, a vertical pressure of 1 gram true. No account has been taken of the buoyancy of the last two additions to the mass of aluminium, because its effect is insensible on our balance.

In practice the aluminium weights used in any experiment never exceed 1 gram by more than one or two tenths; therefore, if they have been simply balanced against the corresponding platinum weights in air, the deduction for buoyancy is insensible; and we have seen that, if the brass weights have been prepared against brass standards in air, the deduction for buoyancy is at the rate of 0·14 milligram per gram when 1 cubic centimetre of air weighs 1·2 milligram per cubic centimetre.

§ 12. *Exposed Stem.*—Let us consider the effect on the resulting value of the specific gravity of a liquid when the correction for the buoyancy of the exposed stem is applied or is neglected. The effect of buoyancy will evidently be the greater, the greater the length of the exposed stem. Let us take the case of the hydrometer suitably loaded, floating at 0 mm., or the lowest division on the stem, both in the distilled water and in the solution. The volume of the exposed stem is 1·25 cubic centimetres in both cases, and the air which it displaces, at 1·2 milligram per cubic centimetre, weighs 1·5 milligram.

In order to avoid complication, we suppose that the necessary "added weights" have been added to the *internal* load of the closed hydrometer, and that the displacing weight of the hydrometer quoted for each immersion is its true weight *in vacuo*, and that nothing which can affect the immersion of the instrument in the liquid is immersed in air excepting the exposed portion of the stem itself. This disengages the effect produced by the buoyancy of the stem from that of every other cause.

Let the weight of the hydrometer so floating at 0 mm. in distilled water be 180·25 grams; and let its weight when floating also at 0 mm. in the solution be 185·25 grams; then, neglecting the buoyancy of the stem, the specific gravity of the solution is 1·027739. But the effect of buoyancy is to reduce the effective weight in both cases by 1·5 milligram, so that the specific gravity of the solution corrected for buoyancy of stem is $\frac{185 \cdot 2485}{180 \cdot 2485} = 1 \cdot 027740$. Therefore, when the whole of the stem is exposed, its buoyancy affects the resulting specific gravity to the extent of only a unit in the sixth decimal place.

§ 13. *Determination of the Weight of the Hydrometer.*—For this purpose the hydrometer is placed on the right-hand pan of the balance in an upright position, and is brought to equilibrium with weights and rider on the left-hand pan. The hydrometer is then removed and equilibrium again established by means of standard weights. These are then replaced by the hydrometer and equilibrium re-established by shifting the rider of the counterpoise if it has been disturbed. The hydrometer is again removed and replaced by standard weights until equilibrium is established. In this way four independent weighings by replacement by standard weights are obtained. The temperature of the air is noted, also the temperature of the wet-bulb thermometer and the height of the barometer. Three such series of weighings are made on different days when the meteorological conditions are different. Each series is treated by itself. In order to obtain the vacuum correction we require to know the weight of the air displaced by the hydrometer and by the weights respectively. The difference of these two weights, the net buoyancy, is the correction to be added to the apparent weight of the hydrometer.

We take as an example of the method the determination of the weight *in vacuo* of hydrometer No. 17.

1st Determination.

5th March 1894.

Barometer = 740·86 mm.

Temperature, dry bulb = 6·15°, wet bulb = 5·1° C.

Whence the vapour tension is 6·03 mm., and the weight of 1 litre of this air is 1·2288 gram.

Four weighings of the hydrometer by replacement with standard weights were made in air. The weights found were :—

	180·7141	grams.
	180·7137	,,
	180·7136	,,
	180·7137	,,
Mean =	180·7138	,,

As the hydrometer floats without added weights in distilled water with only a part of the stem exposed, we take its weight in air, 180·7138 grams, as expressing, to first approximation, its volume in cubic centimetres. The correction for *net buoyancy*, that is, the difference between the weight of air displaced by the hydrometer and that displaced by the weights, is (taking dry air at 6·15° C. and 741 mm.) 0·1961 gram, whence the first approximation to the weight *in vacuo* of the instrument is 180·7138 + 0·1961 = 180·9099 grams.

It was found that by the addition of 1·698 gram to the weight of the hydrometer it floated totally immersed in distilled water at 6·15° C. ; that is, if the weight were diminished ever so little the top of the stem became exposed, and if it were increased ever so little the instrument began to sink to the bottom.

Taking now the first approximation to the weight *in vacuo*, 180·9099 grams, and adding 1·6980 gram, we have the sum 182·6079 grams. This is the first approximation to the weight *in vacuo* of the mass of distilled water which is displaced by the whole hydrometer at a temperature of 6·15° C. Taking the volume of 1 kilogram of water at 6·15° C. to be 1000·034 cubic centimetres, we find the volume of the hydrometer at 6·15° C. to be 182·6139 c.c. ; and this is the volume of air which it displaces at 6·15° C. We have found that 1 litre of the air in the balance-room at the time weighed 1·2288 gram. Therefore the exact weight of the air displaced by the hydrometer when being weighed was 182·6139 × 0·0012288 = 0·22439 gram, and taking the weights as consisting of brass of the density 8·38, we find the weight of air displaced by them to be 0·02650 gram, whence the net buoyancy = 0·19789 gram, and the true weight *in vacuo* of the hydrometer is 180·7138 + 0·1979 = 180·9117 grams.

The weight *in vacuo* was determined on two other days, namely 24th April and 2nd June 1894 ; on each of these days four determinations were made of the weight in air, which on the first of these days weighed 1·2081 gram per litre, and on the second 1·2037 gram per litre. The weights *in vacuo* deduced from these observations were 180·9109 and 180·9113 grams respectively. The mean of the three determinations is 180·9113 grams, which is accepted as the final value of the weight *in vacuo* of hydrometer No. 17. The weights of the other hydrometers, Nos. 21 and 3, were determined in the same way ; the particulars are collected in the following table :—

Determination No.	Date.	Mean Weight of Hydrometer in Air. Grams.	Number of Weighings in Air.	Barometer.	Thermometer. Dry Bulb.	Thermometer. Wet Bulb.	Vapour Pressure. Millimetres.	Weight of 1 litre of this Air. Grams.	Exact net Buoyancy. Grams.	Exact Weight *in vacuo* of Hydrometer. Grams.
						Hydrometer No. 17.				
1	5/3/1894	180·7138	4	740·86	6°·15	5°·10	6·03	1·2288	0·1979	180·9117
2	24/4/1894	180·7165	4	740·46	10·50	8·90	7·70	1·2081	0·1944	180·9109
3	2/6/1894	180·7175	4	747·01	13·90	11·50	8·89	1·2037	0·1938	180·9113
									Mean =	180·9113
						Hydrometer No. 21.				
1	24/11/1893	187·5771	4	749·05	8°·25	6°·70	6·56	1·2327	0·2060	187·7831
2	5/12/1893	187·5764	4	755·40	9·85	8·00	7·07	1·2361	0·2066	187·7830
3	27/ 4/1894	187·5809	4	743·65	13·40	11·05	8·65	1·2006	0·2006	187·7815
									Mean =	187·7825
						Hydrometer No. 3.				
1	9/3/1894	178·1785	4	731·36	9°·45	7°·70	6·98	1·1982	0·1904	178·3689
2	10/5/1894	178·1786	4	740·20	14·10	10·75	7·97	1·1924	0·1895	178·3681
3	2/6/1894	178·1776	4	746·89	13·95	11·50	8·87	1·2032	0·1912	178·3688
									Mean =	178·3686

Final weight *in vacuo* accepted for each hydrometer, No. 17. 180·9113 grams. No. 21. 187·7825 grams. No. 3. 178·3686 grams.

§ 14. As the displacement in distilled water figures in all the determinations of density, we begin by making a number of series of observations of the displacement of the hydrometer in it at the standard temperature chosen.

We take as an example the case of hydrometer No. 17 in distilled water of 15·00° C. as it is given in Table A_1.

Table A_1.—This table gives in detail the data from which is derived the total weight displaced by hydrometer No. 17 when floating at the 50-mm. mark in distilled water at 15·00° C. Each line in the table is distinguished by a letter—*a*, *b*, *c*, etc. In line *a* we have the number and particulars of the hydrometer used. In line *b* is given a reference to the laboratory note-book in which the original observations were entered, followed by the date of the experiment, line *c*. Lines *d* and *i* give the times at which the hydrometer is immersed and removed, while the temperatures of the water at these times are given in lines *e* and *j*, with their mean, $\bar{T}$, in line *k*; the range of temperature over which the series of observations was carried out is shown in line *l*. Although it is intended that the temperature shall be uniform during an experiment, it sometimes happens that it varies, and this has to be provided for in the table. Line f_0 gives the headings *w*, the weights used to sink the hydrometer in the liquid, and R, the reading on the scale of the instrument corresponding to these weights.

Lines f_1 to f_9 give, under *w*, the values of the weights, and under R, the corresponding scale readings, obtained during the series of observations. The value of the mean added weight, $\bar{w}$, is given in line *g*, while the mean reading, $\bar{R}$, is shown in line *h*.

The departure of the mean reading from 50 mm., $50-\bar{R}$, is entered in line *m*; it is given in the headings as $d\bar{r}$. In line *n* we have the weight which is equivalent to $d\bar{r}$, expressed as dw_r (see § 15).

The weight required to immerse the hydrometer to the 50-mm. mark, $w+dw_r$, irrespective of temperature corrections, is shown in line *o*, being that weight which would cause the instrument to float with the scale division at 50 mm. in the plane of the surface of the liquid, at the mean temperature, $\bar{T}$. The difference of the mean temperature, $\bar{T}$, from the standard temperature, T, is given in the line *p*, and is expressed as $\bar{T}-T=d\bar{t}$; the weight corresponding to the difference $d\bar{t}$ is entered in line *q*; it is expressed as dw_t (see § 16).

The total corrected added weight required to immerse the hydrometer to the 50-mm. mark at the standard temperature, T, is $\bar{w}+dw_r+dw_t$, and is given in line *r*.

The total weight of liquid displaced by the hydrometer when floating in the liquid at the 50-mm. mark at the standard temperature, T, is entered in line *s*, and is equal to the weight of the instrument *in vacuo* plus $\bar{w}+dw_r+dw_t$.

Having explained the meaning of the lines, we will proceed to inspect the results of the observations in Table A_1.

After preliminary trial, the first weight added to the hydrometer at the commencement of a series of observations is chosen so that the *mean* of the nine series of immersions or scale readings produced by successive added weights, each increasing by 0·1 gram, shall approximate closely to 50 mm. It is evident that the initial, or first, added weight might be different for each series of observations. In the ten series of observations detailed in Table A_1, however, the first added weight in each case was 0·525 gram, and therefore the nine added weights are given only once, under *w*, in lines f_1 to f_9 of the first column. Each of the ten succeeding columns contains a complete series of observations of the immersions produced by the nine added weights, and the steps in the calculation of the total weight of the hydrometer when floating at the 50-mm. mark at the standard temperature.

§ 15. *Correction for Departure of the Mean Reading from* 50 *mm.*—If the mean reading, $\bar{R}$, be exactly 50 mm., then the weight which must be added to the hydrometer to immerse it to the 50-mm. mark is the mean added weight, $\bar{w}$.

If, however, the mean reading, $\bar{R}$, be less or greater than 50, the mean added weight must be increased or diminished by the weight, dw_r, which would increase or diminish the immersion by the difference $d\bar{r}$ between the observed mean reading, $\bar{R}$, and 50. The calculation of this correction, dw_r, is best explained by taking the series of observations, XVII. 73, Table A_1, as an example. In this case the mean reading, $\bar{R}$, is 50·72, and $d\bar{r} = 50 - 50{\cdot}72 = -0{\cdot}72$ (line *m*).

The immersion in the whole series of observations is increased by 89·2 mm. of the stem by an addition of 0·800 gram. Hence an increase of 1 mm. in the stem immersion is caused by the addition of $\frac{0{\cdot}800}{89{\cdot}2} = 0{\cdot}00897$ gram, and a difference of 0·72 mm. in the immersion must be produced by the weight $0{\cdot}00897 \times -0{\cdot}72 = -0{\cdot}0064$ gram, which is the required correction, dw_r (line *n*).

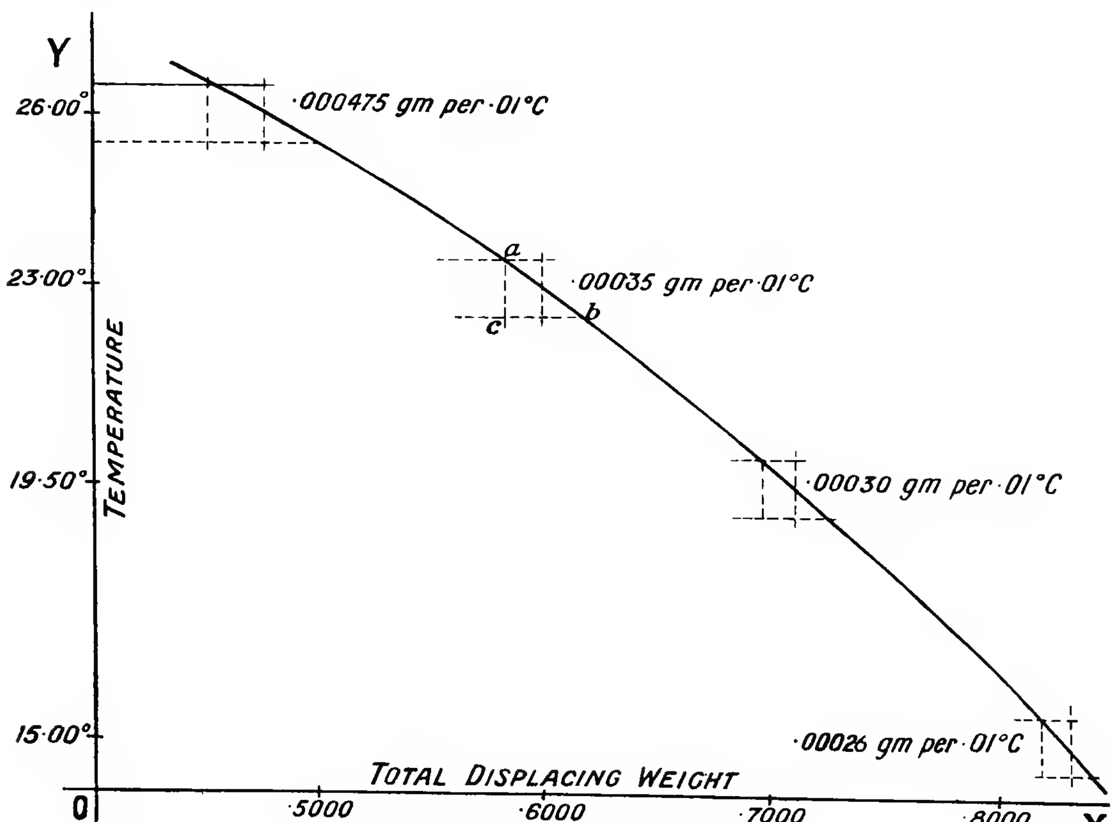
As the mean reading, $\bar{R}$, is in this case greater than 50, the weight required to immerse the hydrometer to 50 mm. must be less than the mean added weight, $\bar{w}$, by the amount of the correction, dw_r, and is, therefore, $0{\cdot}925 - 0{\cdot}0064 = 0{\cdot}9186$ gram (line *o*). If the mean reading, $\bar{R}$, were less than 50 mm., the correction, dw_r, would be calculated in the same manner, but would require to be *added* to the mean added weight.

§ 16. *Correction for Temperature.*—The weight required to be added to the hydrometer to cause it to float at the 50-mm. mark in distilled water of the mean observed temperature of 15·01° C., as found above, is 0·9186 gram. A correction (dw_t, line *q*) must now be applied to reduce the displacement observed at the mean temperature, $\bar{T}$, to the standard temperature, T, which is in this case 15·00° C. Before this can be done we must determine the value of dw_t for 0·01° C. at 15·00° C. This is found as follows. A series of observations is made with the hydrometer in distilled water at various temperatures, the results of which are expressed in a curve, having displacements as abscissæ and temperatures as ordinates. Suppose we wish to find the temperature correction at, say, 23·00° C., we proceed as follows. Draw horizontal lines through T = 23·5° and T = 22·5° C., cutting the curve at *a* and *b* respectively. From *a* drop a perpendicular on *cb*, meeting it at *c*. Then the length *ac* represents 1° C., while *cb* is the difference in the total displacement for this 1° difference in the temperature. Knowing the value of the abscissa OX in grams per unit length, say grams per millimetre, we measure accurately the length *cb* and multiply it by this constant. This gives us the value of dw_t for 1° difference of temperature at 23° C. The value of dw_t per 0·01° C. is simply the former figure divided by 100. This process is repeated at each of the temperatures at which observations are being made.

The value of dw_t in grams per 0·01° C. at 15·00° C. has been taken as 0·00026.

The temperature at the commencement of the observations in our example was 15·00°, and at the end 15·02°, the mean being 15·01°.

The departure of the mean temperature from 15·00° is, $\bar{T} - T = d\bar{t} = 15{\cdot}01 - 15{\cdot}00 = 0{\cdot}01°$ (line *p*). Therefore the amount by which the added weight must be increased for the difference dt is $dw_t = 0{\cdot}00026 \times 1 = 0{\cdot}00026$ gram (line *q*). The mean temperature observed during the time the observations were being made was higher than the standard, so that in this case we must add the correction for temperature to the added weight required to immerse the stem to 50 mm. at 15·00° C.



TEMPERATURE CURVE FOR HYDROMETER No. 17.

Finally, by adding together the mean added weight, $\bar{w}$, the weight, dw_r, for the difference of the mean reading, $\bar{R}$, from 50 mm., and the weight, dw_t, for the difference of the mean observed temperature, $\bar{T}$, from the standard temperature, T, we obtain the weight $\bar{w} + dw_r + dw_t = 0{\cdot}925 + (-0{\cdot}0064) + 0{\cdot}00026 = 0{\cdot}91886$ gram given in line *r*, which must be added to the hydrometer to immerse it to the 50-mm. mark in distilled water of 15·00° C., and by adding this weight to that of the hydrometer *in vacuo* we obtain $180{\cdot}9113 + 0{\cdot}91886 = 181{\cdot}83016$ grams (line *s*), which is the total weight of water displaced by the hydrometer under these conditions.

§ 17. The data for the determination of the total weights of hydrometer No. 17

when floating at the 50-mm. mark in distilled water of 19·50°, 23·00°, and 26·00° C. are given in Tables A_2, A_3, and A_4 respectively.

The mean of the values entered in line *s* of each of the Tables A_1, A_2, A_3, and A_4 is accepted as the correct total weight of hydrometer No. 17 when floating at the 50-mm. mark in distilled water at the temperatures 15·0°, 19·5°, 23·0°, and 26·0° C. respectively; they are collected in Table B with the corresponding values for hydrometers Nos. 21 and 3.

[TABLES.

TABLE

Hydrometer

Details of Determination of the Total Weight of the Hydrometer

Particulars of hydrometer,	*a*	Hydrometer No. 17. J. Y. B., 1893.		
Reference to laboratory note-book,	*b*	...	II. 161	II. 165
			1903	1903
Date of experiment,	*c*	...	Feb. 28	Feb. 28
Time at start,	*d*	...	10.58 a.m.	11.30 a.m.
Temperature at start,	*e*	...	15·00°	15·02°
Added weight, *w*, and reading, R,	f_0	*w*	R	R
First added weight or reading,	f_1	0·525	5·8	6·0
Second ,, ,,	f_2	0·625	17·0	17·1
Third ,, ,,	f_3	0·725	27·8	28·2
Fourth ,, ,,	f_4	0·825	39·1	39·3
Fifth ,, ,,	f_5	0·925	50·2	50·3
Sixth ,, ,,	f_6	1·025	61·3	61·3
Seventh ,, ,,	f_7	1·125	72·5	72·0
Eighth ,, ,,	f_8	1·225	84·8	84·0
Ninth ,, ,,	f_9	1·325	95·0	95·2
Mean added weight, $\bar{w}$,	*g*	0·925		
Mean reading, R,	*h*	...	50·39	50·37
Time at finish,	*i*	...	11.18 a.m.	11.45 a.m.
Final temperature,	*j*	...	15·00°	14·98°
Mean temperature, $\bar{T}$,	*k*	...	15·00°	15·00°
Range of temperature,	*l*	...	0·00	0·02°
Difference of mean reading from 50 mm. $(50-\bar{R}=d\bar{r})$,	*m*	...	−0·39	−0·37
Weight equivalent to displacement, $d\bar{r}$ $(=dw_r)$,	*n*	...	−0·0035	−0·0033
Weight required to immerse hydrometer to 50-mm. mark at mean temperature, $\bar{T}$, $(\bar{w}+dw_r)$,	*o*	...	0·9215	0·9217
Difference of mean temperature, $\bar{T}$, from standard temperature $(\bar{T}-T=d\bar{t})$,	*p*	...	...	...
Correction for difference, $d\bar{t}$ $(=dw_t)$,	*q*	...	...	...
Weight required to immerse hydrometer to 50-mm. mark at standard temperature, T, $(\bar{w}+dw_r+dw_t)$,	*r*	...	0·9215	0·9217
Total weight displaced by hydrometer when floating at 50-mm. mark at standard temperature, T,	*s*	...	181·8328	181·8330

TABLE

Hydrometer

Details of Determination of the Total Weight of the Hydrometer

Particulars of hydrometer,	*a*	Hydrometer No. 17. J.Y.B., 1893.					
Reference to laboratory note-book,	*b*	...	IV. 137	IV. 143	IV. 149	IV. 155	IV. 161
			1903	1903	1903	1903	1903
Date of experiment,	*c*	...	March 23	March 23	March 23	March 24	March 24
Time at start,	*d*	...	11.35 a.m.	2.0 p.m.	3.0 p.m.	12 p.m.	1.51 p.m.
Temperature at start,	*e*	...	19·58°	19·58°	19·50°	19·53°	19·50°
Added weight, *w*, and reading, R,	f_0	*w*	R	R	R	R	R
First added weight or reading,	f_1	0·4	5·1	6·0	5·5	5·3	5·5
Second ,, ,,	f_2	0·5	16·2	17·2	17·0	16·5	17·0
Third ,, ,,	f_3	0·6	27·5	28·0	28·0	27·8	28·0
Fourth ,, ,,	f_4	0·7	38·5	39·0	39·0	39·0	39·2
Fifth ,, ,,	f_5	0·8	50·0	50·0	49·5	50·2	50·1
Sixth ,, ,,	f_6	0·9	61·0	60·8	61·0	61·9	61·0
Seventh ,, ,,	f_7	1·0	72·0	72·5	72·3	72·1	72·5
Eighth ,, ,,	f_8	1·1	83·3	83·0	83·5	83·8	83·8
Ninth ,, ,,	f_9	1·2	94·5	94·5	94·8	95·0	95·0
Mean added weight, $\bar{w}$,	*g*	0·8					
Mean reading, R,	*h*	...	49·79	50·11	50·06	50·17	50·23
Time at finish,	*i*	...	11.52 a.m.	2.15 p.m.	3.15 p.m.	12.15 p.m.	2.5 p.m.
Final temperature,	*j*	...	19·40°	19·48°	19·50°	19·51°	19·50°
Mean temperature, $\bar{T}$,	*k*	...	19·49°	19·53°	19·50°	19·52°	19·50°
Range of temperature,	*l*	...	0·18°	0·10°	0·00	0·02°	0·00
Difference of mean reading from 50 mm. $(50-\bar{R}=d\bar{r})$,	*m*	...	0·21	−0·11	−0·06	−0·17	−0·23
Weight equivalent to displacement, $d\bar{r}$ $(=dw_r)$,	*n*	...	0·0018	−0·0009	−0·0005	−0·0015	−0·0020
Weight required to immerse hydrometer to 50-mm. mark at mean temperature, $\bar{T}$, $(\bar{w}+dw_r)$,	*o*	...	0·8018	0·7991	0·7995	0·7985	0·7980
Difference of mean temperature, $\bar{T}$, from standard temperature $(\bar{T}-T=d\bar{t})$,	*p*	...	−0·01°	0·03°	...	0·02°	...
Correction for difference, $d\bar{t}$ $(=dw_t)$,	*q*	...	−0·00026	0·00078	...	0·0005	...
Weight required to immerse hydrometer to 50-mm. mark at standard temperature, T, $(\bar{w}+dw_r+dw_t)$,	*r*	...	0·8015	0·7998	0·7995	0·7990	0·7980
Total weight displaced by hydrometer when floating at 50-mm. mark at standard temperature, T,	*s*	..	181·7128	181·7111	181·7108	181·7103	181·7093

A_1.

No. 17.

when floating at the 50-*mm. mark in Distilled Water at* 15·00° *C.*

Weight *in vacuo* = 180·9113 grams. T = 15·00° C.

IV. 29	IV. 33	IV. 37	XVII. 73	XVII. 75	XVII. 77	XVII. 79	XVII. 81
1903	1903	1903	1904	1904	1904	1904	1904
March 6	March 6	March 6	Feb. 2	Feb. 2	Feb. 2	Feb. 2	Feb. 2
11.0 a.m.	11.58 a.m.	1.32 p.m.	11.15 a.m.	11.35 a.m.	12.23 p.m.	2.20 p.m.	2.40 p.m.
15·01°	15·00°	15·00°	15·00°	15·02°	15·00°	15·00°	15·00°
R	R	R	R	R	R	R	R
6·1	6·1	6·1	6·0	6·3	6·3	6·3	6·3
17·2	17·1	16·5	17·2	17·5	17·3	17·5	17·5
27·5	27·5	28·2	28·5	29·0	28·8	28·5	29·0
39·8	39·0	39·5	39·8	40·0	39·9	39·9	40·0
50·5	50·2	50·5	50·8	51·0	51·0	51·0	51·1
62·3	62·0	62·0	62·0	62·0	62·0	62·0	62·1
72·5	72·2	73·0	73·0	73·1	73·0	73·0	73·1
84·0	84·0	84·5	84·0	84·2	84·1	84·1	84·1
96·0	95·3	96·0	95·2	95·5	95·3	95·3	95·3
50·65	50·37	50·70	50·72	50·95	50·85	50·84	50·94
11.14 a.m.	12.10 p.m.	1.45 p.m.	11.33 a.m.	11.50 a.m.	12.35 p.m.	2.35 p.m.	2.52 p.m.
14·99°	15·00°	15·00°	15·02°	15·05°	15·00°	15·00°	15·02°
15·00°	15·00°	15·00°	15·01°	15·035°	15·00°	15·00°	15·01°
0·02°	0·00	0·00	0·02°	0·03°	0·00	0·00	0·02°
− 0·65	− 0·37	− 0·70	− 0·72	− 0·95	− 0·85	− 0·84	− 0·94
− 0·0058	− 0·0033	− 0·0062	− 0·0064	− 0·0085	0·0076	− 0·0075	− 0·0084
0·9192	0·9217	0·9188	0·9186	0·9165	0·9174	0·9175	0·9166
...	...	...	0·01°	0·035°	...	...	0·01°
...	...	...	0·0002	0·0009	...	...	0·0002
0·9192	0·9217	0·9188	0·9188	0·9174	0·9174	0·9175	0·9168
181·8305	181·8330	181·8301	181·8301	181·8287	181·8287	181·8288	181·8281

A_2.

No. 17.

when floating at the 50-*mm. mark in Distilled Water at* 19·50° *C.*

Weight *in vacuo* = 180·9113 grams. T = 19·50° C.

IV. 167	VI. 7	VI. 13	VI. 19	VI. 25	VI. 27	VI. 29	VI. 31	VI. 33	VI. 35	VI. 37
1903	1903	1903	1903	1903	1903	1903	1903	1903	1903	1903
March 24	March 25	March 25	March 25	March 26	March 26	March 26	March 26	March 26	March 26	March 26
2.40 p.m.	12.12 p.m.	2.55 p.m.	3.47 p.m.	12.0 p.m.	12.18 p.m.	12.35 p.m.	1.0 p.m.	2.48 p.m.	3.3 p.m.	3.30 p.m.
19·50°	19·53°	19·50°	19·50°	19·50°	19·50°	19·50°	19·50°	19·50°	19·50°	19·51°
R	R	R	R	R	R	R	R	R	R	R
5·8	5·3	5·8	5·8	5·3	6·0	5·5	5·8	5·5	5·8	5·5
17·0	16·5	17·0	17·0	16·5	17·0	16·5	17·0	16·5	17·0	16·5
28·0	27·8	28·0	27·8	27·5	27·5	27·8	28·0	27·5	28·1	27·9
39·0	39·0	39·2	39·1	39·0	39·0	39·0	39·1	39·0	39·3	39·0
50·0	50·1	50·3	50·2	50·0	50·0	50·0	50·2	49·8	50·2	50·3
60·8	61·3	61·0	61·3	61·0	61·0	61·0	61·3	61·0	61·2	61·5
72·0	72·2	72·5	72·5	72·5	71·5	72·0	72·5	72·3	72·5	72·5
83·5	84·5	84·0	83·8	83·5	83·5	83·2	83·5	83·5	83·8	83·5
95·0	95·0	95·0	95·0	94·8	95·0	94·5	95·0	94·5	95·0	94·8
50·12	50·19	50·31	50·27	50·01	50·05	49 94	50·26	49·95	50·32	50·16
2.52 p.m.	12.25 p.m.	3.10 p.m.	4.2 p.m.	12.15 p.m.	12.33 p.m.	12.50 p.m.	1.15 p.m.	3.2 p.m.	3.20 p.m.	3.45 p.m.
19·50°	19·55°	19·53°	19·53°	19·50°	19·50°	19 51°	19·56°	19·50°	19·55°	19·53°
19·50°	19·54°	19·515°	19·515°	19·50°	19·50°	19·50°	19·53°	19·50°	19·525°	19·52°
0·00	0·02°	0·03°	0·03°	0·00	0·00	0·01°	0·06°	0·00	0·05°	0·02°
− 0·12	− 0·19	− 0·31	− 0·27	− 0·01	− 0·05	0·06	− 0·26	0·05	− 0·32	− 0·16
− 0·0010	− 0·0017	− 0·0027	− 0·0024	0·0000	− 0·0004	0·0005	− 0·0023	0·0004	− 0·0028	− 0·0014
0·7990	0·7983	0·7973	0·7976	0·8000	0·7996	0·8005	0·7977	0·8004	0·7972	0·7986
...	0·04°	0·015°	0·015°	...	...	...	0·03°	...	0·025°	0·02°
...	0·0010	0 0004	0·0004	...	...	...	0·0008	...	0·0008	0·0005
0·7990	0·7993	0·7977	0·7979	0·8000	0·7996	0·8005	0·7985	0·8004	0·7979	0·7991
181·7103	181·7106	181·7090	181·7092	181·7113	181·7109	181·7118	181·7098	181·7117	181·7092	181·7104

TABLE

Hydrometer No. 17.

Details of Determination of the Total Weight of the Hydrometer

			Hydrometer No. 17.		
Particulars of hydrometer,	a				
Reference to laboratory note-book,	b	...	XXVIII. 59 1905	XXVIII. 61 1905	XXVIII. 63 1905
Date of experiment,	c	...	June 6	June 6	June 6
Time at start,	d	...	10.52 a.m.	11.42 a.m.	12.25 p.m.
Temperature at start,	e	...	23·05°	23·00°	23·03°
Added weight, *w*, and reading, R,	f_0	*w*	R	R	R
First added weight or reading,	f_1	0·25	2·2	1·8	2·2
Second ,, ,,	f_2	0·35	13·5	12·2	12·5
Third ,, ,,	f_3	0·45	24·8	23·8	24·5
Fourth ,, ,,	f_4	0·55	35·8	35·0	35·8
Fifth ,, ,,	f_5	0·65	47·0	46·2	46·8
Sixth ,, ,,	f_6	0·75	58·0	57·2	57·2
Seventh ,, ,,	f_7	0·85	69·0	68·5	68·8
Eighth ,, ,,	f_8	0·95	80·0	79·5	79·2
Ninth ,, ,,	f_9	1·05	90·8	90·5	91·2
Mean added weight, $\bar{w}$,	g	0·65	...	...	...
Mean reading, $\bar{R}$,	h	...	46·79	46·08	46·47
Time at finish,	i	...	11.15 a.m.	12.00 p.m.	12.40 p.m.
Final temperature,	j	...	22·85°	22·90°	23·02°
Mean temperature, $\bar{T}$,	k	...	22·95°	22·95°	23·025°
Range of temperature,	l	...	0·20°	0·10°	0·01°
Difference of mean reading from 50 mm. ($50 - \bar{R} = d\bar{r}$),	m	...	3·21	3·92	3·53
Weight equivalent to displacement, $d\bar{r}$ ($= dw_r$),	n	...	0·0290	0·0354	0·0319
Weight required to immerse hydrometer to 50-mm. mark at mean temperature, $\bar{T}$, ($\bar{w} + dw_r$),	o	...	0·6790	0·6854	0·6819
Difference of mean temperature, $\bar{T}$, from standard temperature ($\bar{T} - T = d\bar{t}$),	p	...	−0·05°	−0·05°	0·025°
Correction for difference $d\bar{t}$ ($= dw_t$),	q	...	−0·0015	−0·0015	0·0007
Weight required to immerse hydrometer to 50-mm. mark at standard temperature, T, ($= \bar{w} + dw_r + dw_t$),	r	...	0·6775	0·6839	0·6826
Total weight displaced by hydrometer when floating at 50-mm. mark at standard temperature, T,	s	...	181·5888	181·5952	181·5938

TABLE

Hydrometer No. 17.

Details of Determination of the Total Weight of the Hydrometer

			Hydrometer No. 17.
Particulars of hydrometer,	a		
Reference to laboratory note-book,	b	...	VIII. 47 1903
Date of experiment,	c	...	July 2
Time at start,	d	...	3.20 p.m.
Temperature at start,	e	...	26·00°
Added weight, *w*, and reading, R,	f_0	*w*	R
First added weight or reading,	f_1	0·175	6·0
Second ,, ,,	f_2	0·275	17·1
Third ,, ,,	f_3	0·375	28·5
Fourth ,, ,,	f_4	0·475	39·8
Fifth ,, ,,	f_5	0·575	50·8
Sixth ,, ,,	f_6	0·675	62·0
Seventh ,, ,,	f_7	0·775	73·2
Eighth ,, ,,	f_8	0·875	84·3
Ninth ,, ,,	f_9	0·975	95·6
Mean added weight, $\bar{w}$,	g	0·575	
Mean reading, $\bar{R}$,	h	...	50·81
Time at finish,	i	...	3.35 p.m.
Final temperature,	j	...	26·10°
Mean temperature, $\bar{T}$,	k	...	26·05°
Range of temperature,	l	...	0·10°
Difference of mean reading from 50 mm. ($50 - \bar{R} = d\bar{r}$),	m	...	−0·81
Weight equivalent to displacement $d\bar{r}$ ($= dw_r$),	n	...	−0·0072
Weight required to immerse hydrometer to 50-mm. mark at mean temperature, $\bar{T}$, ($\bar{w} + dw_r$),	o	...	0·5678
Difference of mean temperature, $\bar{T}$, from standard temperature ($\bar{T} - T = d\bar{t}$),	p	...	0·05°
Correction for difference $d\bar{t}$ ($= dw_t$),	q	...	0·0018
Weight required to immerse hydrometer to 50-mm. mark at standard temperature, T, ($= \bar{w} + dw_r + dw_t$),	r	...	0·5696
Total weight displaced by hydrometer when floating at 50-mm. mark at standard temperature, T,	s	...	181·4809

A_3.

J. Y. B., 1893.

when floating at the 50-*mm. mark in Distilled Water at* 23·00° *C.*

J. Y. B., 1893. Weight *in vacuo* = 180·9113 grams. T = 23·00° C.

XXVIII. 65	XXVIII. 67	XXVIII. 69	...	XXX. 107	XXX. 111	XXX. 141	XXX. 143	XXX 145	XXX. 147
1905	1905	1905	...	1905	1905	1905	1905	1905	1905
June 6	June 6	June 6	...	July 12	July 12	July 17	July 17	July 17	July 17
1.40 p.m.	2.17 p m.	4.3 p.m.	...	10.35 a.m.	11.20 a.m.	1.54 p.m.	2.22 p.m.	3.15 p.m.	4.0 p.m.
23·00°	23·00°	23·00°	...	22·80°	23·10°	23·00°	23·00°	23·00°	23·00°
R	R	R	*w*	R	R	R	R	R	R
2·0	2·2	1·5	0·275	3·2	5·2	6·2	6·2	5·2	6·2
12·2	12·8	11·5	0·375	15·2	16·2	17·2	17·2	16·2	17·2
23·8	23·5	23·2	0·475	26·2	27·2	27·8	28·0	27·5	28·0
35·2	35·0	34·2	0·575	37·0	37·8	38·2	38·2	38·2	38·2
45·0	46·0	45·2	0·675	48·2	47·8	49·0	49·2	48·5	49·2
55·8	57·5	55·8	0·775	59·2	60·2	60·0	60·2	60·0	60·2
68·2	68·5	67·8	0·875	70·2	71·2	71·2	71·2	71·2	71·2
79·2	78·8	78·5	0·975	81·2	82·2	82·2	82·2	82·2	82·5
90·2	90·0	89·5	1·075	93·2	94·2	94·2	94·2	93·2	94·2
...	...	...	0·675						
45·73	46·03	45·25	...	48·177	49·111	49·555	49·622	49·133	49·655
2.0 p.m.	2.30 p.m.	4.23 p.m.	...	10.57 a.m.	11.40 a.m.	2.8 p.m.	2.40 p.m.	3.33 p.m.	4.10 p.m.
23·00°	23·00°	23·00°	...	23·00°	23·22°	23·00°	23·00°	23·00°	23·00°
23·00°	23·00°	23·00°	...	22·90°	23·16°	23·00°	23·00°	23·00°	23·00°
0·00	0·00	0·00	...	0·20°	0·12°	0·00	0·00	0·00	0·00
4·27	3·97	4·75	...	1·823	0·889	0·445	0·378	0·867	0·345
0·0386	0·0360	0·0430	...	0·0164	0·0080	0·0040	0·0034	0·0078	0·0031
0·6386	0·6860	0·6930	...	0·6914	0·6830	0·6790	0·6784	0·6828	0·6781
...	...	...	...	−0·10°	0·16°	...	...	...	...
...	...	...	...	−0·0031	0·0049	...	...	...	...
0·6886	0·6860	0·6930	...	0·6883	0·6879	0·6790	0·6784	0·6828	0·6781
181·5999	181·5973	181·6043	...	181·5996	181·5992	181·5903	181·5897	181·5941	181·5894

A_4.

J. Y. B., 1893.

when floating at the 50-*mm. mark in Distilled Water at* 26·00° *C.*

J. Y. B., 1893. Weight *in vacuo* = 180·9113 grams. T = 26·00° C.

VIII. 49	VIII. 51	VIII. 53	VIII. 55	VIII. 57	VIII. 59	VIII. 61	VIII. 63
1903	1903	1903	1903	1903	1903	1903	1903
July 2	July 2	July 2	July 3	July 3	July 3	July 3	July 3
3.35 p.m.	3.55 p.m.	4.15 p.m.	12.15 p.m.	12.30 p.m.	12.45 p.m.	1.5 p.m.	11.50 a.m.
26·10°	26·10°	26·00°	26·01°	26·00°	26·00°	26·00°	26·00°
R	R	R	R	R	R	R	R
6·8	6·8	7·0	6·3	6·3	6·3	6·3	6·3
17·8	17·8	18·2	17·0	17·0	17·1	17·1	16·3
29·2	29·3	29·5	27·8	28·5	28·5	28·2	27·0
40·5	40·5	40·5	39·8	39·2	39·0	39·3	39·2
51·5	51·5	51·8	50·8	50·8	50·5	50·0	50·1
62·8	62·5	63·0	61·3	61·8	61·8	61·8	61·3
74·0	74·0	74·0	73·0	73·0	72·3	72·2	73·0
85·0	85·0	85·0	84·0	84·2	84·0	84·5	84·0
96·2	96·2	96·2	95·0	95·3	95·5	95·5	95·0
51·53	51·51	51·68	50·55	50·67	50·55	50·54	50·24
3.50 p.m.	4.10 p.m.	4.30 p.m.	12.28 p.m.	12.45 p.m.	1.0 p.m.	1.15 p.m.	12.10 p.m.
26·10°	26·00°	26·00°	25·95°	25·98°	26·00°	26·00°	25·80°
26·10°	26·05°	26·00°	25·98°	25·99°	26·00°	26·00°	25·90°
0·00	0·10°	0·00	0·06°	0·02°	0·00	0·00	0·20°
−1·53	−1·51	−1·68	−0·55	−0·67	−0·55	−0·54	−0·24
−0·0137	−0·0135	−0·0152	−0·0049	−0·0060	−0·0049	−0·0048	−0·2100
0·5613	0·5615	0·5598	0·5701	0·5690	0·5701	0·5702	0 5729
0·10°	0·05°	...	−0·02°	−0·01°	...	...	−0·10°
0·0036	0·0018	...	−0·0007	−0·0003	...	...	−0·0036
0·5649	0·5633	0·5598	0·5693	0·5687	0·5701	0·5702	0·5693
181·4762	181·4746	181·4711	181·4806	181·4800	181·4814	181·4815	181·4806

TABLE

Hydrometer

Details of Determination of the Total Weight of the Hydrometer

Particulars of hydrometer,	a		Hydrometer No. 21. J. Y. B., 1893.				
Reference to laboratory note-book,	b	...	II. 17 1903	IV. 139 1903	IV. 145 1903	IV. 151 1903	IV. 157 1903
Date of experiment,	c	...	Feb. 10	March 23	March 23	March 23	March 24
Time at start,	d	...	1.15 p.m.	12 p.m.	2.20 p.m.	3.15 p.m.	12.25 p.m.
Temperature at start,	e	...	19·8°	19·55°	19·55°	19·50°	19·51°
Added weight, w, and reading, R,	f_0	w	R	R	R	R	R
First added weight or reading,	f_1	0·4	4·2	4·0	3·5	3·8	3·5
Second ,, ,,	f_2	0·5	16·0	15·3	15·1	15·5	15·1
Third ,, ,,	f_3	0·6	28·0	27·1	27·0	27·2	26·2
Fourth ,, ,,	f_4	0·7	39·1	39·0	38·5	39·0	38·5
Fifth ,, ,,	f_5	0·8	50·0	51·0	50·5	50·8	50·5
Sixth ,, ,,	f_6	0·9	62·5	62·3	62·2	62·5	62·0
Seventh ,, ,,	f_7	1·0	74·2	74·2	74·0	74·3	73·5
Eighth ,, ,,	f_8	1·1	86·0	86·0	86·0	86·0	85·2
Ninth ,, ,,	f_9	1·2	97·3	97·5	97·5	97·5	97·5
Mean added weight, $\overline{w}$,	g	0·8					
Mean reading, $\overline{R}$,	h	...	50·81	50·71	50·47	50·73	50·22
Time at finish,	i	...	1.28 p.m.	12.15 p.m.	2.35 p.m.	3.30 p.m.	12.40 p.m.
Final temperature,	j	...	19·50°	19·49°	19·45°	19·51°	19·50°
Mean temperature, $\overline{T}$,	k	...	19·65°	19·52°	19·50°	19·505°	19·505°
Range of temperature,	l	...	0·30°	0·06°	0·10°	0·01°	0·01°
Difference of mean reading from 50 mm. ($50-\overline{R}=d\overline{r}$),	m	...	−0·81	−0·71	−0·47	−0·73	−0·22
Weight equivalent to displacement $d\overline{r}$ ($=dw_r$),	n	...	−0·0069	−0·0060	−0·0040	−0·0062	−0·0018
Weight required to immerse hydrometer to 50-mm. mark at mean temperature, $\overline{T}$, ($\overline{w}+dw_r$),	o	...	0·7931	0·7940	0·7960	0·7938	0·7982
Difference of mean temperature, $\overline{T}$, from standard temperature ($\overline{T}-T=d\overline{t}$),	p	...	0·15°	0·02°	...	0·005°	0·005°
Correction for difference $d\overline{t}$ ($=dw_t$)	q	...	0·0040	0·0005	...	0·0001	0·0001
Weight required to immerse hydrometer to 50-mm. mark at standard temperature, T, ($=\overline{w}+dw_r+dw_t$),	r	...	0·7971	0·7945	0·7960	0·7939	0·7983
Total weight displaced by hydrometer when floating at 50-mm. mark at standard temperature, T.	s	...	188·5796	188·5770	188·5785	188·5764	188·5808

A_5.

No. 21.

when floating at the 50-*mm. mark in Distilled Water at* 19·50° *C.*

Weight *in vacuo* = 187·7825 grams. T = 19·50° C.

IV. 163	IV. 169	VI. 9	VI. 15	VI. 21	VI. 39	VI. 41	VI. 43	VI. 45	VI. 47	VI. 49
1903	1903	1903	1903	1903	1903	1903	1903	1903	1903	1903
March 24	March 24	March 25	March 25	March 25	March 27	March 27	March 27	March 27	March 27	March 27
2.8 p.m.	2.53 p.m.	2.40 p.m.	3.15 p.m.	4.3 p.m.	2.0 p.m.	2.15 p.m.	2.40 p.m.	3.0 p.m.	3.18 p.m.	3.38 p.m.
19·50°	19·50°	19·50°	19·50°	19·53°	19·50°	19·50°	19·53°	19·50°	19·50°	19·50°
R	R	R	R	R	R	R	R	R	R	R
3·8	3·8	3·1	4·0	4·0	3·5	3·3	4·2	4·0	4·0	4·0
15·5	15·5	15·0	15·5	16·0	15·3	15·2	16·0	15·8	15·8	15·5
27·2	27·5	26·8	27·5	28·0	26·8	27·2	28·0	27·5	27·5	27·5
39·0	39·1	38·5	39·1	39·3	38·5	39·0	39·3	39·1	39·0	39·2
50·8	51·0	50·5	51·0	51·0	50·5	51·0	51·0	51·0	51·0	51·0
62·5	62·8	61·8	62·5	62·8	62·3	62·5	63·0	62·8	62·2	63·0
74·3	74·5	74·0	74·2	74·8	74·2	74·5	75·0	74·5	74·5	74·5
86·0	86·0	86·0	86·0	86·3	86·0	86·0	86·5	85·8	86·0	86·2
97·5	98·0	97·5	97·5	98·0	97·5	98·0	98·0	97·5	98·0	98·0
50·73	50·91	50·35	50·81	51·13	50·51	50·74	51·22	50·88	50·88	50·98
2.23 p.m.	3.10 p.m	2.55 p.m.	3.30 p.m.	4.18 p.m.	2.13 p.m.	2.30 p.m.	2.55 p.m.	3.15 p.m.	3.35 p.m.	3.55 p.m.
19·50°	19·55°	19·50°	19·50°	19·59°	19·50°	19·55°	19·51°	19·50°	19·52°	19·58°
19·50°	19·525°	19·50°	19·50°	19·56°	19·50°	19·525°	19·52°	19·50°	19·51°	19·54°
0·00	0·05°	0·00	0·00	0·06°	0·00	0·05°	0·02°	0·00	0·02°	0·08°
−0·73	−0·91	−0·35	−0·81	−1·13	−0·51	−0·74	−1·22	−0·88	−0·88	−0·98
−0·0062	−0·0077	−0·0030	−0·0069	−0·0096	−0·0043	−0·0063	−0·0104	−0·0075	−0·0075	−0·0083
0·7938	0·7923	0·7970	0·7931	0·7904	0·7957	0·7937	0·7896	0·7925	0·7925	0·7917
...	0·025°	...	...	0·06°	...	0·025°	0·02°	...	0·01°	0·04°
...	0·0006	...	...	0·0016	...	0·0006	0·0005	...	0·0002	0·0011
0·7938	0·7929	0·7970	0·7931	0·7920	0·7957	0·7943	0·7901	0·7925	0·7927	0·7928
188·5763	188·5754	188·5795	188·5756	188·5745	188·5782	188·5768	188·5726	188·5750	188·5752	188·5753

TABLE B.

Observed Total Weights of Hydrometers when floating at the 50-mm. mark in Distilled Water at various Temperatures.

Hydrometer.	Temperature. °C.	Mean Total Weight. Grams.	Number of Series of Observations.	Maximum Departure from the Mean.	Probable Error of Arithmetical Mean.
					±
17	15·00	181·8304	10	0·0049	0·0006
	19·50	181·7105	16	0·0023	0·0002
	23·00	181·5952	12	0·0093	0·0010
	26·00	181·4785	9	0·0074	0·0008
21	15·00	188·7012	12	0·0085	0·0007
	19·50	188·5767	16	0·0040	0·0003
	23·00	188·4570	15	0·0107	0·0011
	26·00	188·3363	9	0·0031	0·0005
3	15·00	179·3116	12	0·0117	0·0012
	19·50	179·1987	12	0·0040	0·0005
	23·00	179·0757	4	0·0006	0·0001
	26·00	178·9697	16	0·0038	0·0002

SECTION III.—DETERMINATION OF THE SPECIFIC GRAVITY OF A SALINE SOLUTION.

§ 18. When determining the specific gravity of a saline solution by the hydrometric method, it is necessary first to find the weight which must be added to the hydrometer to immerse it to the 50-mm. mark when floating in the saline solution at the chosen standard temperature. This added weight is found by a series of observations in exactly the same manner as with the hydrometer in distilled water (see § 14 *et seq.*).

The details of three series of observations with hydrometer No. 17 in a solution of $\frac{1}{8}$ gram-molecule of cæsium chloride in 1000 grams of water at 19·50° are given as an example in Table C. This table is arranged in the same manner as Table A_1.

The correction for the difference of the mean immersion from 50 mm. is calculated in the same manner as has been explained in connection with the determination of the displacement of hydrometer No. 17 in distilled water.

Taking the series XX. 79 as an example, we find that an increase of 0·8 gram in the added weight increases the stem immersion from 6·0 to 93·5 mm., that is, 87·5 mm.; hence each mm. increase in the stem immersion is caused by an addition of $0·800/87·5 = 0·00914$ gram. The difference of the mean reading from 50 mm. in the series in question is $50 - 49·62 = 0·38$, which is equivalent to an added weight of $0·00914 \times 0·38 = 0·0034$ gram.

As the mean reading is less than 50 mm., the hydrometer is not sufficiently immersed in the solution; therefore the added weight is too small and must be increased. The weight to be added is 0·0034 gram; the resultant weight which must be added to the hydrometer to immerse it to the 50-mm. mark in the solution at the mean observed temperature is therefore $3·7 + 0·0034 = 3·7034$ grams (line *o*).

As in the three series detailed in Table C the mean observed temperature is identical with the standard temperature, 19·50°, no temperature correction is required, and the weight 3·7034 grams is that which is, in the first case, required to immerse the hydrometer to the 50-mm. mark in the saline solution of 19·50°.

Adding this weight to that of hydrometer No. 17 *in vacuo*, we get 180·9113 + 3·7034 = 184·6147 grams, entered on line *s*; this is the total weight of the hydrometer when floating at the 50-mm. mark in the solution under experiment.

Dividing this number by the displacement of the hydrometer when floating at the 50-mm. mark in distilled water of the same temperature, 19·5° C., namely, 181·7105 grams, line *s'*, Table C, we obtain 1·015982 as the specific gravity of the solution. The specific gravity calculated from each series of observations is given at the foot of the corresponding column, in line *t*.

Table C.

Hydrometer No. 17.

Details of the Determination of the Total Weight of the Hydrometer when floating at the 50-*mm. mark in a solution of* $\frac{1}{8}$ *gram-molecule of Cæsium Chloride in* 1000 *grams of water at* 19·50° *C., and the Specific Gravity of the Solution.*

Particulars of hydrometer,	*a*	Hyd. No. 17.	J. Y. B., 1893	Weight *in vacuo* = 180·9113 grms.	
Reference to laboratory note-books,	*b*	...	XX. 79	XX. 87	XX. 91
Date of experiment,	*c*	...	April 22, 1904	April 22, 1904	April 22, 1904
Time at start,	*d*	...	2.00 p.m.	3.18 p.m.	3.50 p.m.
Temperature at start,	*e*	...	19·50°	19·50°	19·50°
Added weight, *w*, and reading, R,	f_0	*w*	R	R	R
First added weight or reading, R,	f_1	3·3	6·0	5·5	5·5
Second " " "	f_2	3·4	16·5	16·5	16·5
Third " " "	f_3	3·5	28·0	27·5	27·5
Fourth " " "	f_4	3·6	38·5	38·3	38·5
Fifth " " "	f_5	3·7	49·8	49·5	49·5
Sixth " " "	f_6	3·8	60·5	60·3	60·3
Seventh " " "	f_7	3·9	71·3	71·1	71·1
Eighth " " "	f_8	4·0	82·5	82·0	82·5
Ninth " " "	f_9	4·1	93·5	93·0	93·5
Mean added weight, $\bar{w}$,	*g*	3·7			
Mean reading, $\bar{R}$,	*h*	...	49·62	49·30	49·43
Time at finish,	*i*	...	2.15 p.m.	3.35 p.m.	4.05 p.m.
Final temperature,	*j*	...	19·50°	19·50°	19·50°
Mean temperature, $\bar{T}$,	*k*	...	19·50°	19·50°	19·50°
Range of temperature,	*l*	...	0·00	0·00	0·00
Difference of mean reading from 50 mm. ($50 - \bar{R} = d\bar{r}$),	*m*	...	0·38	0·70	0·57
Weight equivalent to displacement $d\bar{r}$ ($= dw_r$),	*n*	...	0·0034	0·0063	0·0052
Weight required to immerse hydrometer to 50-mm. mark at mean temperature, $\bar{T}$, ($\bar{w} + dw_r$),	*o*	...	3·7034	3·7063	3·7052
Difference of mean temperature, $\bar{T}$, from standard temperature, T, ($\bar{T} - T = d\bar{t}$),	*p*	...	0·00	0·00	0·00
Correction for difference $d\bar{t}$ ($= dw_t$),	*q*	...	0·00	0·00	0·00

Table C—*continued.*

Weight required to immerse hydrometer to 50-mm. mark at standard temperature, T, $(=\bar{w}+dw_r+dw_t)$,	r	...	3·7034	3·7063	3·7052
Total weight of solution displaced by hydrometer when immersed to 50-mm. mark at standard temperature, T,	s	...	184·6147	184·6176	184·6165
Corresponding weight in distilled water	s'	...	181·7105	181·7105	181·7105
Specific gravity of solution $\frac{s}{s'}$	t	...	1·015982	1·015998	1·015992

§ 19. *Influence of the Meniscus.*—It may be here pointed out that the weight which causes the immersion of the hydrometer is its own weight plus that of the liquid meniscus which it carries on the stem above the line of flotation. It is impossible to measure or weigh this exactly; but we are concerned only with the question if, and to what extent, it can affect the exactness of the determination of the specific gravity of our solutions.

The weight of the meniscus depends on the surface-tension of the liquid, and this has been determined for distilled water and for a few saline solutions. The results are given in Tables Nos. 124 to 129 of the Smithsonian collection of Physical Tables, of which a new edition has been recently published.* In these tables only one salt belonging to either of the enneads MR or MRO_3 is included, namely, chloride of potassium. For the solutions of the remaining seventeen salts there are no experimental data. We are therefore unable to state exactly the effect which would be produced on the specific gravities of their solutions if the weights of the meniscuses of the distilled water and of the solution had been taken into account. Nevertheless, it is worth while to study the influence of the meniscus on the specific gravity of the solutions of the salts quoted in the table, because light will thereby be thrown on the probable extent of its influence on the specific gravity of the solutions of the salts of the two enneads.

When we determine the specific gravity of a solution, we first immerse the hydrometer in distilled water, and add (if necessary) small weights to the top of the stem until it floats at a certain division on the stem, say 50; and we call the total displacing weight H. We then immerse the hydrometer in the solution, having the same temperature as the distilled water had, and add weights to the top of the stem until the instrument floats in the solution at the same division, 50; and we call the total displacing weight H′. Then the specific gravity of the solution is $S=\frac{H'}{H}$. Here we have not taken account either of the meniscus of the distilled water or of that of the solution. But each of these meniscuses exerts the pressure of its own weight and contributes to the displacing weight of the instrument when it floats at 50 in the distilled water and the solution respectively. Let h = the weight of the meniscus of

* *Smithsonian Physical Tables*, 5th revised edition, 1910.

distilled water, and let h' = that of the solution, then the two displacing weights are $H+h$ and $H'+h'$, and the specific gravity is $S_h = \frac{H'+h'}{H+h}$. The data given in the Smithsonian Tables, Nos. 124 and 126, enable us to arrive at the value of h for distilled water, Table No. 124, and at those of h' for certain solutions of the salts included in Table No. 126. In the following table the effect of introducing the weight of the meniscus in the hydrometric determination of the specific gravity of some of the solutions contained in Smithsonian Table No. 126 has been calculated. The tabular specific gravity (col. *b*) furnished by the Smithsonian Tables is taken to represent the hydrometric specific gravity arrived at without taking into account the influence of the meniscus. From this, and using the surface-tension of distilled water, and that of the solutions furnished by the Smithsonian Tables given in columns *d* and *e*, the hydrometric specific gravity of the same solution, having regard to the weight of the meniscus, is calculated and entered in column *l*. The details of this calculation are as follows. In column *a* we have the formula of the salt in solution, in column *b* the specific gravity of the solution. In column *c* is the temperature, and in column *e* is the surface-tension of the solution at that temperature. The surface-tension of distilled water at the same temperature is given in column *d*. In columns *d* and *e* the values of the surface-tension are expressed in dynes per centimetre. Accepting 1/981 gram as the pressure which balances the force of 1 dyne, the weight, in milligrams, of liquid lifted by 1 centimetre of glass is obtained by multiplying the entries in the fourth column by 1·019. If the circumference of the stem of our hydrometer were exactly 1 centimetre, this would be the weight of the meniscus.

The circumference of the stem of hydrometer No. 17, the one which has been most frequently used, is 1·062 centimetre; therefore, to get the weight in milligrams of the meniscus supported by it, we must multiply the entry in column *d* by $1·019 \times 1·062 = 1·0822$.

Consider the first line of the table. The salt dissolved is NaCl. Taking the total weight of hydrometer No. 17 when floating at the 50th division in distilled water to be 181·7105 grams (col. *h*), and multiplying this number by 1·036 (col. *b*), the tabular specific gravity of the least concentrated solution of NaCl quoted in the table, we obtain 188·2521 (col. *i*) as the displacing weight of this hydrometer when floating at the 50th division in NaCl solution of the specific gravity 1·0360. The surface-tension of distilled water at 20° C. is given (col. *d*) as 72·8 dynes per centimetre. Multiplying this by 1·0822, we obtain 78·8 milligrams (col. *f*) as the weight of the meniscus of distilled water. Similarly, multiplying 77·6, the surface-tension of the NaCl solution in dynes per centimetre, by 1·0822, we obtain 84·0 milligrams (col. *g*) as the weight of the meniscus of this solution supported by the stem of hydrometer No. 17. We have, then, after taking account of the influence of meniscus, for the displacing weight of the hydrometer when floating in distilled water, $181·7105 + 0·0788 = 181·7893$ grams (col. *j*), and for that of the hydrometer when floating at the same division in NaCl solution, $188{\cdot}2521 + 0{\cdot}0840 = 188{\cdot}3361$ grams (col. *k*). Dividing this number by 181·7893, we obtain for the specific gravity of the solution, amended for the weight of meniscus,

$$S_h = \frac{188{\cdot}3361}{181{\cdot}7893} = 1{\cdot}036013 \quad \text{(col. } l\text{)}.$$

The difference between the two specific gravities is 1·3 in the fifth place. This difference is higher than the average. In the case of the nearly saturated solution of NaCl, sp. gr. = 1·1932, the difference is only 1 in the fifth place. In the corresponding solutions of KCl the difference in the dilute solution is 1, and in the concentrated solution 2, in the fifth place. In the case of $CaCl_2$ both solutions are of considerable concentration; the difference in the more concentrated is 2, and in the less concentrated 1·7, in the fifth place. In the case of $MgCl_2$ the difference in the dilute solution is 1, and in the concentrated solution 0·4, in the fifth place. It is therefore evident that the specific gravity of solutions determined by the hydrometric method is not affected by an error due to the influence of the meniscus which calls for correction.

The specific gravity of the less concentrated NaCl solution is 1·036, that of average oceanic sea-water is not greater than 1·027; therefore the influence of the meniscus in sea-water of highest concentration would be not greater than 1 in the fifth place. *In all my hydrometric work on sea-water the influence of the meniscus has been disregarded, because I recognised from the beginning that the ratio of the weights of the meniscus must be the same as that of the masses of the liquids displaced by the same immersed portion of the hydrometer, unless their volumes differed considerably, and of this there was no evidence.*

a.	b.	c.	d.	e.	1·0822d. f.	1·0822e. g.	h.	h × b. i.	h + f. j.	i + g. k.	k/j. l.	l − b. m.
Salt dissolved.	Without Meniscus. Specific Gravity of Solution.	Temp. °C.	Surface-tension, dynes per cm.		Weight in Milligrams of Meniscus.		Without Meniscus. Weight of Hydrometer in		With Meniscus. Weight of Hydrometer in		With Meniscus. Specific Gravity of Solution.	Difference.
			H_2O.	Solution.	H_2O.	Solution.	H_2O.	Solution.	H_2O.	Solution.		
NaCl	1·03600	20	72·8	77·6	78·8	84·0	181·7105	188·2521	181·7893	188·3361	1·036013	+0·000013
NaCl	1·19320	20	72·8	85·8	78·8	92·8	181·7105	216·8168	181·7893	216·9096	1·193190	−0·000010
KCl	1·04630	15	73·5	78·2	79·5	84·6	181·7105	190·1237	181·7900	190·2083	1·046310	−0·000010
KCl	1·16990	15	73·5	82·8	79·5	89·6	181·7105	212·5831	181·7900	212·6726	1·169880	−0·000020
$MgCl_2$	1·03620	15	73·5	78·0	79·5	84·4	181·7105	188·2884	181·7900	188·3728	1·036210	+0·000010
$MgCl_2$	1·23380	15	73·5	90·1	79·5	97·5	181·7105	224·1943	181·7900	224·2918	1·233796	−0·000004
$CaCl_2$	1·27730	19	72·9	90·2	78·9	97·6	181·7105	232·0987	181·7894	232·1963	1·277283	−0·000017
$CaCl_2$	1·35110	19	72·9	95·0	78·9	102·8	181·7105	245·5090	181·7894	245·6118	1·351080	−0·000020

§ 20. *Serial Determination of the Specific Gravity of a Saline Solution.*—The method by which the specific gravity of a saline solution is determined with the hydrometer is given in another place (see § 18), and it is there shown how the total displacement of the instrument when floating in the solution up to the 50-mm.

mark, and at the standard temperature, is obtained. In that method the mean values of the added weights and the corresponding scale readings are utilised to obtain one value of the specific gravity of the solution. The conditions of the method are that exactly the same volume of the hydrometer shall be immersed in the solution and the distilled water, the temperature of both being the same, and the total weight required to cause the instrument to float at 50 mm. in the saline solution divided by the total weight required to float it at the 50-mm. mark in distilled water gives the specific gravity of the solution.

In an experiment the hydrometer is loaded with nine successive weights increasing by 0·1 gram in each step, and the reading of the stem is observed after each addition. It is evident that if we take the first of the readings and find the weight that would have to be added to the hydrometer when it was floating at the same level in distilled water, the ratio of the two total weights would give a value for the specific gravity of the solution. In a similar way we might use the second scale reading as the basis for another computation of the specific gravity; and, in general, each of the nine values of the scale readings, obtained by the addition of successive increments of 0·1 gram to the stem of the hydrometer, furnishes a separate basis for determining the specific gravity. Proceeding in this manner, the accompanying table has been compiled.

Table giving Specific Gravity of 1/32 *RbCl* + 1000 *grms.* H_2O *at* 19·5° *C.*

(*a*) Time: 2.00 p.m.

w.	*t.*	dw_t.	W.	R.	W_{H_2O}.	S.
0·91	19·5000	...	181·821300	5·9	181·314344	1·002796
1·01	·4975	·000065	·921235	17·0	·413622	798
1·11	·4950	130	182·021170	28·2	·514932	789
1·21	·4925	195	·121105	39·3	·613497	795
1·31	·4900	260	·221040	50·5	·715553	781
1·41	·4875	325	·320975	61·5	·814891	783
1·51	·4850	390	·420910	72·3	·911848	798
1·61	·4825	455	·520845	83·5	182·010671	800
1·71	·4800	520	·620780	94·9	·112200	792
					Mean	1·002793

(*b*) Time: 2.31 p.m.

w.	*t.*	dw_t.	W.	R.	W_{H_2O}.	S.
0·91	19·5000	...	181·821300	6·5	181·319701	1·002766
1·01	·5062	·000161	·921461	17·9	·421691	54
1·11	·5124	322	182·021622	29·0	·522019	52
1·21	·5186	484	·121783	40·0	·619795	64
1·31	·5248	645	·221944	51·1	·721003	61
1·41	·5310	806	·322105	62·1	·820278	60
1·51	·5372	967	·422266	73·1	·918880	67
1·61	·5434	·001128	·522427	84·2	182·016885	77
1·71	·5500	1300	·622588	95·5	·117518	73
					Mean	1·002764

(c) Time: 3.02 p.m.

w.	t.	dw_t.	W.	R.	W_{H_2O}.	S.
0·91	19·5000	0·000000	181·821300	6·2	181·317012	1·002782
1·01	,,	,,	·921300	17·5	·418105	73
1·11	,,	,,	182·021300	29·0	·522019	50
1·21	,,	,,	·121300	40·0	·619795	61
1·31	,,	,,	·221300	51·0	·720107	57
1·41	,,	,,	·321300	62·0	·819382	59
1·51	,,	,,	·421300	73·0	·917984	66
1·61	,,	,,	·521300	84·0	182·015092	81
1·71	,,	,,	·621300	95·2	·114828	81
					Mean	1·002768

(d) Time: 3.30 p.m.

w.	t.	dw_t.	W.	R.	W_{H_2O}.	S.
0·91	19·5200	·000520	181·821820	6·5	181·319701	1·002769
1·01	,,	,,	·921820	17·8	·420794	61
1·11	,,	,,	182·021820	29·0	·522019	53
1·21	,,	,,	·121820	40·0	·619795	64
1·31	,,	,,	·221820	51·0	·720107	59
1·41	,,	,,	·321820	62·0	·819382	63
1·51	,,	,,	·421820	73·0	·917984	66
1·61	,,	,,	·521820	84·2	182·016885	74
1·71	,,	,,	·621820	95·3	·115724	99
					Mean	1·002765

Dealing first with sub-table (*a*), we see that the temperature at the commencement was 19·5° C., and at the end of the series of observations 19·48°, giving a difference of 0·02° fall during the time that the observations were being made. As nine separate observations were made, we can assume that the temperature fell by equal amounts between each two observations, so that if we take as the first temperature 19·5°, and then successively subtract 0·0025° for each reading, the last will be taken at 19·48° C. As the temperature falls, the density of the solution rises, so that in order to compensate for the fall of temperature, the weight, dw_t, corresponding to this departure of temperature from the standard must be subtracted from the total observed weight of the hydrometer. The values of dw_t are placed in the column headed dw_t. Under w are given the small weights which were placed on the stem of the hydrometer in order to immerse the instrument. Under W is given the total weight of the hydrometer, corrected for temperature, that causes it to float at the scale reading, R, at 19·5° C. In column W_{H_2O} is given the corresponding total weight when the hydrometer is immersed in distilled water of 19·5° C. The value found for the specific gravity for each added weight is given under S.

The values found for the specific gravity in this series are greater than those found in series (*b*), (*c*), or (*d*), but it will be noticed that the values agree very well *inter se*, the maximum difference, or amplitude, between any two values being only 19 in the sixth decimal place. For series *b*, *c*, and *d* the amplitudes are 25, 32, and 46 respectively; or if we take the difference between the highest and the lowest value of the specific gravity in any of the three series, the amplitude is only 49 in the sixth decimal place.

Attention should be called to the number of figures after the decimal place given in columns t, dw_t, W, and W_{H_2O}, which is greater than is consistent with the probability of experimental error, but they are inserted in order to facilitate the better understanding of the various steps in the process of correcting for temperature, and to avoid any ambiguity which might arise were the temperatures only given to two places of decimals, and the corresponding weights to only four decimal places. The effect of inclusion or exclusion of the last two decimal places in W and W_{H_2O} has no appreciable effect upon the value of the sixth decimal figure in the specific gravity.

Twenty-seven salts—KCl, RbCl, CsCl, NaCl; KBr, RbBr, CsBr; KI, RbI, CsI; $KClO_3$, $RbClO_3$, $CsClO_3$; $KBrO_3$, $RbBrO_3$, $CsBrO_3$; KIO_3, $RbIO_3$, $CsIO_3$; KNO_3, $RbNO_3$, $CsNO_3$, $LiNO_3$, $NaNO_3$, $Sr(NO_3)_2$, $Ba(NO_3)_2$, and $Pb(NO_3)_2$—have been dealt with in solutions varying in concentration from 1/2 to 1/1024 gram-molecule per 1000 grams of water, and at temperatures of 15°, 19·5°, 23°, and 26° C.

§ 21. *Statistics relating to the Range of Variation of Temperature during a Series of Observations.*—In the following table statistics of the variations of the temperature of the liquid, while a total of 1316 series of observations was made with hydrometers Nos. 17 and 21—namely, 837 with No. 17, and 479 with No. 21—are given. In 68 per cent. of the series made with No. 17 there was no sensible

A.		B.	C.		A.		B.	C.	
Number of Series of Observations made with Hydrometer No.		During which the Range of Variation of Temperature was	The Numbers under A expressed as Percentages.		Number of Series of Observations made with Hydrometer No.		During which the Range of Variation of Temperature was	The Numbers under A expressed as Percentages.	
17	21		17	21	17	21		17	21
		° C.					° C.		
568	265	0·00	68·0	55·2	2	0	0·14	0·2	0·0
48	51	0·01	5·7	10·6	3	1	0·15	0·4	0·2
68	54	0·02	8·1	11·3	1	1	0·16	0·12	0·2
19	23	0·03	2·3	5·0	1	1	0·17	0·12	0·2
12	16	0·04	1·4	3·3	0	1	0·18	0·00	0·2
34	18	0·05	4·0	3·8	0	1	0·19	0·00	0·2
7	8	0·06	0·8	1·7	6	2	0·20	0·71	0·4
1	3	0·07	0·12	0·6	1	0	0·21	0·12	0·0
13	10	0·08	1·6	2·1	1	0	0·24	0·12	0·0
4	1	0·09	0·5	0·2	0	1	0·28	0·00	0·2
44	18	0·10	5·2	3·8	1	0	0·30	0·12	0·0
1	0	0·11	0·12	0·0					
1	3	0·12	0·12	0·6					
1	1	0·13	0·12	0·2	Total, 837	479		100·00	100·00

Departure of the Mean Temperature from the Standard Temperature during a Series of Observations.

A.		B.	C.		A.		B.	C.	
Number of Series of Observations made with Hydrometer No.		During which the Departure of the Mean Temperature from the Standard Temperature was	The Numbers under A expressed as Percentages.		Number of Series of Observations made with Hydrometer No.		During which the Departure of the Mean Temperature from the Standard Temperature was	The Numbers under A expressed as Percentages.	
17	21	° C.	17	21	17	21	° C.	17	21
554	260	0·000	66·3	54·2	2	3	0·060	0·2	0·6
35	39	0·005	4·2	8·2	1	3	0·065	0·1	0·6
69	56	0·010	8·2	11·7	1	1	0·070	0·1	0·2
19	26	0·015	2·3	5·5	2	0	0·075	0·2	0·0
17	18	0·020	2·0	3·8	1	4	0·080	0·1	0·9
32	12	0·025	3·8	2·5	0	0	0·085	0·0	0·0
10	17	0·030	1·2	3·5	3	0	0·090	0·4	0·0
3	3	0·035	0·4	0·6	1	0	0·095	0·1	0·0
19	9	0·040	2·3	1·9	6	1	0·100	0·7	0·2
3	2	0·045	0·4	0·4	2	0	0·120	0·2	0·0
52	23	0·050	6·2	4·8					
5	2	0·055	0·6	0·4	Total, 837	479		100·0	100·0

variation of temperature, and the same was the case in 55·2 per cent. of the series made with No. 21. If we consider the series made with both hydrometers for which the variation of temperature was not greater than 0·05°, the percentages are almost identical, namely, 89·5 for No. 17 and 89·2 for No. 21.

The maximum departure of the mean temperature from the standard, during any single series of observations, was 0·12° C.; the mean departure was 0·0075° C.

The maximum range of temperature while a series of nine observations was being made was 0·30° C.; the mean range of temperature for the 1316 series of nine observations each was 0·018° C.

These statistics show that the efforts made to secure constancy of temperature were successful.

Section IV.—The Control of the Temperature of the Laboratory.

§ 22. A laboratory is an inhabited room, and in Northern Europe the temperature of such apartments lies generally between 12° and 20° C. Consequently we find that a large amount of specific gravity work has been done at 15° C. by Gerlach, at 19·5° C. by Kremers, and at 17·5° C. by others; while the calorimetric work by Julius Thomsen, extending over the last half-century, was all done at the temperature 18° C.

It is always possible to raise the temperature of a laboratory or a dwelling-room, but it is not easy to lower it below the atmospheric temperature outside of the house. The temperature of the atmosphere in our latitudes rises often above 20° C. in summer, and then a temperature as low as 19·5° C. cannot be maintained. Similarly, in cold winters it is difficult to keep even a fairly well-heated room constantly at as high a temperature as 19·5°.

We see then that, although the temperature 19·5° C. may be very suitable and be very easily maintained during the greater part of the year, this may not be the case under the extreme conditions of mid-winter and mid-summer. It is therefore necessary, besides the particular temperature selected as the basis of the mean temperature of the locality, to have a higher temperature to meet the case of a warm summer, and a lower temperature to meet that of a cold winter. When the heat outside is such that the temperature of the air of the laboratory naturally rises above 19·5°, I use 23° C. as the particular temperature at which all my observations are made. On the rare occasions when this temperature is too low, I use 26° C., and this is a very useful temperature in tropical countries. At sea, even in equatorial regions, the highest particular temperature that would be required is 30° C. In winter the temperature of a laboratory or other inhabited room should never fall below 12° C. In these circumstances 15° C. has been adopted as the particular temperature.

The room used as laboratory should be of moderate dimensions, rather small than large, because it must be occupied only by the experimenter, and he must have absolute control over it. It should be illuminated by sky light, and the direct rays of the sun must be absolutely excluded. In our latitudes this means that the window must have a northern exposure. The room should be furnished with central heating, preferably by hot *water*.

When the particular temperature required for the liquid, the specific gravity of which is to be determined, is 19·5° C., the temperature of the air should be maintained at 19·0° to 19·3°, but this depends on the room. When the temperature of the liquid has been brought exactly to 19·5°, it will remain constant at this temperature when the temperature of the air in the neighbourhood of the cylinder is at 19·2° or 19·3°, because the heat which is removed from it by convection is equal to that supplied to it by radiation, principally from the experimenter himself.

The constant temperature of the liquid in the cylinder is the integral effect of a number of separate elements, of which the principal are the temperature of the air within and without the room and that of the experimenter. In mild weather, when the temperature of the air outside is about 14° to 17° C., it is generally very easy to regulate the temperature of the air in the laboratory so that that of the liquid in the cylinder may remain constantly at 19·5° for the duration of the experiment. In cold weather, however, when the temperature of the air outside may be 8° or 10° or even more below the temperature of the room, the liquid is apt to experience sensible cooling by radiation to the outside through the glass of the window. (Of course the window must always be shut when experiments are being made, because the slightest draught striking the exposed stem of the hydrometer disturbs the reading as well as the temperature.) In these circumstances it is often difficult to maintain its temperature at 15° for the duration of the experiment.

§ 23. It was by an accident, which for the moment was annoying, that I found a very simple and efficacious means of counteracting this loss of heat on the part of the liquid.

At Edinburgh, in December 1902, I was making some observations on the specific gravity of various saline solutions, and had got the temperature of the laboratory so that I had no difficulty in carrying out the determinations without change of temperature of the liquid. In latitude 56° in December, even in clear weather, night begins to fall early in the afternoon, and, in the middle of the work, I had to light the gas in order to be able to continue it. The gas jet was fully a metre above the cylinder, and the light which it gave, though sufficient, was far from brilliant. When I had finished the series of the usual number, nine individual observations, and removed the hydrometer, I took the temperature of the liquid, expecting to find that it had remained sensibly constant as before; but, instead of this, it showed a rise of several tenths of a degree. As the gas had been burning for only a few minutes, it was impossible for it, in the time, to raise the temperature of the air of the laboratory so that it, in its turn, could raise that of the liquid in the cylinder by any sensible amount; and, in fact, on regarding the thermometer used for indicating the temperature of the air, it was found that this had remained unchanged. For the moment I was perplexed; the shortness of the interval between the lighting of the gas jet and the production of the heat-effect on the liquid in the cylinder situated at a considerable distance below it puzzled me. However, after some reflection *I perceived that it must be an effect of radiation from the luminous flame of the gas jet, which, as soon as it was lighted, dispatched its heat-rays in all directions and with the velocity of light.* It was this radiation that penetrated the cylinder and raised the temperature of the liquid in so short a time; and this effect could be produced by no other agency.

Further reflection showed me that the agency which can raise the temperature of the liquid in this way can also prevent it falling. I gave effect to this idea by attaching a luminous gas lamp by an india-rubber tube to one of the gas cocks on the working bench of the laboratory, and placed it between the cylinder and the window. The gas burner was at a height above the table inferior to that of the top of the cylinder, and the standard which carried it could be shifted towards or away from the cylinder at will.

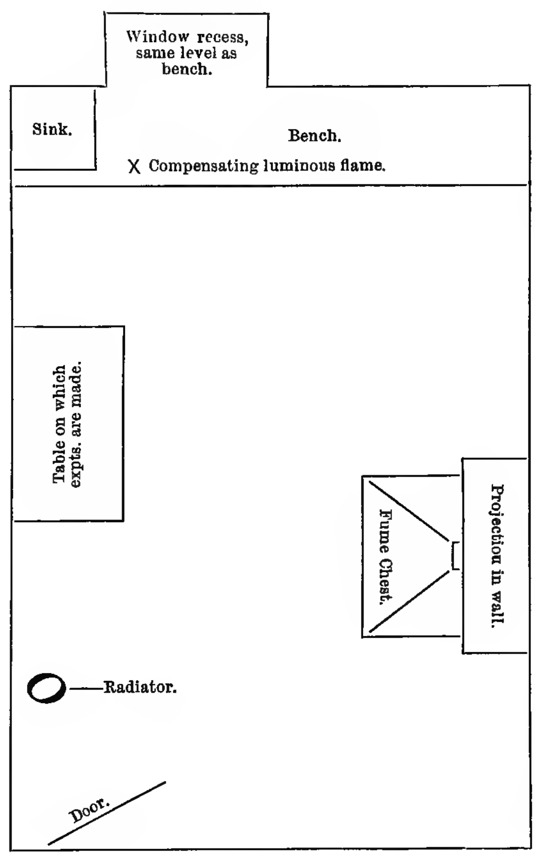
It must be remembered that the provision of this gas flame was not for the purpose of heating the air and raising or maintaining its temperature up to a certain degree; it was to supply the cylinder and liquid with heat, and that without warming the intervening air; and the heat so communicated from a distance was to be regulated so as exactly to make good that which they were dissipating.

As the combustion of the gas necessarily heated air which went upwards, it was important to secure the supply of radiant heat with the least possible combustion of gas. This was effected by reducing the flame so as to give the smallest possible flame which was sufficiently luminous to furnish the necessary radiation at a distance which was found to be convenient, generally from 50 to 75 centimetres.

§ 24. The room in which the work is carried out is 16 × 8 by 10 feet high, the 16-feet length being almost due north and south. The room is lighted by a fairly large window on the north wall, and there is a door in the south wall. There is a small fume chest projecting half way along the east wall, closed in by a casement window.

Against the west wall is a strong, well-made table about 48 × 27 inches, whereat all the observations are made.

The room is fitted with gas and electric light, and a bench runs the whole width of the room under the window, and is fitted with gas fittings. The room is warmed by a steam radiator. The following plan gives an idea of the general disposition of the room :—



For a typical working day the 6th December 1911 has been selected.

On arriving at the laboratory about 10.0 A.M. the room temperature was 17·6° C., although the radiator was working at full pressure.

The meteorological conditions, such as pressure and relative humidity, were noted and recorded for the purpose of reducing weights "to vacuo" where solutions were to be prepared.

The temperature of the air by this time was about 18·5° C., and the day was cold. A bunsen was lit in the fume chest, which is at the back of the experimenter when seated at the table making observations. The door of the fume chest being closed, the heat from the bunsen is very evenly distributed into the room.

The two hydrometers which were used in the experiments were taken from their cases, were each immersed in a small cylinder containing distilled water at about 19·6° C., and left in to attain a temperature about 19·50° C., a temperature at which the specific gravity observations were to be made.

The solution used on this occasion was a $\frac{1}{16}$ gram-molecule solution of the potassium chloride and iodide mixed in equimolecular proportions, the molecular weight assigned being the mean between the molecular weight of potassium chloride and that of potassium iodide. The weight of salt represented by $\frac{1}{16}$ gram-molecule was dissolved in 1000 grams of water and was prepared overnight.

The bottle of solution had been standing near the radiator for some time to attain a temperature near to 19·50° C.

The solution was now poured into the cylinder used for the experiments, and the quantity was such that when the largest hydrometer was immersed to its fullest extent the surface of the solution was fully an inch below the rim of the cylinder, a precaution which obviates difficulties in reading likely to be occasioned by irregularities in the glass occurring near the top.

The cylinder containing the solution was then placed on the table at a convenient altitude for making observations (the foot of the cylinder resting on a thickness of sixteen folds of soft German filter paper to form a non-conducting surface), and the temperature, as observed with a standard thermometer divided into $\frac{1}{10}$ths inch, each division being of such a size as to enable one to read $\frac{1}{100}$ths of a degree in temperature with comparative ease, was 18·70° C., the air temperature by this time being 19·0° C.

An expeditious and effective method for rapidly raising the temperature of the cylinder and contents, by stroking the side of the cylinder with the palms of the hands, was adopted, and by this means the temperature was quickly raised to 19·50° C. exactly.

The time was then 10.50 a.m., and the room temperature 19·1 C., so the bunsen was lowered somewhat, and the radiator turned to half way.

On removing the hydrometers from their respective cylinders and drying them, the temperature of the water in which they had been immersed was 19·35° C. in both cases, so that the hydrometers were presumably at that temperature.

The temperature of the solution was still at 19·50° C., and the air temperature 19·20° C., so that the conditions were suitable for commencing observations.

After removing the thermometer from the solution and immersing it in one of the cylinders of distilled water, the hydrometer No. 17 was taken from its case and gently lowered into the solution, and an initial added weight placed on the top of the stem of the hydrometer, the time of commencement of the experiment being noted.

Nine successive readings, as the results of addition of nine weights, eight of them having the value of 0·1 gram each, were taken, and after the ninth observation the thermometer was taken out of the distilled water and dried; the hydrometer was removed from the experimental solution and put into the cylinder containing the distilled water, and the temperature of the experimental solution observed by immersing the thermometer and gently stirring the solution.

The hydrometer was then dried and replaced in its case.

To complete the experimental data, the air temperature and time of completion of the experiment were noted.

The following is the record of the experiment:—

$\frac{1}{16}$ gram-molecule solution $K\frac{Cl+I}{2}$.

Hydrometer 17.

Air temperature . . = 19·20° C.
Initial solution temperature = 19·50° C.
Time . . . . = 11.3 a.m.

Added Weight.	Reading.
1·32 grams.	5·2 millimetres.
1·42 ,,	16·3 ,,
1·52 ,,	27·5 ,,
1·62 ,,	38·4 ,,
1·72 ,,	49·4 ,,
1·82 ,,	59·2 ,,
1·92 ,,	70·1 ,,
2·02 ,,	82·1 ,,
2·12 ,,	93·0 ,,

Mean added weight . = 1·72 gram.
Mean reading . . = 49·02 millimetres.
Air temperature . = 19·30° C.
Final solution temperature = 19·50° C.
Time . . = 11.16 a.m.

It will be seen that the air temperature rose 0·1° C. during the experiment, which occupied 13 minutes, and the solution temperature remained constant; so the bunsen in the fume chest was lowered to a further extent, and then preparations for the next experiment, conducted in precisely the same manner with hydrometer No. 3, were made.

As this section deals only with temperature conditions, the following table has been drawn up to show the temperature conditions which prevailed during the experiments which were made on the $\frac{1}{16}$ and $\frac{1}{32}$ gram-molecule solution of the potassium salt of the mixed halides (chloride and iodide), these being the two solutions experimented upon during the day:—

Hydrometer used.	Number of Experiment.	Quantity of Salt in 1000 grams of Water, expressed in gram-molecules. m.	Initial Air Temperature.	Initial Solution Temperature.	Time of Commencement of Experiment.	Final Air Temperature.	Final Solution Temperature.	Time of Completion of Experiment.	Duration of Experiment in Minutes.
			° C.	° C.		° C.	° C.		
No. 17	1	$\frac{1}{16}$	19·20	19·50	11.3 a.m.	19·30	19·50	11.16 a.m.	13
„ 3	2	„	19·30	19·50	11.20 „	19·30	19·50	11.34 „	14
„ 17	3	„	19·30	19·50	11.40 „	19·30	19·50	11.54 „	14
„ 3	4	„	19·30	19·50	11.59 „	19·30	19·50	12.12 p.m.	13
„ 17	5	„	19·30	19·50	12.17 p.m.	19·30	19·50	12.31 „	14
„ 3	6	„	19·30	19·50	12.36 „	19·30	19·50	12.50 „	14
No. 17	7	$\frac{1}{32}$	19·30	19·50	1.45 p.m.	19·30	19·50	1.57 p.m.	12
„ 3	8	„	19·30	19·50	2.5 „	19·30	19·50	2.17 „	12
„ 17	9	„	19·30	19·50	2.22 „	19·30	19·50	2.34 „	12
„ 3	10	„	19·35	19·50	2.45 „	19·35	19·50	2.59 „	14
„ 17	11	„	19·30	19·50	3.4 „	19·35	19·50	3.16 „	12
„ 3	12	„	19·30	19·50	3.22 „	19·30	19·50	3.35 „	13
„ 17	13	„	19·30	19·50	3.42 „	19·30	19·50	3.56 „	14

It will be seen that the initial and final solution temperatures were constant to within 0·01° C. throughout the series of experiments. There were slight variations in the air temperature of the room, the widest range being from 19·20° C. to 19·35° C. The rise was occasioned by turning on the radiator full for a few minutes and opening the door for fresh air, but no change occurred in solution temperature, so that latitude can be given in the range of air temperatures; but from experience it is not advisable to go below 19·2° C. unless direct radiation can be supplied to the solution, as shown in the earlier part of this section, the source of which can be effectively controlled.

In the conditions which obtain in this laboratory, it is possible to conduct a series of experiments extending over the day and to maintain the temperature of each solution constant for at least fourteen minutes if the temperature of the air is kept 0·3° lower than that of the solution.

§ 25. While the conditions which have been described are all essential for complete success in hydrometric work from the point of view of constant temperature, it may, and does occasionally, happen that, even after adopting all the precautions mentioned above, a series of observations will be taken, and then the solution temperature will be found to have changed, and with this change there has been a deviation in the value of specific gravity, certainly in the most extreme case amounting to only a few units in the 5th decimal place; but the deviation coupled with the temperature change has, in the most recent work, justified its elimination from the remaining series of observations which are perfect, in that the results of the other series agree *inter se* and no change in solution temperature has occurred.

An example of such an occurrence happened on 4th April 1911, when a series of hydrometric observations was made upon a solution containing $\frac{7}{64}$ gram-molecule NaCl in 1000 grams water. It was the first series of the day, and although the initial and final air temperatures for this experiment were both 19·30° C., the solution temperature changed from 19·50° C. at the commencement of the experiment to 19·41° C. when the observations were completed. The value of the specific gravity as calculated from this experiment was 1·004542. This value was not included in the accepted results, and the second series of experiments with the same hydrometer, where no change of temperature occurred in the solution during the experiment, gave a specific gravity value of 1·004570, while the mean of the whole series was 1·004579.

This variation in the solution temperature could only be due to the temperature of the hydrometer itself being considerably lower than that of the solution, and this factor operates in the twofold manner of lowering the solution temperature, and by virtue of the fact that there is a contraction in the volume of the hydrometer at a lower temperature, the added weight to sink it to a given scale division is less than at the higher temperature, so that the specific gravity value is lower than that obtained when the hydrometer is at the standard temperature. This is indubitably the explanation of the change in the solution temperature during the experiment quoted above, and may not improbably account for the observation that the first reading of the day is sometimes not comparable with the later results obtained in observations made on the same solution.

It is not possible to directly ascertain the temperature of the hydrometer, and since it is necessary that it should be acclimatised to the experimental temperature, *the precaution of immersing the hydrometers in distilled water at the experimental temperature for some time before commencing hydrometric observation is important.* It ensures that the hydrometer shall be at the experimental temperature and that its volume shall be normal for the given temperature.

The water value of one of the hydrometers is 11 gram-degrees centigrade. It is possible to calculate the temperature of the hydrometer which would reduce the solution temperature from 19·50° C. to 19·41° C. in the instance mentioned above, assuming no loss of heat due to radiation (this loss is negligible in any case under the conditions of experiment). The weight of the water in solution is about 600 grams (specific heat = 1). Hence, applying the principle of the determination of specific heat by the method of mixtures, the temperature of the hydrometer to produce this must be 14·59° C.

The effect of immersing the hydrometer at that temperature was to make the earlier readings lower than they would have been if the hydrometer were at normal temperature; but the hydrometer is expanding, because it is in a warmer medium, so that the later readings in the same series of observations would approach the values that would have been obtained if the hydrometer had been at normal temperature.

It is not difficult to show that the hydrometer was at this temperature (14·59° C.) at the beginning of the experiment, since we have its coefficient of expansion, namely, 0·003 c.c. per degree difference of temperature.

If we assume a mean hydrometer temperature of 17·00° C.—which is the mean between the initial and final hydrometer temperatures—then the volume of solution not displaced on account of the shrinkage in bulk of the hydrometer due to lower temperature = $0·003 \times 2·5 = 0·0075$ c.c., and its weight is 0·00753 gram (specific gravity of solution being 1·004542).

Hence, in correcting this value of specific gravity, the weight of solution displaced by the hydrometer is 182·53030 grams + 0·00753 gram = 182·53783 grams. The weight of distilled water displaced = 181·70496 grams. The specific gravity is therefore 1·004583, which closely agrees with the mean specific gravity of 1·004579.

The difference of scale reading occasioned by this difference due to temperature, and which is equivalent to an added weight of 0·00753 gram, is 0·82 mm., and is quite an appreciable quantity when it affects all the readings in a series to this extent.

The importance, therefore, of the precaution for commencing the series of experiments with the hydrometers at the standard temperature by the simple device of immersing them in water at, or a little above, the standard temperature for some time, and drying them before commencing the experiments, is at once apparent.

In the following section the experimental work is embodied in a number of tables. Full explanation is given for each class of tables. Supplemental work which was done during the preparation of this memoir is described and discussed in later sections.

[TABLES.

SECTION V.—TABLES.

§ 26. A. GENERAL TABLES, giving the Facts of Observation; namely, W, the weight, in grams, of the solution which contains *m* gram-molecules of the salt dissolved in 1000 grams of water; S, the specific gravity of this solution at the temperature, T, referred to that of distilled water at the same temperature as unity. From these data, Δ, the displacement of the solution, is obtained by the equation $\Delta = W/S$, and it is expressed in grams of water having the temperature T. The symbol adopted for this unit is G_T. The numbers printed in *italics* refer to specific gravities observed at the temperature printed in *italics*.

TRIAD OF CHLORIDES.

TABLE No. 1.

POTASSIUM CHLORIDE. KCl = 74·6.

T = 19·5° and *23·0°* C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/2	1037·3000	1·022977	1014·001		(3·0060385)*
1/4	1018·6500	1·011670	1006·899	7·102	0·0030522
1/8	1009·3250	1·005889	1003·416	3·483	0·0015052
1/16	1004·6625	1·002973	1001·684	1·732	0·0007500
1/32	1002·3312	1·001489	1000·841	0·843	0·0003658
1/64	1001·1656	1·000741	1000·423	0·418	0·0001811
1/128	1000·5828	1·000365	1000·217	0·206	0·0000895
1/256	1000·2914	1·000193	1000·098	0·119	0·0000517
1/512	1000·1457	1·000082	1000·064	0·034	0·0000149
1/16	*1004·6625*	*1·002924*	*1001·733*		

TABLE No. 2.

RUBIDIUM CHLORIDE. RbCl = 121·0.

T = 19·5° and *23·0°* C.

1/2	1060·5000	1·043144	1016·637		(3·0071662)
1/4	1030·2500	1·021868	1008·202	8·435	0·0036185
1/8	1015·1250	1·011023	1004·057	4·145	0·0017892
1/16	1007·5625	1·005531	1002·020	2·037	0·0008819
1/32	1003·7812	1·002772	1001·006	1·014	0·0004397
1/64	1001·8906	1·001400	1000·489	0·517	0·0002241
1/128	1000·9453	1·000707	1000·238	0·251	0·0001090
1/256	1000·4726	1·000350	1000·122	0·116	0·0000502
1/512	1000·2363	1·000163	1000·073	0·049	0·0000215
1/16	*1007·5625*	*1·005485*	*1002·066*		

TABLE No. 3.

CÆSIUM CHLORIDE. CsCl = 168·5.

T = 19·5° and *23·0°* C.

1/2	1084·2500	1·062572	1020·401		(3·0087709)
1/4	1042·1250	1·031739	1010·066	10·335	0·0044198
1/8	1021·0625	1·015994	1004·989	5·077	0·0021887
1/16	1010·5312	1·008036	1002·475	2·514	0·0010875
1/32	1005·2656	1·004035	1001·225	1·250	0·0005417
1/64	1002·6328	1·002027	1000·604	0·621	0·0002694
1/128	1001·3164	1·001025	1000·291	0·313	0·0001362
1/256	1000·6582	1·000514	1000·144	0·147	0·0000636
1/512	1000·3291	1·000249	1000·079	0·065	0·0000280
1/16	*1010·5312*	*1·007954*	*1002·557*		

* The entry in brackets in the column *d* log Δ gives log Δ for the solution of highest concentration in the series. With it and the following values of *d* log Δ in the same column the values of log Δ for each solution in the series can be calculated.

A. General Tables, giving the Facts of Observation in the columns under *m*, W, and S.

TRIAD OF BROMIDES.

Table No. 4.
POTASSIUM BROMIDE. KBr = 119·1.
T = 19·5° and *23·0°* C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G^T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/2	1059·5500	1·041278	1017·547		(3·0075546)
1/4	1029·7750	1·020903	1008·690	8·857	0·0037967
1/8	1014·8875	1·010528	1004·314	4·376	0·0018882
1/16	1007·4437	1·005279	1002·153	2·161	0·0009346
1/32	1003·7218	1·002638	1001·081	1·072	0·0004648
1/64	1001·8609	1·001306	1000·554	0·527	0·0002287
1/128	1000·9304	1·000652	1000·278	0·276	0·0001195
1/256	1000·4652	1·000325	1000·139	0·139	0·0000602
1/512	1000·2326	1·000158	1000·074	0·065	0·0000283
1/16	*1007·4437*	*1·005306*	*1002·126*		

Table No. 5.
RUBIDIUM BROMIDE. RbBr = 165·5.
T = 19·5° C.

1/2	1082·7500	1·061247	1020·262		(3·0087116)
1/4	1041·3750	1·031081	1009·983	10·279	0·0043972
1/8	1020 6875	1·015669	1004·941	5·042	0·0021737
1/16	1010·3437	1·007868	1002·456	2·485	0·0010751
1/32	1005·1718	1·003945	1001·222	1·234	0·0005350
1/64	1002·5859	1·001957	1000·627	0·595	0·0002578
1/128	1001·2929	1·000984	1000·308	0·319	0·0001385
1/256	1000·6464	1·000457	1000·189	0·119	0·0000518
1/512	1000 3232	1·000233	1000·090	0·099	0·0000430
1/1024	1000·1616	1·000079	1000·082	0·008	0·0000033

Table No. 6.
CÆSIUM BROMIDE. CsBr = 213·0.
T = 19·5° C.

1/2	1106·5000	1·080935	1023·650		(3·0101517)
1/4	1053·2500	1·041011	1011·756	11·894	0·0050756
1/8	1026·6250	1·020702	1005·802	5·954	0·0025632
1/16	1013·3125	1·010409	1002·873	2·929	0·0012666
1/32	1006·6562	1·005182	1001·466	1·407	0·0006097
1/64	1003·3281	1·002631	1000·695	0·771	0·0003346
1/128	1001·6640	1·001270	1000·393	0·302	0·0001309
1/256	1000·8320	1·000607	1000·224	0·169	0·0000732
1/512	1000·4160	1·000308	1000·108	0·116	0·0000507
1/1024	1000·2080	1·000145	1000·063	0·045	0·0000195

A. General Tables, giving the Facts of Observation in the columns under *m*, W, and S.

TRIAD OF IODIDES.

Table No. 7.

POTASSIUM IODIDE. KI = 166·1.

T = 19·5° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1	1166·1000	1·114617	1046·189		(3·0196102)
3/4	1124·5750	1·087124	1034·449	11·740	0·0049008
1/2	1083·0500	1·058929	1022·778	11·671	0·0049276
1/4	1041·5250	1·029906	1011·281	11·497	0·0049095
1/8	1020·7625	1·015104	1005·574	5·707	0·0024579
1/16	1010·3812	1·007588	1002·772	2·802	0·0012119
1/32	1005·1906	1·003790	1001·395	1·377	0·0005967
1/64	1002·5953	1·001899	1000·695	0·700	0·0003038
1/128	1001·2976	1·000950	1000·347	0·348	0·0001509
1/256	1000·6488	1·000480	1000·168	0·179	0·0000775
1/512	1000·3244	1·000235	1000·089	0·079	0·0000345
1/1024	1000·1622	1·000122	1000·040	0·049	0·0000212

Table No. 8.

RUBIDIUM IODIDE. RbI = 212·5.

T = 19·5° C.

1/2	1106·2500	1·078421	1025·805		(3·0110649)
1/4	1053·1250	1·039778	1012·836	12·969	0·0055256
1/8	1026·5625	1·020010	1006·424	6·412	0·0027583
1/16	1013·2812	1·010046	1003·203	3·221	0·0013921
1/32	1006·6406	1·005030	1001·602	1·601	0·0006934
1/64	1003·3203	1·002505	1000·813	0·789	0·0003423
1/128	1001·6601	1·001237	1000·422	0·391	0·0001695
1/256	1000·8300	1·000612	1000·218	0·204	0·0000888
1/512	1000·4150	1·000272	1000·143	0·075	0 0000325
1/1024	1000·2075	1·000146	1000·061	0·082	0·0000353

Table No. 9.

CÆSIUM IODIDE. CsI = 260·0.

T = 19·5° C.

1/2	1130·0000	1·097427	1029·681		(3·0127028)
1/4	1065·0000	1·049480	1014·788	14·893	0·0063273
1/8	1032·5000	1·024973	1007·343	7·445	0·0031978
1/16	1016·2500	1·012529	1003·675	3·668	0·0015845
1/32	1008·1250	1·006299	1001·814	1·861	0·0008057
1/64	1004·0625	1·003120	1000·939	0·875	0·0003795
1/128	1002·0312	1·001546	1000·484	0·455	0·0001975
1/256	1001·0156	1·000738	1000·277	0·207	0·0000898
1/512	1000·5078	1·000272	1000·235	0·042	0·0000181
1/1024	1000·2539	1·000100	1000·153	0·082	0·0000355

A. General Tables, giving the Facts of Observation in the columns under m, W, and S.

TRIAD OF IODIDES.

Table No. 10.

POTASSIUM IODIDE. KI = 166·1.

T = 23·0° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/2	1083·0500	1·058639	1023·058	-	(3·0099005)
1/4	1041·5250	1·029717	1011·467	11·591	0·0049486
1/8	1020·7625	1·014990	1005·687	5·780	0·0024888
1/16	1010·3812	1·007544	1002·816	2·871	0·0012416
1/32	1005·1906	1·003761	1001·424	1·392	0·0006034
1/64	1002·5953	1·001919	1000·675	0·749	0·0003249
1/128	1001·2976	1·000950	1000·347	0·328	0·0001420
1/256	1000·6488	1·000497	1000·152	0·195	0·0000849

Table No. 11.

RUBIDIUM IODIDE. RbI = 212·5.

T = 23·0° C.

1/8	1026·5625	1·020075	1006·360		(3·0027534)
1/16	1013·2812	1·010092	1003·157	3·203	0·0013943
1/32	1006·6406	1·005043	1001·589	1·568	0·0006793
1/64	1003·3203	1·002555	1000·763	0·826	0·0003582
1/128	1001·6601	1·001277	1000·382	0·381	0·0001653
1/256	1000·8300	1·000653	1000·176	0·206	0·0000893

Table No. 12.

CÆSIUM IODIDE. CsI = 260·0.

T = 23·0° and *26·0°* C.

1/8	1032·5000	1·025081	1007·237		(3·0031318)
1/16	1016·2500	1·012637	1003·608	3·629	0·0015675
1/32	1008·1250	1·006341	1001·772	1·836	0·0007950
1/64	1004·0625	1·003163	1000·896	0·876	0·0003799
1/128	1002·0312	1·001596	1000·434	0·462	0·0002005
1/256	1001·0156	1·000814	1000·201	0·233	0·0001010
1/16	*1016·2500*	*1·012623*	*1003·582*		

A. General Tables, giving the Facts of Observation in the columns under m, W, and S.

TRIAD OF NITRATES.

Table No. 13.

POTASSIUM NITRATE. $KNO_3 = 101 \cdot 1$.

T = 19·5° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. Gr.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	dΔ.	d log Δ.
1/2	1050·5500	1·030564	1019·410		(3·0083492)
1/4	1025·2750	1·015533	1009·593	9·817	0·0042028
1/8	1012·6375	1·007873	1004·727	4·866	0·0020983
1/16	1006·3187	1·003968	1002·342	2·385	0·0010319
1/32	1003·1593	1·002013	1001·144	1·198	0·0005194
1/64	1001·5796	1·001004	1000·575	0·569	0·0002463
1/128	1000·7898	1·000509	1000·281	0·294	0·0001283

Table No. 14.

RUBIDIUM NITRATE. $RbNO_3 = 147 \cdot 5$.

T = 19·5° C.

1/2	1073·7500	1·050634	1022·002		(3·0094517)
1/4	1036·8750	1·025698	1010·897	11·105	0·0047448
1/8	1018·4375	1·012973	1005·394	5·503	0·0023704
1/16	1009·2188	1·006597	1002·604	2·790	0·0012068
1/32	1004·6094	1·003355	1001·250	1·354	0·0005870
1/64	1002·3047	1·001750	1000·553	0·697	0·0003024
1/128	1001·1523	1·000920	1000·232	0·321	0·0001393
1/256	1000·5762	1·000458	1000·118	0·114	0·0000495

Table No. 15.

CÆSIUM NITRATE. $CsNO_3 = 195 \cdot 0$.

T = 19·5° C.

1/4	1048·7500	1·035619	1012·679		(3·0054720)
1/8	1024·3750	1·017961	1006·301	6·378	0·0027440
1/16	1012·1875	1·009041	1003·118	3·183	0·0013757
1/32	1006·0937	1·004585	1001·501	1·617	0·0007006
1/64	1003·0468	1·002247	1000·798	0·703	0·0003049
1/128	1001·5234	1·001146	1000·376	0·422	0·0001830
1/256	1000·7617	1·000604	1000·157	0·219	0·0000952

A. General Tables, giving the Facts of Observation in the columns under *m*, W, and S.

TRIAD OF CHLORATES.

Table No. 16.

POTASSIUM CHLORATE. $KClO_3 = 122·6$.

T = 19·5° and *23·0°* C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. Gr.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/4	1030·6500	1·019081	1011·352		(3·0049025)
1/8	1015·3250	1·009638	1005·632	5·720	0·0024631
1/16	1007·6625	1·004863	1002·785	2·847	0·0012311
1/32	1003·8312	1·002490	1001·337	1·448	0·0006276
1/64	1001·9156	1·001253	1000·661	0·676	0·0002933
1/128	1000·9578	1·000633	1000·324	0·337	0·0001463
1/256	1000·4789	1·000320	1000·158	0·166	0·0000719
1/512	1000·2394	1·000182	1000·057	0·101	0·0000440
1/16	*1007·6625*	*1·004759*	*1002·889*		

Table No. 17.

RUBIDIUM CHLORATE. $RbClO_3 = 169·0$.

T = 19·5° and *23·0°* C.

1/4	1042·2500	1·029153	1012·726		(3·0054919)
1/8	1021·1250	1·014679	1006·353	6·373	0·0027417
1/16	1010·5625	1·007356	1003·183	3·170	0·0013700
1/32	1005·2813	1·003691	1001·584	1·599	0·0006926
1/64	1002·6406	1·001863	1000·775	0·809	0·0003406
1/128	1001·3203	1·000919	1000·400	0·375	0·0001626
1/256	1000·6602	1·000459	1000·200	0·200	0·0000867
1/512	1000·3301	1·000218	1000·111	0·089	0·0000386
1/16	*1010·5625*	*1·007331*	*1003·204*		

Table No. 18.

CÆSIUM CHLORATE. $CsClO_3 = 216·5$.

T = 19·5° and *23·0°* C.

1/4	1054·1250	1·039043	1014·515		(3·0062587)
1/8	1027·0625	1·019686	1007·233	7·282	0·0031285
1/16	1013·5312	1·009825	1003·669	3·564	0·0015394
1/32	1006·7656	1·004953	1001·804	1·865	0·0008066
1/64	1003·3828	1·002409	1000·971	0·833	0·0003624
1/128	1001·6914	1·001216	1000·476	0·495	0·0002149
1/256	1000·8457	1·000552	1000·292	0·184	0·0000795
1/512	1000·4228	1·000210	1000·212	0·080	0·0000348
1/16	*1013·5312*	*1·009886*	*1003·609*		

A. General Tables, giving the Facts of Observation in the columns under *m*, W, and S.

TRIAD OF BROMATES.

Table No. 19.

POTASSIUM BROMATE. $KBrO_3 = 167{\cdot}1$.

T = 19·5° and *23·0°* C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/4	1041·7750	1·030144	1011·290		(3·0048760)
1/8	1020·8875	1·015227	1005·576	5·714	0·0024613
1/16	1010·4438	1·007662	1002·761	2·815	0·0012173
1/32	1005·2219	1·003846	1001·370	1·391	0·0006026
1/64	1002·6109	1·001921	1000·688	0·682	0·0002959
1/128	1001·3055	1·000958	1000·347	0·341	0·0001482
1/256	1000·6527	1·000476	1000·176	0·171	0·0000740
1/512	1000·3264	1·000237	1000·089	0·087	0·0000379
1/16	*1010·4438*	*1·007568*	*1002·854*		

Table No. 20.

RUBIDIUM BROMATE. $RbBrO_3 = 213{\cdot}5$.

T = 19·5° and *23·0°* C.

1/16	1013·3438	1·010255	1003·057		(3·0013258)
1/32	1006·6719	1·005123	1001·541	1·516	0·0006571
1/64	1003·3359	1·002566	1000·767	0·774	0·0003356
1/128	1001·6680	1·001260	1000·407	0·360	0·0001563
1/256	1000·8340	1·000642	1000·191	0·216	0·0000935
1/512	1000·4170	1·000320	1000·096	0·095	0·0000411
1/16	*1013·3438*	*1·010162*	*1003·149*		

Table No. 21.

CÆSIUM BROMATE. $CsBrO_3 = 261{\cdot}0$.

T = 19·5° and *23·0°* C.

1/16	1016·3125	1·012756	1003·511		(3·0015225)
1/32	1008·1562	1·006377	1001·767	1·744	0·0007554
1/64	1004·0781	1·003211	1000·864	0·903	0·0003918
1/128	1002·0390	1·001617	1000·421	0·443	0·0001920
1/256	1001·0195	1·000784	1000·235	0·186	0·0000810
1/512	1000·5097	1·000375	1000·134	0·101	0·0000437
1/16	*1016·3125*	*1·012759*	*1003·508*		

A. General Tables, giving the Facts of Observation in the columns under *m*, W, and S.

TRIAD OF IODATES.

Table No. 22.

POTASSIUM IODATE. $KIO_3 = 214{\cdot}1$.

T = 19·5° and *23·0°* C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/4	1053·5250	1·044302	1008·832		(3·0038187)
1/8	1026·7625	1·022327	1004·339	4·493	0·0019383
1/16	1013·3812	1·011169	1002·189	2·150	0·0009307
1/32	1006·6906	1·005589	1001·096	1·093	0·0004739
1/64	1003·3453	1·002760	1000·584	0·512	0·0002221
1/128	1001·6726	1·001403	1000·269	0·315	0·0001365
1/256	1000·8363	1·000709	1000·127	0·142	0·0000617
1/512	1000·4181	1·000361	1000·057	0·070	0·0000304
1/16	*1013·3812*	*1·011147*	*1002·210*		

Table No. 23.

RUBIDIUM IODATE. $RbIO_3 = 260{\cdot}5$.

T = 19·5° and *23·0°* C.

1/16	1016·2812	1·013677	1002·576		(3·0011173)
1/32	1008·1406	1·006856	1001·276	1·300	0·0005636
1/64	1004·0703	1·003405	1000·661	0·615	0·0002669
1/128	1002·0351	1·001690	1000·344	0·317	0·0001377
1/256	1001·0175	1·000827	1000·190	0·154	0·0000669
1/512	1000·5087	1·000436	1000·072	0·118	0·0000510
1/16	*1016·2812*	*1·013625*	*1002·618*		

Table No. 24.

CÆSIUM IODATE. $CsIO_3 = 308{\cdot}0$.

T = 19·5° and *23·0°* C.

1/16	1019·2500	1·016299	1002·903		(3·0012590)
1/32	1009·6250	1·008142	1001·471	1·432	0·0006383
1/64	1004·8125	1·004023	1000·786	0·685	0·0002970
1/128	1002·4062	1·001948	1000·457	0·329	0·0001426
1/256	1001·2031	1·000930	1000·272	0·185	0·0000802
1/512	1000·6015	1·000449	1000·152	0·120	0·0000524
1/16	*1019·2500*	*1·016226*	*1002·976*		

A. General Tables, giving the Facts of Observation in the columns under m, W, and S.

Table No 25.

POTASSIUM CHLORIDE. KCl = 74·6.

T = 15·0° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	$W/S = \Delta$.	$d\Delta$.	$d \log \Delta$.
1/2	1037·3000	1·023167	1013·813		(3·0059576)
1/4	1018·6500	1·011900	1006·671	7·142	0·0030702
1/8	1009·3250	1·005912	1003·393	3·278	0·0014163
1/16	1004·6625	1·002972	1001·685	1·708	0·0007399
1/32	1002·3312	1·001487	1000·842	0·843	0·0003653
1/64	1001·1656	1·000716	1000·449	0·393	0·0001708
1/128	1000·5828	1·000365	1000·218	0·231	0·0001003

Table No. 26.

SODIUM CHLORIDE. NaCl = 58·5.

T = 15·0° C.

1/2	1029·2500	1·020564	1008·510		(3·0036805)
1/4	1014·6250	1·010433	1004·148	4·362	0·0018825
1/8	1007·3125	1·005258	1002·043	2·105	0·0009114
1/16	1003·6562	1·002650	1001·003	1·040	0·0004508
1/32	1001·8281	1·001322	1000·505	0·498	0·0002162
1/64	1000·9140	1·000655	1000·259	0·246	0·0001069
1/128	1000·4570	1·000322	1000·135	0·124	0·0000538

Table No. 27.

RUBIDIUM BROMIDE. RbBr = 165·5.

T = 23·0° C.

1/8	1020·6875	1·015740	1004·870		(3·0021101)
1/16	1010·3437	1·007895	1002·429	2·441	0·0010563
1/32	1005·1718	1·003993	1001·174	1·255	0·0005441
1/64	1002·5859	1·001968	1000·616	0·558	0·0002420
1/128	1001·2929	1·000986	1000·306	0·310	0·0001343

Table No. 28.

CÆSIUM BROMIDE. CsBr = 213·0.

T = 23·0° C.

1/8	1026·6250	1·020672	1005·832		(3·0025257)
1/16	1013·3125	1·010386	1002·896	2·936	0·0012698
1/32	1006·6562	1·005246	1001·403	1·493	0·0006470
1/64	1003·3281	1·002634	1000·692	0·711	0·0003084
1/128	1001·6640	1·001332	1000·331	0·361	0·0001563

A. GENERAL TABLES, giving the Facts of Observation in the columns under *m*, W, and S.

TABLE No. 29.

LITHIUM NITRATE. $LiNO_3 = 69{\cdot}0$.

T = 19·5° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/2	1034·5000	1·019707	1014·507		(3·0062551)
1/4	1017·2500	1·010033	1007·145	7·362	0·0031631
1/8	1008·6250	1·005031	1003·565	3·580	0·0015418
1/16	1004·3125	1·002548	1001·759	1·806	0·0007865
1/32	1002·1562	1·001290	1000·864	0·895	0·0003882
1/64	1001·0781	1·000654	1000·423	0·441	0·0001914
1/128	1000·5390	1·000336	1000·203	0·220	0·0000957

TABLE No. 30.

SODIUM NITRATE. $NaNO_3 = 85{\cdot}0$.

T = 15·0° and *19·5°* C.

1/16	1005·3125	1·003453	1001·852		(3·0008039)
1/32	1002·6562	1·001715	1000·939	0·913	0·0003962
1/64	1001·3281	1·000863	1000·464	0·475	0·0002061
1/128	1000·6640	1·000431	1000·232	0·232	0·0001005
1/2	*1042·5000*	*1·027810*	*1014·292*		*(3·0061632)*
1/4	*1021·2500*	*1·014123*	*1007·027*	*7·265*	*0·0031217*
1/8	*1010·6250*	*1·007119*	*1003·480*	*3·547*	*0·0015325*
1/16	*1005·3125*	*1·003588*	*1001·718*	*1·762*	*0·0007635*
1/32	*1002·6562*	*1·001802*	*1000·852*	*0·866*	*0·0003752*
1/64	*1001·3281*	*1·000900*	*1000·427*	*0·425*	*0·0001847*

TABLE No. 31.

POTASSIUM NITRATE. $KNO_3 = 101{\cdot}1$.

T = 15·0° C.

1/2	1050·5500	1·030874	1019·087		(3·0082113)
1/4	1025·2750	1·015700	1009·427	9·660	0·0041364
1/8	1012·6375	1·007974	1004·623	4·804	0·0020717
1/16	1006·3187	1·003966	1002·343	2·280	0·0009865
1/32	1003·1593	1·001974	1001·183	1·160	0·0005032
1/64	1001·5796	1·000985	1000·594	0·589	0·0002554
1/128	1000·7898	1·000490	1000·299	0·295	0·0001277

A. General Tables, giving the Facts of Observation in the columns under *m*, W, and S.

Table No. 32.

RUBIDIUM NITRATE. $RbNO_3 = 147 \cdot 5$.

T = 23·0° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/4	1036·8750	1·025590	1011·003		(3·0047526)
1/8	1018·4375	1·013065	1005·303	5·700	0·0024556
1/16	1009·2188	1·006584	1002·617	2·686	0·0011618
1/32	1004·6094	1·003354	1001·251	1·366	0·0005922
1/64	1002·3047	1·001731	1000·573	0·678	0·0002941
1/128	1001·1523	1·000955	1000·197	0·376	0·0001631
1/256	1000·5762	1·000404	1000·172	0·025	0·0000109

Table No. 33.

CÆSIUM NITRATE. $CsNO_3 = 195 \cdot 0$.

T = 23·0° C.

m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/8	1024·3750	1·017943	1006·318		(3·0027353)
1/16	1012·1875	1·009035	1003·124	3·194	0·0013804
1/32	1006·0937	1·004536	1001·550	1·574	0·0006818
1/64	1003·0468	1·002288	1000·757	0·793	0·0003445
1/128	1001·5234	1·001186	1000·337	0·420	0·0001822
1/256	1000·7617	1·000580	1000·181	0·156	0·0000673

Table No. 34.

STRONTIUM NITRATE. $Sr(NO_3)_2 = 211 \cdot 6$.

T = 15·0° C.

m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/32	1006·6125	1·005344	1001·261		(3·0005477)
1/64	1003·3062	1·002673	1000·631	0·630	0·0002734
1/128	1001·6531	1·001351	1000·302	0·329	0·0001430
1/256	1000·8265	1·000666	1000·160	0·142	0·0000615
1/512	1000·4132	1·000329	1000·084	0·076	0·0000331

A. General Tables, giving the Facts of Observation in the columns under *m*, W, and S.

Table No. 35.

BARIUM NITRATE. $Ba(NO_3)_2 = 261{\cdot}0$.

$T = 15{\cdot}0°$ C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/32	1008·1562	1·006719	1001·427		(3·0006197)
1/64	1004·0781	1·003377	1000·699	0·728	0·0003164
1/128	1002·0390	1·001693	1000·345	0·354	0·0001535
1/256	1001·0195	1·000836	1000·183	0·152	0·0000700
1/512	1000·5097	1·000422	1000·088	0·095	0·0000416
1/1024	1000·2548	1·000218	1000·036	0·052	0·0000224

Table No. 36.

BARIUM NITRATE. $Ba(NO_3)_2 = 261{\cdot}0$.

$T = 19{\cdot}5°$ C.

1/16	1016·3125	1·013302	1002·971		(3·0012884)
1/32	1008·1562	1·006697	1001·449	1·522	0·0006592
1/64	1004·0781	1·003367	1000·708	0·741	0·0003216
1/128	1002·0390	1·001710	1000·328	0·380	0·0001651
1/256	1001·0195	1·000856	1000·163	0·165	0·0000716
1/512	1000·5097	1·000433	1000·076	0·087	0·0000376
1/1024	1000·2548	1·000205	1000·049	0·027	0·0000115

Table No. 37.

LEAD NITRATE. $Pb(NO_3)_2 = 331{\cdot}0$.

$T = 19{\cdot}5°$ C.

1/16	1020·6875	1·017788	1002·849		(3·0012356)
1/32	1010·3437	1·008947	1001·384	1·465	0·0006347
1/64	1005·1718	1·004504	1000·664	0·720	0·0003124
1/128	1002·5859	1·002250	1000·335	0·329	0·0001429
1/256	1001·2929	1·001128	1000·165	0·170	0·0000739
1/512	1000·6464	1·000577	1000·069	0·096	0·0000413
1/1024	1000·3232	1·000300	1000·023	0·046	0·0000203

§ 27. B. Tables giving particulars relating to the exactness of the determinations of the specific gravity given in Tables A, in cases where two hydrometers have been used.

In these tables we have, under S_{21}, the mean specific gravity derived from s_{21} series of observations made with Hydrometer No. 21; under S_{17}, the mean specific gravity derived from s_{17} series of observations with Hydrometer No. 17; under S_3, the mean specific gravity derived from s_3 series of observations with Hydrometer No. 3; and under S the mean of the sum, $\bar{s}$, of these series of observations. Under r_0 we have the probable error of $\bar{S}$ calculated by the method of least squares; and under d, the maximum departure of the mean of any individual series from $\bar{S}$. Numbers under r_0 and d represent units in the sixth decimal place.

Table No. 38.

POTASSIUM CHLORIDE. KCl = 74·6.

T = 19·5° and *23·0°* C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	*d.*
1/2	1·022986	4	1·022969	4	1·022977	8	2·9	18
1/4	1·011674	4	1·011665	4	1·011670	8	3·0	20
1/8	1·005895	4	1·005883	4	1·005889	8	2·1	16
1/16	1·002980	3	1·002967	3	1·002973	6	3·6	18
1/32	1·001494	3	1·001485	4	1·001489	7	2·2	14
1/64	1·000756	4	1·000730	4	1·000741	8	3·1	27
1/128	1·000368	3	1·000361	3	1·000365	6	1·5	10
1/256	1·000199	4	1·000188	4	1·000193	8	2·2	14
1/512	1·000088	4	1·000076	4	1·000082	8	2·2	14
1/16			*1·002924*	*3*			*0·3*	*1*

Table No. 39.

RUBIDIUM CHLORIDE. RbCl = 121·0.

T = 19·5° and *23·0°* C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	*d.*
1/2	1·043150	4	1·043138	4	1·043144	8	2·2	19
1/4	1·021875	4	1·021858	3	1·021868	7	2·7	13
1/8	1·011027	4	1·011019	4	1·011023	8	1·5	10
1/16	1·005530	4	1·005531	4	1·005531	8	2·5	15
1/32	1·002776	4	1·002767	3	1·002772	7	2·4	13
1/64	1·001398	4	1·001402	3	1·001400	7	2·5	13
1/128	1·000710	4	1·000705	4	1·000707	8	1·8	12
1/256	1·000349	3	1·000350	4	1·000350	7	1·6	12
1/512	1·000163	3	1·000163	3	1·000163	6	0·6	3
1/16			*1·005485*	*3*			*1·8*	*5*

Table No. 40.

CÆSIUM CHLORIDE. CsCl = 168·5.

T = 19·5° and *23·0°* C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	*d.*
1/2	1·062581	3	1·062566	4	1·062572	7	3·1	19
1/4	1·031742	4	1·031734	3	1·031739	7	1·6	7
1/8	1·015996	4	1·015991	3	1·015994	7	1·5	12
1/16	1·008044	3	1·008030	4	1·008036	7	3·2	20
1/32	1·004040	4	1·004029	3	1·004035	7	1·6	7
1/64	1·002032	4	1·002021	4	1·002027	8	2·0	14
1/128	1·001026	4	1·001024	3	1·001025	7	1·8	15
1/256	1·000517	4	1·000511	4	1·000514	8	1·6	13
1/512	1·000253	4	1·000244	3	1·000249	7	1·7	12
1/16			*1·007954*	*3*			*2·0*	*6*

B. Tables giving the Probable Error (r_0) of the Mean Specific Gravity ($\bar{S}$); and the greatest Departure (d) of the mean of any individual series from $\bar{S}$; both r_0 and d being expressed in units of the sixth decimal place.

Table No. 41.

POTASSIUM BROMIDE. KBr = 119·1.

T = 19·5° and *23·0°* C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/2	1·041284	4	1·041272	4	1·041278	8	2·8	17
1/4	1·020915	4	1·020890	4	1·020903	8	3·6	24
1/8	1·010534	4	1·010521	4	1·010528	8	2·4	18
1/16	1·005288	4	1·005271	4	1·005279	8	3·8	26
1/32	1·002649	3	1·002630	4	1·002638	7	2·7	13
1/64	1·001313	3	1·001301	4	1·001306	7	3·1	23
1/128	1·000650	3	1·000654	4	1·000652	7	1·6	11
1/256	1·000336	4	1·000314	4	1·000325	8	3·1	20
1/512	1·000163	2	1·000153	2	1·000158	4	6·6	28
1/16			*1·005306*	*4*			*2·0*	*9*

Table No. 42.*

RUBIDIUM BROMIDE. RbBr = 165·5.

T = 19·5° C.

m.	S_3.	s_3.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/2	1·061235	3	1·061259	3	1·061247	6	4·8	26
1/4	1·031081	3	1·031080	3	1·031081	6	1·3	6
1/8	1·015674	3	1·015665	4	1·015669	7	6·0	30
1/16	1·007865	3	1·007870	3	1·007868	6	1·0	6
1/32	1·003941	3	1·003949	3	1·003945	6	2·7	17
1/64	1·001949	3	1·001964	3	1·001957	6	3·1	16
1/128	1·000980	2	1·000988	2	1·000984	4	3·5	15
1/256	1·000478	2	1·000446	4	1·000457	6	5·1	22
1/512	1·000227	2	1·000238	3	1·000233	5	2·3	11
1/1024	1·000090	3	1·000070	4	1·000079	7	3·9	28

Table No. 43.

CÆSIUM BROMIDE. CsBr = 213·0.

T = 19·5° C.

m.	S_3.	s_3.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/2	1·080943	4	1·080923	3	1·080935	7	3·5	24
1/4	1·041024	3	1·040997	3	1·041011	6	5·5	17
1/8	1·020708	3	1·020695	3	1·020702	6	2·9	14
1/16	1·010407	3	1·010411	3	1·010409	6	2·0	10
1/32	1·005203	3	1·005160	3	1·005182	6	6·8	34
1/64	1·002638	3	1·002622	2	1·002631	5	3·3	16
1/128	1·001269	2	1·001271	4	1·001270	6	3·9	18
1/256	1·000595	3	1·000619	3	1·000607	6	3·7	28
1/512	1·000317	3	1·000296	3	1·000308	6	5·9	26
1/1024	1·000135	3	1·000155	3	1·000145	6	3·3	16

* In Tables Nos. 42, 43, 45, and 46 Hydrometer No. 3 has been used in place of No. 21.

B. TABLES giving the Probable Error (r_0) of the Mean Specific Gravity ($\bar{S}$); and the greatest Departure (d) of the mean of any individual series from $\bar{S}$; both r_0 and d being expressed in units of the sixth decimal place.

TABLE No. 44.

POTASSIUM IODIDE. KI=166·1.

T=19·5° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1	1·114619	1	1·114617	2	1·114617	3	3·0	8
3/4	1·087124	3	1·087124	3	1·087124	6	2·8	17
1/2	1·058929	2	1·058929	3	1·058929	5	2·2	10
1/4	1·029912	1	1·029904	2	1·029906	3	1·3	6
1/8	1·015104	2	1·015103	3	1·015104	5	5·3	22
1/16	1·007593	2	1·007583	2	1·007588	4	2·7	12
1/32	1·003794	2	1·003786	2	1·003790	4	2·2	9
1/64	1·001906	2	1·001893	2	1·001899	4	2·8	10
1/128	1·000951	2	1·000949	2	1·000950	4	2·6	10
1/256	1·000481	2	1·000479	2	1·000480	4	3·2	13
1/512	1·000237	3	1·000232	3	1·000235	6	2·0	13
1/1024	1·000132	2	1·000115	3	1·000122	5	3·5	14

TABLE No. 45.

RUBIDIUM IODIDE. RbI=212·5.

T=19·5° C.

m.	S_3.	s_3.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/2	1·078425	5	1·078416	5	1·078421	10	2·6	21
1/4	1·039781	3	1·039774	3	1·039778	6	4·6	25
1/8	1·020020	4	1·020009	4	1·020010	8	4·2	27
1/16	1·010058	3	1·010033	3	1·010046	6	4·2	23
1/32	1·005038	3	1·005032	3	1·005030	6	5·1	25
1/64	1·002503	2	1·002506	4	1·002505	6	4·1	23
1/128	1·001231	3	1·001240	4	1·001237	7	4·6	23
1/256	1·000602	3	1·000621	3	1·000612	6	4·3	23
1/512	1·000272	3	1·000269	3	1·000272	6	2·5	12
1/1024	1·000149	3	1·000139	3	1·000146	6	3·4	20

TABLE No. 46.

CÆSIUM IODIDE. CsI=260·0.

T=19·5° C.

1/2	1·097435	3	1·097420	3	1·097427	6	2·6	15
1/4	1·049485	3	1·049477	3	1·049480	6	4·0	19
1/8	1·024973	3	1·024973	4	1·024973	7	2·1	13
1/16	1·012527	3	1·012532	3	1·012529	6	2·2	11
1/32	1·006272	2	1·006307	3	1·006299	5	8·0	35
1/64	1·003109	3	1·003130	3	1·003120	6	3·7	21
1/128	1·001549	3	1·001543	3	1·001546	6	1·7	11
1/256	1·000733	2	1·000742	3	1·000738	5	2·8	13
1/512	1·000281	3	1·000263	3	1·000272	6	3·1	16
1/1024	1·000114	2	1·000093	4	1·000100	6	3·9	15

B. Tables giving the Probable Error (r_0) of the Mean Specific Gravity ($\bar{S}$); and the greatest Departure (d) of the mean of any individual series from $\bar{S}$; both r_0 and d being expressed in units of the sixth decimal place.

Table No. 47.

POTASSIUM IODIDE. KI = 166·1.

T = 23·0° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/2	1·058643	2	1·058636	3	1·058639	5	1·8	9
1/4	1·029724	3	1·029705	2	1·029717	5	3·4	14
1/8	1·014981	2	1·014999	2	1·014990	4	3·6	12
1/16	1·007543	3	1·007536	2	1·007544	5	4·2	18
1/32	1·003767	2	1·003754	2	1·003761	4	2·5	8
1/64	1·001919	2	1·001918	2	1·001919	4	2·9	12
1/128	1·000958	2	1·000945	4	1·000950	6	2·4	9
1/256	1·000498	2	1·000495	2	1·000497	4	3·4	14

Table No. 48.

RUBIDIUM IODIDE. RbI = 212·5.

T = 23·0° C.

1/8	1·020084	4	1·020055	2	1·020075	6	4·3	20
1/16	1·010091	2	1·010092	3	1·010092	5	0·8	3
1/32	1·005046	2	1·005040	3	1·005043	5	1·7	8
1/64	1·002565	3	1·002545	3	1·002555	6	3·6	17
1/128	1·001275	3	1·001279	3	1·001277	6	1·5	8
1/256	1·000670	3	1·000635	3	1·000653	6	4·5	24

Table No. 49.

CÆSIUM IODIDE. CsI = 260·0.

T = 23·0 and *26·0°* C.

1/8	1·025089	2	1·025075	3	1·025081	5	4·1	15
1/16	1·012644	7	1·012628	6	1·012637	13	2·6	23
1/32	1·006332	3	1·006355	2	1·006341	5	4·2	14
1/64	1·003165	3	1·003160	2	1·003163	5	1·3	7
1/128	1·001600	2	1·001593	2	1·001596	4	1·3	4
1/256	1·000826	2	1·000804	3	1·000814	5	4·4	13
1/16	*1·012635*	*4*	*1·012606*	*3*	*1·012623*	*7*	*4·1*	*25*

B. Tables giving the Probable Error (r_0) of the Mean Specific Gravity ($\overline{S}$); and the greatest Departure (d) of the mean of any individual series from $\overline{S}$; both r_0 and d being expressed in units of the sixth decimal place.

Table No. 50.

POTASSIUM CHLORIDE. KCl=74·6.

T=15·0° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\overline{S}$.	$\overline{s}$.	r_0.	*d*.
1/2	1·023170	4	1·023164	4	1·023167	8	2·0	20
1/4	1·011899*	3*	1·011900	3	1·011900	6	1·1	5
1/8	1·005912	4	1·005912	3	1·005912	7	1·7	14
1/16	1·002977	4	1·002967	4	1·002972	8	1·6	14
1/32	1·001493	4	1·001480	4	1·001487	8	2·1	13
1/64	1·000723	4	1·000706	3	1·000716	7	3·0	21
1/128	1·000365	4	1·000365	4	1·000365	8	2·0	10

Table No. 51.

SODIUM CHLORIDE. NaCl=58·5.

T=15·0° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\overline{S}$.	$\overline{s}$.	r_0.	*d*.
1/2	1·020576	4	1·020552	4	1·020564	8	3·9	25
1/4	1·010436	4	1·010429	3	1·010433	7	2·5	14
1/8	1·005265	3	1·005253	4	1·005258	7	2·4	19
1/16	1·002654	4	1·002645	4	1·002650	8	1·7	11
1/32	1·001331	4	1·001312	4	1·001322	8	2·6	17
1/64	1·000658	3	1·000652	4	1·000655	7	2·6	14
1/128	1·000324	4	1·000320	4	1·000322	8	0·7	6

Table No. 52.

RUBIDIUM BROMIDE. RbBr=165·5.

T=23·0° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\overline{S}$.	$\overline{s}$.	r_0.	*d*.
1/8	1·015751	2	1·015729	2	1·015740	4	4·2	11
1/16	1·007902	2	1·007888	2	1·007895	4	2·7	8
1/32	1·004002	2	1·003983	2	1·003993	4	3·7	10
1/64	1·001977	2	1·001962	3	1·001968	5	2·9	8
1/128	1·000972	2	1·000999	2	1·000986	4	5·2	14

Table No. 53.

CÆSIUM BROMIDE. CsBr=213·0.

T=23·0° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\overline{S}$.	$\overline{s}$.	r_0.	*d*.
1/8	1·020681	2	1·020662	2	1·020672	4	3·9	14
1/16	1·010395	2	1·010376	2	1·010386	4	3·7	10
1/32	1·005238	2	1·005251	3	1·005246	5	3·8	16
1/64	1·002636	3	1·002630	2	1·002634	5	1·9	11
1/128	1·001328	2	1·001335	3	1·001332	5	1·6	8

* These observations were made with Hydrometer No. 3.

B. TABLES giving the Probable Error (r_0) of the Mean Specific Gravity ($\bar{S}$); and the greatest Departure (d) of the mean of any individual series from $\bar{S}$; both r_0 and d being expressed in units of the sixth decimal place.

TABLE No. 54.

LITHIUM NITRATE. $LiNO_3 = 69·0$.

T = 19·5° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/2	1·019726	2	1·019688	2	1·019707	4	7·4	20
1/4	1·010026	2	1·010039	2	1·010033	4	2·5	7
1/8	1·005025	2	1·005036	3	1·005031	5	1·9	7
1/16	1·002535	2	1·002561	2	1·002548	4	5·1	13
1/32	1·001285	2	1·001294	2	1·001290	4	1·8	5
1/64	1·000649	2	1·000658	2	1·000654	4	1·8	5
1/128	1·000339	3	1·000332	2	1·000336	5	1·6	6

TABLE No. 55.

SODIUM NITRATE. $NaNO_3 = 85·0$.

T = 15·0° and *19·5°* C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/16	1·003455	4	1·003450	4	1·003453	8	2·1	17
1/32	1·001724	4	1·001706	4	1·001715	8	2·7	20
1/64	1·000866	4	1·000859	3	1·000863	7	2·4	12
1/128	1·000437	4	1·000425	4	1·000431	8	2·1	19
1/2	*1·027814*	*3*	*1·027806*	*4*	*1·027810*	*7*	*2·1*	*15*
1/4	*1·014130*	*4*	*1·014115*	*4*	*1·014123*	*8*	*3·1*	*19*
1/8	*1·007122*	*4*	*1·007117*	*4*	*1·007119*	*8*	*2·4*	*15*
1/16	*1·003593*	*4*	*1·003583*	*4*	*1·003588*	*8*	*1·9*	*16*
1/32	*1·001805*	*4*	*1·001798*	*4*	*1·001802*	*8*	*1·3*	*10*
1/64	*1·000907*	*3*	*1·000893*	*3*	*1·000900*	*6*	*2·6*	*15*

TABLE No. 56.

POTASSIUM NITRATE. $KNO_3 = 101·1$.

T = 19·5° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/2	1·030565	2	1·030562	2	1·030564	4	3·6	13
1/4	1·015533	2	1·015533	2	1·015533	4	3·7	13
1/8	1·007880	3	1·007866	3	1·007873	6	2·5	17
1/16	1·003978	6	1·003958	6	1·003968	12	3·1	22
1/32	1·002015	2	1·002010	2	1·002013	4	4·7	15
1/64	1·001011	3	1·000996	3	1·001004	6	2·5	11
1/128	1·000513	3	1·000503	2	1·000509	5	3·9	20

TABLE No. 57.

STRONTIUM NITRATE. $Sr(NO_3)_2 = 211·6$.

T = 15·0° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/32	1·005349	3	1·005338	3	1·005344	6	2·4	15
1/64	1·002675	4	1·002672	4	1·002673	8	1·4	9
1/128	1·001351	4	1·001350	4	1·001351	8	1·5	8
1/256	1·000675	4	1·000657	4	1·000666	8	2·8	16
1/512	1·000336	4	1·000323	4	1·000329	8	2·0	12

B. TABLES giving the Probable Error (r_0) of the Mean Specific Gravity ($\bar{S}$); and the greatest Departure (d) of the mean of any individual series from $\bar{S}$; both r_0 and d being expressed in units of the sixth decimal place.

TABLE No. 58.

BARIUM NITRATE. $Ba(NO_3)_2 = 261 \cdot 0$.

T = 15·0° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/32	1·006718	6	1·006719	6	1·006719	12	1·0	11
1/64	1·003381	4	1·003373	4	1·003377	8	1·8	10
1/128	1·001694	4	1·001692	4	1·001693	8	1·8	14
1/256	1·000844	4	1·000827	4	1·000836	8	3·1	17
1/512	1·000424	4	1·000419	4	1·000422	8	2·1	14
1/1024	1·000221	3	1·000217	4	1·000218	7	1·8	13

TABLE No. 59.

BARIUM NITRATE. $Ba(NO_3)_2 = 261 \cdot 0$.

T = 19·5° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/16	1·013304	6	1·013303	7	1·013302	13	1·4	15
1/32	1·006698	4	1·006695	4	1·006697	8	2·1	17
1/64	1·003370	4	1·003364	3	1·003367	7	3·0	23
1/128	1·001717	4	1·001703	3	1·001710	7	2·3	15
1/256	1·000856	8	1·000856	8	1·000856	16	1·3	17
1/512	1·000430	3	1·000435	4	1·000433	7	2·2	19
1/1024	1·000214	7	1·000196	7	1·000205	14	2·4	23

TABLE No. 60.

LEAD NITRATE. $Pb(NO_3)_2 = 331 \cdot 0$.

T = 19·5° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/16	1·017791	3	1·017784	3	1·017788	6	2·8	14
1/32	1·008948	3	1·008946	3	1·008947	6	0·2	13
1/64	1·004508	4	1·004502	3	1·004504	7	2·5	14
1/128	1·002262	3	1·002238	3	1·002250	6	6·4	40
1/256	1·001134	3	1·001123	4	1·001128	7	1·8	11
1/512	1·000589	4	1·000564	4	1·000577	8	3·8	24
1/1024	1·000305	4	1·000293	3	1·000300	7	2·1	13

TABLE No. 61.

POTASSIUM NITRATE. $KNO_3 = 101 \cdot 1$.

T = 15·0° C.

m.	S_{21}.	s_{21}.	S_{17}.	s_{17}.	$\bar{S}$.	$\bar{s}$.	r_0.	d.
1/2	1·030857	2	1·030891	2	1·030874	4	11·6	49
1/4	1·015701*	3*	1·015698	2	1·015700	5	1·5	7
1/8	1·007971*	3*	1·007977	3	1·007974	6	1·7	12
1/16	1·003970	4	1·003962	4	1·003966	8	1 6	13
1/32	1·001980	4	1·001968	4	1·001974	8	1·9	15
1/64	1·000989	4	1·000982	4	1·000985	8	1·6	12
1/128	1·000494	4	1·000487	4	1·000490	8	1·6	10

* These observations were made with Hydrometer No. 3.

§ 28. C. Tables giving a Summary of the Specific Gravities of the Solutions of different Salts at different Temperatures.

CHLORIDES, BROMIDES, AND IODIDES.

Table No. 62.

CHLORIDES. MCl.

M=	Na.	K.	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	15·0° C.		19·5° C.			23·0° C.		
m.	Specific Gravity.		Specific Gravity.			Specific Gravity.		
1/2	1·020564	1·023167	1·022977	1·043144	1·062572			
1/4	1·010433	1·011900	1·011670	1·021868	1·031739			
1/8	1·005258	1·005912	1·005889	1·011023	1·015994			
1/16	1·002650	1·002972	1·002973	1·005531	1·008036	1·002924	1·005485	1·007954
1/32	1·001322	1·001487	1·001489	1·002772	1·004035			
1/64	1·000655	1·000716	1·000741	1·001400	1·002027			
1/128	1·000322	1·000365	1·000365	1·000707	1·001025			
1/256			1·000193	1·000350	1·000514			
1/512			1·000082	1·000163	1·000249			

Table No. 63.

BROMIDES. MBr.

M=	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	19·5° C.			23·0° C.		
m.	Specific Gravity.			Specific Gravity.		
1/2	1·041278	1·061247	1·080935			
1/4	1·020903	1·031081	1·041011			
1/8	1·010528	1·015669	1·020702		1·015740	1·020672
1/16	1·005279	1·007868	1·010409	1·005306	1·007895	1·010386
1/32	1·002638	1·003945	1·005182		1·003993	1·005246
1/64	1·001306	1·001957	1·002631		1·001968	1·002634
1/128	1·000652	1·000984	1·001270		1·000986	1·001332
1/256	1·000325	1·000457	1·000607			
1/512	1·000158	1·000233	1·000308			
1/1024		1·000079	1·000145			

Table No. 64.

IODIDES. MI.

M=	K.	Rb.	Cs.	K.	Rb.	Cs.	Cs.
T=	19·5° C.			23·0° C.			26·0° C.
m.	Specific Gravity.			Specific Gravity.			Specific Gravity.
1/2	1·058929	1·078421	1·097427	1·058639			
1/4	1·029906	1·039778	1·049480	1·029717			
1/8	1·015104	1·020010	1·024973	1·014990	1·020075	1·025081	
1/16	1·007588	1·010046	1·012529	1·007544	1·010092	1·012637	1·012623
1/32	1·003790	1·005030	1·006299	1·003761	1·005043	1·006341	
1/64	1·001899	1·002505	1·003120	1·001919	1·002555	1·003163	
1/128	1·000950	1·001237	1·001546	1·000950	1·001277	1·001596	
1/256	1·000480	1·000612	1·000738	1·000497	1·000653	1·000814	
1/512	1·000235	1·000272	1·000272				
1/1024	1·000122	1·000146	1·000100				

C. Tables giving a Summary of the Specific Gravities of the Solutions of different Salts at different Temperatures.

Table No. 65.

NITRATES. $M'NO_3$ and $M''(NO_3)_2$.

M' or M'' =	Na.	K.	Sr''.	Ba''.	Li.	Na.	Ba''.	Pb''.	Rb.	Cs.
T =	15·0° C.				19·5° C.				23·0° C.	
m.	Specific Gravity.				Specific Gravity.				Specific Gravity.	
1/2		1·030874			1·019707	1·027810				
1/4		1·015700			1·010033	1·014123			1·025590	
1/8		1·007974			1·005031	1·007119			1·013065	1·017943
1/16	1·003453	1·003966			1·002548	1·003588	1·013302	1·017788	1·006584	1·009035
1/32	1·001715	1·001974	1·005344	1·006719	1·001290	1·001802	1·006697	1·008947	1·003354	1·004536
1/64	1·000863	1·000985	1·002673	1·003377	1·000654	1·000900	1·003367	1·004504	1·001731	1·002288
1/128	1·000431	1·000490	1·001351	1·001693	1·000336		1·001710	1·002250	1·000955	1·001186
1/256			1·000666	1·000836			1·000856	1·001128	1·000404	1·000580
1/512			1·000329	1·000422			1·000433	1·000577		
1/1024				1·000218			1·000205	1·000300		

Table No. 66.

TRIADS OF NITRATES, CHLORATES, BROMATES, AND IODATES. MRO_3.

RO_3 =	NO_3.			ClO_3.			BrO_3.			IO_3.		
M =	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
T =	19·5° C.			19·5° C. and *23·0°* C.			19·5° C. and *23·0°* C.			19·5° C. and *23·0°* C.		
m.	Specific Gravity.			Specific Gravity.			Specific Gravity.			Specific Gravity.		
1/2	1·030564	1·050634										
1/4	1·015533	1·025698	1·035619	1·019081	1·029153	1·039043	1·030144			1·044302		
1/8	1·007873	1·012973	1·017961	1·009638	1·014679	1·019686	1·015227			1·022327		
1/16	1·003968	1·006597	1·009041	1·004863	1·007356	1·009825	1·007662	1·010255	1·012756	1·011169	1·013677	1·016299
1/32	1·002013	1·003355	1·004585	1·002490	1·003691	1·004953	1·003846	1·005123	1·006377	1·005589	1·006856	1·008142
1/64	1·001004	1·001750	1·002247	1·001253	1·001863	1·002409	1·001921	1·002566	1·003211	1·002760	1·003405	1·004023
1/128	1·000509	1·000920	1·001146	1·000633	1·000919	1·001216	1·000958	1·001260	1·001617	1·001403	1·001690	1·001948
1/256		1·000458	1·000604	1·000320	1·000459	1·000552	1·000476	1·000642	1·000784	1·000709	1·000827	1·000930
1/512				1·000182	1·000218	1·000210	1·000237	1·000320	1·000375	1·000361	1·000436	1·000449
1/16				*1·004759*	*1·007331*	*1·009886*	*1·007568*	*1·010162*	*1·012759*	*1·011147*	*1·013625*	*1·016226*

C. Tables giving a Summary of the Specific Gravities of the Solutions of different Salts at different Temperatures.

POTASSIUM, RUBIDIUM, AND CÆSIUM SALTS.

Table No. 67.

POTASSIUM SALTS. KR and KRO_3.

R or RO_3=	Cl.	NO_3.	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	Cl.	Br.	I.	ClO_3.	BrO_3.	IO_3.
T=	15·0° C.		19·5° C.							23·0° C.					
m.	Specific Gravity.		Specific Gravity.							Specific Gravity.					
1/2	1·023167	1·030874	1·022977	1·041278	1·058929	1·030564						1·058639			
1/4	1·011900	1·015700	1·011670	1·020903	1·029906	1·015533	1·019081	1·030144	1·044302			1·029717			
1/8	1·005912	1·007974	1·005889	1·010528	1·015104	1·007873	1·009638	1·015227	1·022327			1·014990			
1/16	1·002972	1·003966	1·002973	1·005279	1·007588	1·003968	1·004863	1·007662	1·011169	1·002924	1·005306	1·007544	1·004759	1·007568	1·011147
1/32	1·001487	1·001974	1·001489	1·002638	1·003790	1·002013	1·002490	1·003846	1·005589			1·003761			
1/64	1·000716	1·000985	1·000741	1·001306	1·001899	1·001004	1·001253	1·001921	1·002760			1·001919			
1/128	1·000365	1·000490	1·000365	1·000652	1·000950	1·000509	1·000633	1·000958	1·001403			1·000950			
1/256			1·000193	1·000325	1·000480		1·000320	1·000476	1·000709			1·000497			
1/512			1·000082	1·000158	1·000235		1·000182	1·000237	1·000361						
1/1024					1·000122										

Table No. 68.

RUBIDIUM SALTS. RbR and $RbRO_3$.

R or RO_3=	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.
T=	19·5° C.							23·0° C.						
m.	Specific Gravity.							Specific Gravity.						
1/2	1·043144	1·061247	1·078421	1·050634										
1/4	1·021868	1·031081	1·039778	1·025698	1·029153						1·025590			
1/8	1·011023	1·015669	1·020010	1·012973	1·014679				1·015740	1·020075	1·013065			
1/16	1·005531	1·007868	1·010046	1·006597	1·007356	1·010255	1·013677	1·005485	1·007895	1·010092	1·006584	1·007331	1·010162	1·013625
1/32	1·002772	1·003945	1·005030	1·003355	1·003691	1·005123	1·006856		1·003993	1·005043	1·003354			
1/64	1·001400	1·001957	1·002505	1·001750	1·001863	1·002566	1·003405		1·001968	1·002555	1·001731			
1/128	1·000707	1·000984	1·001237	1·000920	1·000919	1·001260	1·001690		1·000986	1·001277	1·000955			
1/256	1·000350	1·000457	1·000612	1·000458	1·000459	1·000642	1·000827			1·000653	1·000404			
1/512	1·000163	1·000233	1·000272		1·000218	1·000320	1·000436							
1/1024		1·000079	1·000146											

Table No. 69.

CÆSIUM SALTS. CsR and $CsRO_3$.

R or RO_3=	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	I.
T=	19·5° C.							23·0° C.							26·0° C.
m.	Specific Gravity.							Specific Gravity.							Specific Gravity.
1/2	1·062572	1·080935	1·097427												
1/4	1·031739	1·041011	1·049480	1·035619	1·039043										
1/8	1·015994	1·020702	1·024973	1·017961	1·019686				1·020672	1·025081	1·017943				
1/16	1·008036	1·010409	1·012529	1·009041	1·009825	1·012756	1·016299	1·007954	1·010386	1·012637	1·009035	1·009886	1·012759	1·016226	1·012623
1/32	1·004035	1·005182	1·006299	1·004585	1·004953	1·006377	1·008142		1·005246	1·006341	1·004536				
1/64	1·002027	1·002631	1·003120	1·002247	1·002409	1·003211	1·004023		1·002634	1·003163	1·002288				
1/128	1·001025	1·001270	1·001546	1·001146	1·001216	1·001617	1·001948		1·001332	1·001596	1·001186				
1/256	1·000514	1·000607	1·000738	1·000604	1·000552	1·000784	1·000930			1·000814	1·000580				
1/512	1·000249	1·000308	1·000272		1·000210	1·000375	1·000449								
1/1024		1·000145	1·000100												

C. Tables giving a Summary of the Specific Gravities of the Solutions of different Salts at different Temperatures.

Table No. 70.

The Ennead, MR:—CHLORIDES, BROMIDES, AND IODIDES OF POTASSIUM, RUBIDIUM, AND CÆSIUM.

M=		K.	Rb.	Cs.	M=		K.	Rb.	Cs.
T=		19·5° C.			T=		19·5° C.		
m.	R.	Specific Gravity.			*m.*	R.	Specific Gravity.		
1/2	Cl	1·022977	1·043144	1·062572	1/64	Cl	1·000741	1·001400	1·002027
	Br	1·041278	1·061247	1·080935		Br	1·001306	1·001957	1·002631
	I	1·058929	1·078421	1·097427		I	1·001899	1·002505	1·003120
1/4	Cl	1·011670	1·021868	1·031739	1/128	Cl	1·000365	1·000707	1·001025
	Br	1·020903	1·031081	1·041011		Br	1·000652	1·000984	1·001270
	I	1·029906	1·039778	1·049480		I	1·000950	1·001237	1·001546
1/8	Cl	1·005889	1·011023	1·015994	1/256	Cl	1·000193	1·000350	1·000514
	Br	1·010528	1·015669	1·020702		Br	1·000325	1·000457	1·000607
	I	1·015104	1·020010	1·024973		I	1·000480	1·000612	1·000738
1/16	Cl	1·002973	1·005531	1·008036	1/512	Cl	1·000082	1·000163	1·000249
	Br	1·005279	1·007868	1·010409		Br	1·000158	1·000233	1·000308
	I	1·007588	1·010046	1·012529		I	1·000235	1·000272	1·000272
1/32	Cl	1·001489	1·002772	1·004035	1/1024	Cl			
	Br	1·002638	1·003945	1·005182		Br		1·000079	1·000145
	I	1·003790	1·005030	1·006299		I	1·000122	1·000146	1·000100

Table No. 71.

The Ennead, MRO_3:—CHLORATES, BROMATES, AND IODATES OF POTASSIUM, RUBIDIUM, AND CÆSIUM.

M=		K.	Rb.	Cs.	K.	Rb.	Cs.
T=		19·5° C.			23·0° C.		
m.	RO_3.	Specific Gravity.			Specific Gravity.		
1/4	ClO_3	1·019081	1·029153	1·039043			
	BrO_3	1·030144					
	IO_3	1·044302					
1/8	ClO_3	1·009638	1·014679	1·019686			
	BrO_3	1·015227					
	IO_3	1·022327					
1/16	ClO_3	1·004863	1·007356	1·009825	1·004759	1·007331	1·009886
	BrO_3	1·007662	1·010255	1·012756	1·007568	1·010162	1·012759
	IO_3	1·011169	1·013677	1·016299	1·011147	1·013625	1·016226
1/32	ClO_3	1·002490	1·003691	1·004953			
	BrO_3	1·003846	1·005123	1·006377			
	IO_3	1·005589	1·006856	1·008142			
1/64	ClO_3	1·001253	1·001863	1·002409			
	BrO_3	1·001921	1·002566	1·003211			
	IO_3	1·002760	1·003405	1·004023			
1/128	ClO_3	1·000633	1·000919	1·001216			
	BrO_3	1·000958	1·001260	1·001617			
	IO_3	1·001403	1·001690	1·001948			
1/256	ClO_3	1·000320	1·000459	1·000552			
	BrO_3	1·000476	1·000642	1·000784			
	IO_3	1·000709	1·000827	1·000930			
1/512	ClO_3	1·000182	1·000218	1·000210			
	BrO_3	1·000237	1·000320	1·000375			
	IO_3	1·000361	1·000436	1·000449			

§ 29. D. Tables giving a Summary of the Increments of Displacement, v, caused by the Dissolution of m grm.-mol. Salt in 1000 grams Water at different Temperatures. $v = \Delta - 1000$.

CHLORIDES, BROMIDES, AND IODIDES.

Table No. 72.

CHLORIDES. MCl.

M=	Na.	K.	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	15·0° C.		19·5° C.			23·0° C.		
m.	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*
1/2	8·510	13·813	14·001	16·637	20·401			
1/4	4·148	6·671	6·899	8·202	10·066			
1/8	2·043	3·393	3·416	4·057	4·989			
1/16	1·003	1·685	1·684	2·020	2·475	1·733	2·066	2·557
1/32	0·505	0·842	0·841	1·006	1·225			
1/64	0·259	0·449	0·423	0·489	0·604			
1/128	0·135	0·218	0·217	0·238	0·291			
1/256			0·098	0·122	0·144			
1/512			0·064	0·073	0·079			

Table No. 73.

BROMIDES. MBr.

M=	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	19·5° C.			23·0° C.		
m.	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*
1/2	17·547	20·262	23·650			
1/4	8·690	9·983	11·756			
1/8	4·314	4·941	5·802		4·870	5·832
1/16	2·153	2·456	2·873	2·126	2·429	2·896
1/32	1·081	1·222	1·466		1·174	1·403
1/64	0·554	0·627	0·695		0·616	0·692
1/128	0·278	0·308	0·393		0·306	0·331
1/256	0·139	0·189	0·224			
1/512	0·074	0·090	0·108			
1/1024		0·082	0·063			

Table No. 74.

IODIDES. MI.

M=	K.	Rb.	Cs.	K.	Rb.	Cs.	Cs.
T=	19·5° C.			23·0° C.			26·0° C.
m.	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*
1/2	22·778	25·805	29·681	23·058			
1/4	11·281	12·836	14·788	11·467			
1/8	5·574	6·424	7·343	5·687	6·360	7·237	
1/16	2·772	3·203	3·675	2·816	3·157	3·608	3·582
1/32	1·395	1·602	1·814	1·424	1·589	1·772	
1/64	0·695	0·813	0·939	0·675	0·763	0·896	
1/128	0·347	0·422	0·484	0·347	0·382	0·434	
1/256	0·168	0·218	0·277	0·152	0·176	0·201	
1/512	0·089	0·143	0·235				
1/1024	0·040	0·061	0·153				

D. TABLES giving a Summary of the Increments of Displacement, v, caused by the Dissolution of m grm.-mol. Salt in 1000 grams Water at different Temperatures. $v = \Delta - 1000$.

TABLE No. 75.

NITRATES. $M'NO_3$ and $M''(NO_3)_2$.

M′ or M″=	Na.	K.	Sr″	Ba″.	Li.	Na.	Ba″.	Pb″.	Rb.	Cs.
T=	15·0° C.				19·5° C.				23·0° C.	
m.	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*
1/2		19·087			14·507	14·292				
1/4		9·427			7·145	7·027			11·003	
1/8		4·623			3·565	3·480			5·303	6·318
1/16	1·852	2·343			1·759	1·718	2·971	2·849	2·617	3·124
1/32	0·939	1·183	1·261	1·427	0·864	0·852	1·449	1·384	1·251	1·550
1/64	0·464	0·594	0·631	0·699	0·423	0·427	0·708	0·664	0·573	0·757
1/128	0·232	0·299	0·302	0·345	0·203		0·328	0·335	0·197	0·337
1/256			0·160	0·183			0·163	0·165	0·172	0·181
1/512			0·084	0·088			0·076	0·069		
1/1024				0·036			0·049	0·023		

TABLE No. 76.

TRIADS OF NITRATES, CHLORATES, BROMATES, AND IODATES. MRO_3.

RO_3=	NO_3.			ClO_3.			BrO_3.			IO_3.		
M=	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	19·5° C.			19·5° C. and *23·0°* C.			19·5° C. and *23·0°* C.			19·5° C. and *23·0°* C.		
m.	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*
1/2	19·410	22·002										
1/4	9·593	10·897	12·679	11·352	12·726	14·515	11·290			8·832		
1/8	4·727	5·394	6·301	5·632	6·353	7·233	5·576			4·339		
1/16	2·342	2·604	3·118	2·785	3·183	3·669	2·761	3·057	3·511	2·189	2·576	2·903
1/32	1·144	1·250	1·501	1·337	1·584	1·804	1·370	1·541	1·767	1·096	1·276	1·471
1/64	0·575	0·553	0·798	0·661	0·775	0·971	0·688	0·767	0·864	0·584	0·661	0·786
1/128	0·281	0·232	0·376	0·324	0·400	0·476	0·347	0·407	0·421	0·269	0·344	0·457
1/256		0·118	0·157	0·158	0·200	0·292	0·176	0·191	0·235	0·127	0·190	0·272
1/512				0·057	0·111	0·212	0·089	0·096	0·134	0·057	0·072	0·152
1/16				*2·889*	*3·204*	*3·609*	*2·854*	*3·149*	*3·508*	*2·210*	*2·618*	*2·976*

D. Tables giving a Summary of the Increments of Displacement, v, caused by the Dissolution of m grm.-mol. Salt in 1000 grams Water at different Temperatures. $v = \Delta - 1000$.

POTASSIUM, RUBIDIUM, AND CÆSIUM SALTS.

Table No. 77.

POTASSIUM SALTS. KR and KRO_3.

R or RO_3=	Cl.	NO_3.	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	Cl.	Br.	I.	ClO_3.	BrO_3.	IO_3.
T=	15·0° C.		19·5° C.							23·0° C.					
m.	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*
1/2	13·813	19·087	14·001	17·547	22·778	19·410						23·058			
1/4	6·671	9·427	6·899	8·690	11·281	9·593	11·352	11·290	8·832			11·467			
1/8	3·393	4·623	3·416	4·314	5·574	4·727	5·632	5·576	4·339			5·687			
1/16	1·685	2·343	1·684	2·153	2·772	2·342	2·785	2·761	2·189	1·733	2·126	2·816	2·889	2·854	2·210
1/32	0·842	1·183	0·841	1·081	1·395	1·144	1·337	1·370	1·096			1·424			
1/64	0·449	0·594	0·423	0·554	0·695	0·575	0·661	0·688	0·584			0·675			
1/128	0·218	0·299	0·217	0·278	0·347	0·281	0·324	0·347	0·269			0·347			
1/256			0·098	0·139	0·168		0·158	0·176	0·127			0·152			
1/512			0·064	0·074	0·089		0·057	0·089	0·057						
1/1024					0·040										

Table No. 78.

RUBIDIUM SALTS. RbR and $RbRO_3$.

R or RO_3=	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.
T=	19·5° C.							23·0° C.						
m.	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*
1/2	16·637	20·262	25·805	22·002										
1/4	8·202	9·983	12·836	10·897	12·726						11·003			
1/8	4·057	4·941	6·424	5·394	6·353				4·870	6·360	5·303			
1/16	2·020	2·456	3·203	2·604	3·183	3·057	2·576	2·066	2·429	3·157	2·617	3·204	3·149	2·618
1/32	1·006	1·222	1·602	1·250	1·584	1·541	1·276		1·174	1·589	1·251			
1/64	0·489	0·627	0·813	0·553	0·775	0·767	0·661		0·616	0·763	0·573			
1/128	0·238	0·308	0·422	0·232	0·400	0·407	0·344		0·306	0·382	0·197			
1/256	0·122	0·189	0·218	0·118	0·200	0·191	0·190			0·176	0·172			
1/512	0·073	0·090	0·143		0·111	0·096	0·072							
1/1024		0·082	0·061											

Table No. 79.

CÆSIUM SALTS. CsR and $CsRO_3$.

R or RO_3=	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	CsI.
T=	19·5° C.							23·0° C.							26·0° C.
m.	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*	*v.*
1/2	20·401	23·650	29·681												
1/4	10·066	11·756	14·788	12·679	14·515										
1/8	4·989	5·802	7·343	6·301	7·233				5·832	7·237	6·318				
1/16	2·475	2·873	3·675	3·118	3·669	3·511	2·903	2·557	2·896	3·608	3·124	3·609	3·508	2·976	3·582
1/32	1·225	1·466	1·814	1·501	1·804	1·767	1·471		1·403	1·772	1·550				
1/64	0·604	0·695	0·939	0·798	0·971	0·864	0·786		0·692	0·896	0·757				
1/128	0·291	0·393	0·484	0·376	0·476	0·421	0·457		0·331	0·434	0·337				
1/256	0·144	0·224	0·277	0·157	0·292	0·235	0·272			0·201	0·181				
1/512	0·079	0·108	0·235		0·212	0·134	0·152								
1/1024		0·063	0·153												

D. TABLE giving a Summary of the Increments of Displacement, v, caused by the Dissolution of m grm.-mol. Salt in 1000 grams Water at different Temperatures. $v = \Delta - 1000$.

TABLE No. 80.

THE ENNEAD, MR :—CHLORIDES, BROMIDES, AND IODIDES OF POTASSIUM, RUBIDIUM, AND CÆSIUM.

M =		K.	Rb.	Cs.	K.	Rb.	Cs.
T =		19·5° C.			23·0° C.		
m.	R.	*v*.	*v*.	*v*.	*v*.	*v*.	*v*.
1/2	Cl	14·001	16·637	20·401			
	Br	17·547	20·262	23·650			
	I	22·778	25·805	29·681	23·058		
1/4	Cl	6·899	8·202	10·066			
	Br	8·690	9·983	11·756			
	I	11·281	12·836	14·788	11·467		
1/8	Cl	3·416	4·057	4·989			
	Br	4·314	4·941	5·802		4·870	5·832
	I	5·574	6·424	7·343	5·687	6·360	7·237
1/16	Cl	1·684	2·020	2·475	1·733	2·066	2·557
	Br	2·153	2·456	2·873	2·126	2·429	2·896
	I	2·772	3·203	3·675	2·816	3·157	3·608
1/32	Cl	0·841	1·006	1·225			
	Br	1·081	1·222	1·466		1·174	1·403
	I	1·395	1·602	1·814	1·424	1·589	1·772
1/64	Cl	0·423	0·489	0·604			
	Br	0·554	0·627	0·695		0·616	0·692
	I	0·695	0·813	0·939	0·675	0·763	0·896
1/128	Cl	0·217	0·238	0·291			
	Br	0·278	0·308	0·393		0·306	0·331
	I	0·347	0·422	0·484	0·347	0·382	0·434
1/256	Cl	0·098	0·122	0·144			
	Br	0·139	0·189	0·224			
	I	0·168	0·218	0·277	0·152	0·176	0·201
1/512	Cl	0·064	0·073	0·079			
	Br	0·074	0·090	0·108			
	I	0·089	0·143	0·235			
1/1024	Cl						
	Br		0·082	0·063			
	I	0·040	0·061	0·153			

D. TABLE giving a Summary of the Increments of Displacement, v, caused by the Dissolution of m grm.-mol. Salt in 1000 grams Water at different Temperatures. $v = \Delta - 1000$.

TABLE No. 81.

THE ENNEAD, MRO_3:—CHLORATES, BROMATES, AND IODATES OF POTASSIUM, RUBIDIUM, AND CÆSIUM.

M=		K.	Rb.	Cs.	K.	Rb.	Cs.
T=			19·5° C.			23·0° C.	
m.	RO_3.	*v*.	*v*.	*v*.	*v*.	*v*.	*v*.
1/4	ClO_3	11·352	12·726	14·515			
	BrO_3	11·290					
	IO_3	8·832					
1/8	ClO_3	5·632	6·353	7·233			
	BrO_3	5·576					
	IO_3	4·339					
1/16	ClO_3	2·785	3·183	3·669	2·889	3·204	3·609
	BrO_3	2·761	3·057	3·511	2·854	3·149	3·508
	IO_3	2·189	2·576	2·903	2·210	2·618	2·976
1/32	ClO_3	1·337	1·584	1·804			
	BrO_3	1·370	1·541	1·767			
	IO_3	1·096	1·276	1·471			
1/64	ClO_3	0·661	0·775	0·971			
	BrO_3	0·688	0·767	0·864			
	IO_3	0·584	0·661	0·786			
1/128	ClO_3	0·324	0·400	0·476			
	BrO_3	0·347	0·407	0·421			
	IO_3	0·269	0·344	0·457			
1/256	ClO_3	0·158	0·200	0·292			
	BrO_3	0·176	0·191	0·235			
	IO_3	0·127	0·190	0·272			
1/512	ClO_3	0·057	0·111	0·212			
	BrO_3	0·089	0·096	0·134			
	IO_3	0·057	0·072	0·152			

§ 30. E. Tables giving the Values of *v/m*, that is, the Mean Increments of Displacement (Tables D), calculated for the Dissolution of 1 grm.-mol. Salt in 1000 grams Water at different Temperatures.

CHLORIDES, BROMIDES, AND IODIDES.

Table No. 82.

CHLORIDES. MCl.

M=	Na.	K.	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	15·0° C.		19·5° C.			23·0° C.		
m.	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*
1/2	17·02	27·63	28·00	33·27	40·80			
1/4	16·59	26·68	27·59	32·80	40·26			
1/8	16·34	27·14	27·26	32·45	39·91			
1/16	16·05	26·96	26·95	32·32	39·60	27·72	33·06	40·91
1/32	16·17	26·97	26·92	32·20	39·22			
1/64	16·69	28·75	27·12	31·34	38·70			
1/128	17·31	27·92	27·86	30·54	37·24			
1/256			25·21	31·46	36·94			
1/512			32·77	37·47	40·85			

Table No. 83.

BROMIDES. MBr.

M=	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	19·5° C.			23·0° C.		
m.	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*
1/2	35·09	40·52	47·30			
1/4	34·76	39·93	47·02			
1/8	34·51	39·52	46·42		38·96	46·66
1/16	34·45	39·30	45·97	34·01	38·87	46·33
1/32	34·79	39·10	46·93		37·67	44·89
1/64	35·45	40·17	44·49		39·44	44·28
1/128	35·64	39 51	50·38		39·27	42·47
1/256	35·76	48·46	57·54			
1/512	38·14	46·18	55·29			
1/1024		84·48	64·51			

Table No. 84.

IODIDES. MI.

M=	K.	Rb.	Cs.	K.	Rb.	Cs.	Cs.
T=	19·5° C.			23·0° C.			26·0° C.
m.	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*
1/2	45·55	51·71	59·36	46·11			
1/4	45·12	51·34	59·15	45·86			
1/8	44·59	51·39	58·74	45·50	50·88	57·89	
1/16	44·25	51·24	58·80	45·06	50·51	57·73	57·32
1/32	44·65	51·28	58·06	45·57	50·86	56·72	
1/64	44·58	52·04	60·12	43·20	48·87	57·39	
1/128	44·45	54·09	62·00	44·50	48·97	55·62	
1/256	43·18	55·80	71·01	39·91	45·23	51·68	
1/512	45·61	73·21	120·67				
1/1024	41·16	62·97	157·49				

E. TABLES giving the Values of v/m, that is, the Mean Increments of Displacement (Tables D), calculated for the Dissolution of 1 grm.-mol. Salt in 1000 grams Water at different Temperatures.

TABLE No. 85.

NITRATES. $M'NO_3$ or $M''(NO_3)_2$.

M' or M''=	Na.	K.	Sr''.	Ba''.	Li.	Na.	Ba''.	Pb''.	Rb.	Cs.
T=	15·0° C.				19·5° C.				23·0° C.	
m.	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*
1/2		38·17			29·01	28·58				
1/4		37·71			28·58	28·11			44·01	
1/8		36·98			28·52	27·84			42·42	50·54
1/16	29·64	37·49			28·15	27·48	47·53	45·58	41·88	49·98
1/32	30·05	37·85	40·37	45·69	27·66	27·28	46·39	44·31	40·04	49·62
1/64	29·71	38·02	40·42	44·74	27·10	27·34	45·34	42·53	36·69	48·43
1/128	29·79	38·38	38·66	44·16	25·98		41·99	41·59	25·26	43·13
1/256			41·08	47·07			41·72	42·29	43·94	46·51
1/512			43·05	45·05			39·11	35·78		
1/1024				37·37			50·89	23·55		

TABLE No. 86.

TRIADS OF NITRATES, CHLORATES, BROMATES, AND IODATES. MRO_3.

RO_3=	NO_3.			ClO_3.			BrO_3.			IO_3.		
M=	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	19·5° C.			19·5° C. and *23·0°* C.			19·5° C. and *23·0°* C.			19·5° C. and *23·0°* C.		
m.	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*
1/2	38·82	44·00										
1/4	38·37	43·59	50·71	45·40	50·90	58·06	45·16			35·33		
1/8	37·81	43·15	50·41	45·06	50·82	57·86	44·61			34·71		
1/16	37·48	41·67	49·89	44·57	50·93	58·71	44·17	48·91	56·69	35·02	41·22	46·45
1/32	36·62	40·05	48·04	42·80	50·69	57·73	43·86	49·31	56·56	35·16	40·85	47·07
1/64	36·80	35·41	51·09	42·34	49·61	62·17	44·07	49·10	55·32	37·37	42·33	50·32
1/128	35·96	29·73	48·21	41·53	51·26	60·95	44·42	52·09	54·00	36·77	44·07	58·76
1/256		30·25	40·29	40·55	51·37	74·98	45·20	49·04	59·21	32·56	48·69	69·83
1/512				29·63	57·24	108·90	45·67	49·56	68·86	29·23	37·17	77·82
1/16				*46·23*	*51·27*	*57·75*	*45·67*	*50·37*	*56·13*	*35·41*	*41·90*	*47·61*

E. Tables giving the Values of v/m, that is, the Mean Increments of Displacement (Tables D), calculated for the Dissolution of 1 grm.-mol. Salt in 1000 grams Water at different Temperatures.

POTASSIUM, RUBIDIUM, AND CÆSIUM SALTS.

Table No. 87.

POTASSIUM SALTS. KR and KRO_3.

R or RO_3=	Cl.	NO_3.	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	Cl.	Br.	I.	ClO_3.	BrO_3.	IO_3.
T=	15·0° C.		19·5° C.							23·0° C.					
m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.
1/2	27·63	38·17	28·00	35·09	45·55	38·82						46·11			
1/4	26·68	37·71	27·59	34·76	45·12	38·37	45·40	45·16	35·33			45·86			
1/8	27·14	36·98	27·26	34·51	44·59	37·81	45·06	44·61	34·71			45·50			
1/16	26·96	37·49	26·95	34·45	44·25	37·48	44·57	44·17	35·02	27·72	34·01	45·06	46·23	45·67	35·41
1/32	26·97	37·85	26·92	34·79	44·65	36·62	42·80	43·86	35·16			45·57			
1/64	28·75	38·02	27·12	35·45	44·58	36·80	42·34	44·07	37·37			43·20			
1/128	27·92	38·38	27·86	35·64	44·45	35·96	41·53	44·42	36·77			44·50			
1/256			25·21	35·76	43·18		40·55	45·20	32·56			39·91			
1/512			32·77	38·14	45·61		29·63	45·67	29·23						
1/1024					41·16										

Table No. 88.

RUBIDIUM SALTS. RbR and $RbRO_3$.

R or RO_3=	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.
T=	19·5° C.							23·0° C.						
m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.
1/2	33·27	40·52	51·71	44·00										
1/4	32·80	39·93	51·34	43·59	50·90						44·01			
1/8	32·45	39·52	51·39	43·15	50·82				38·96	50·88	42·42			
1/16	32·32	39·30	51·24	41·67	50·93	48·91	41·22	33·06	38·87	50·51	41·88	51·27	50·37	41·90
1/32	32·20	39·10	51·28	40·05	50·69	49·31	40·85		37·67	50·86	40·04			
1/64	31·34	40·17	52·04	35·41	49·61	49·10	42·33		39·44	48·87	36·69			
1/128	30·54	39·51	54·09	29·73	51·26	52·09	44·07		39·27	48·97	25·26			
1/256	31·46	48·46	55·80	30·25	51·37	49·04	48·69			45·23	43·94			
1/512	37·47	46·18	73·21		57·24	49·56	37·17							
1/1024		84·48	62·97											

Table No. 89.

CÆSIUM SALTS. CsR and $CsRO_3$.

R or RO_3=	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	Cl.	Br.	I.	NO_3.	ClO_3.	BrO_3.	IO_3.	I.
T=	19·5° C.							23·0° C.							26·0° C.
m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.
1/2	40·80	47·30	59·36												
1/4	40·26	47·02	59·15	50·71	58·06										
1/8	39·91	46·42	58·74	50·41	57·86				46·66	57·89	50·54				
1/16	39·60	45·97	58·80	49·89	58·71	56·69	46·45	40·91	46·33	57·73	49·98	57·75	56·13	47·61	57·32
1/32	39·22	46·93	58·06	48·04	57·73	56·56	47·07		44·89	56·72	49·62				
1/64	38·70	44·49	60·12	51·09	62·17	55·32	50·32		44·28	57·39	48·43				
1/128	37·24	50·38	62·00	48·21	60·95	54·00	58·76		42·47	55·62	43·13				
1/256	36·94	57·54	71·01	40·29	74·98	59·21	69·83			51·68	46·51				
1/512	40·85	55·29	120·67		108·90	68·86	77·82								
1/1024		64·51	157·49												

E. Table giving the Values of v/m, that is, the Mean Increments of Displacement (Tables D), calculated for the Dissolution of 1 grm.-mol. Salt in 1000 grams Water at different Temperatures.

Table No. 90.

The Ennead, MR:—CHLORIDES, BROMIDES, AND IODIDES OF POTASSIUM, RUBIDIUM, AND CÆSIUM.

M=		K.	Rb.	Cs.	K.	Rb.	Cs.
T=		19·5° C.			23·0° C.		
m.	R.	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*
1/2	Cl	28·00	33·27	40·80			
	Br	35·09	40·52	47·30			
	I	45·55	51·71	59·36	46·11		
1/4	Cl	27·59	32·80	40·26			
	Br	34·76	39·93	47·02			
	I	45·12	51·34	59·15	45·86		
1/8	Cl	27·26	32·45	39·91			
	Br	34·51	39·52	46·42		38·96	46·66
	I	44·59	51·39	58·74	45·50	50·88	57·89
1/16	Cl	26·95	32·32	39·60	27·72	33·06	40·91
	Br	34·45	39·30	45·97	34·01	38·87	46·33
	I	44·25	51·24	58·80	45·06	50·51	57·73
1/32	Cl	26·92	32·20	39·22			
	Br	34·79	39·10	46·93		37·67	44·89
	I	44·65	51·28	58·06	45·57	50·86	56·72
1/64	Cl	27·12	31·34	38·70			
	Br	35·45	40·17	44·49		39·44	44·28
	I	44·58	52·04	60·12	43·20	48·87	57·39
1/128	Cl	27·86	30·54	37·24			
	Br	35·64	39·51	50·38		39·27	42·47
	I	44·45	54·09	62·00	44·50	48·97	55·62
1/256	Cl	25·21	31·46	36·94			
	Br	35·76	48·46	57·54			
	I	43·18	55·80	71·01	39·91	45·23	51·68
1/512	Cl	32·77	37·47	40·85			
	Br	38·14	46·18	55·29			
	I	45·61	73·21	120·67			
1/1024	Cl						
	Br		84·48	64·51			
	I	41·16	62·97	157·49			

E. TABLE giving the Values of v/m, that is, the Mean Increments of Displacement (Tables D), calculated for the Dissolution of 1 grm.-mol. Salt in 1000 grams Water at different Temperatures.

TABLE No. 91.

THE ENNEAD, MRO_3:—CHLORATES, BROMATES, AND IODATES OF POTASSIUM, RUBIDIUM, AND CÆSIUM.

M=		K.	Rb.	Cs.	K.	Rb.	Cs.
T=		19·5° C.			23·0° C.		
m.	RO_3.	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*	*v/m.*
1/4	ClO_3	45·40	50·90	58 06			
	BrO_3	45·16					
	IO_3	35·31					
1/8	ClO_3	45·06	50·82	57·86			
	BrO_3	44·61					
	IO_3	34·71					
1/16	ClO_3	44·57	50·93	58·71	46·23	51·27	57·75
	BrO_3	44·17	48·91	56·69	45·67	50·37	56·13
	IO_3	35·02	41·22	46·45	35·41	41·90	47·61
1/32	ClO_3	42·80	50·69	57·73			
	BrO_3	43·86	49·31	56·56			
	IO_3	35·16	40·85	47·07			
1/64	ClO_3	42·34	49·61	62·17			
	BrO_3	44·07	49·10	55·32			
	IO_3	37·37	42·33	50·32			
1/128	ClO_3	41·53	51·26	60·95			
	BrO_3	44·42	52·09	54·00			
	IO_3	36·77	44·07	58·76			
1/256	ClO_3	40 55	51·37	74·98			
	BrO_3	45·20	49·04	59·21			
	IO_3	32·56	48·69	69·83			
1/512	ClO_3	29·63	57·24	108·90			
	BrO_3	45·67	49·56	68·86			
	IO_3	29·23	37·17	77·82			

STRONG SOLUTIONS.

§ 31. A. General Tables giving, in the columns under *m*, W, and S, the Facts of Observation relating to concentrated Solutions of the Salts of the Ennead (K, Rb, Cs, Cl, Br, I), the Specific Gravity of which has been determined with the Specific Gravity Bottle or Pyknometer.

TRIAD OF CHLORIDES.

Table No. 92.

POTASSIUM CHLORIDE. KCl = 74·6.

T = 19·5° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1	1074·6	1·0449	1028·42		
				30·45	·012672
2	1149·2	1·0853	1058·87		
				31·65	·012791
3	1223·8	1·1222	1090·52		
				32·41	·012719
4	1298·4	1·1562	1122·93		
					(3·050352)

Table No. 93.

RUBIDIUM CHLORIDE. RbCl = 121·0

T = 19·5° C.

1/2	1060·5	1·0426	1017·17		
				17·72	·007503
1	1121·0	1·0832	1034·89		
				36·54	·015066
2	1242·0	1·1592	1071·43		
				38·23	·015228
3	1363·0	1·2283	1109·66		
				37·52	·014442
4	1484·0	1·2936	1147·18		
				38·11	·014190
5	1605·0	1·3541	1185·29		
				40·56	·014613
6	1726·0	1·4084	1225·85		
				40·43	·014092
7	1847·0	1·4586	1266·28		
				19·70	·006704
7·5	1907·5	1·4833	1285·98		
					(3·109235)

Table No. 94.

CÆSIUM CHLORIDE. CsCl = 168·5.

T = 19·5° C.

1/2	1084·25	1·0616	1021·37		
				23·02	·009679
1	1168·5	1·1188	1044·39		
				45·22	·018408
2	1337·0	1·2270	1089·61		
				46·98	·018333
3	1505·5	1·3245	1136·59		
				49·47	·018502
4	1674·0	1·4113	1186·06		
				45·82	·016462
5	1842·5	1·4956	1231·88		
				51·43	·017763
6	2011·0	1·5670	1283·31		
				47·72	·015856
7	2179·5	1·6374	1331·03		
				47·78	·015316
8	2348·0	1·7025	1378·81		
				51·78	·016010
9	2516·5	1·7590	1430·59		
					(3·155515)

STRONG SOLUTIONS.

A. General Tables giving the Facts of Observation in the columns under m, W, and S.

TRIAD OF BROMIDES.

Table No. 95.

POTASSIUM BROMIDE. KBr = 119·1.

T = 19·5° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	dΔ.	d log Δ.
1	1119·1	1·0808	1035·44	37·05	·015273
2	1238·2	1·1545	1072·49	38·22	·015207
3	1357·3	1·2220	1110·71	38·87	·014932
4	1476·4	1·2843	1149·57	38·59	·014336
5	1595·5	1·3425	1188·16		(3·074873)

Table No. 96.

RUBIDIUM BROMIDE. RbBr = 165·5.

T = 19·5° C.

m.	W.	S.	W/S = Δ.	dΔ.	d log Δ.
1/2	1082·75	1·0613	1020·21	21·05	·008869
1	1165·5	1·1193	1041·26	42·45	·017353
2	1331·0	1·2281	1083·71	43·85	·017226
3	1496·5	1·3272	1127·56	44·67	·016873
4	1662·0	1·4178	1172·23	45·33	·016477
5	1827·5	1·5009	1217·56	46·11	·016108
6	1993·0	1·5772	1263·57		(3·101599)

Table No. 97.

CÆSIUM BROMIDE. CsBr = 213·0.

T = 21·4° C.

m.	W.	S.	W/S = Δ.	dΔ.	d log Δ.
1	1213·0	1·1590	1046·54	46·91	·019046
2	1426·0	1·3041	1093·45	50·57	·018696
3	1639·0	1·4326	1144·02	47·08	·017495
4	1852·0	1·5548	1191·10	51·06	·018232
5	2065·0	1·6624	1242·16		(3·094181)

STRONG SOLUTIONS.

A. General Tables giving the Facts of Observation in the columns under *m*, W, and S.

TRIAD OF IODIDES.

Table No. 98.

POTASSIUM IODIDE. KI = 166·1.

T = 19·5° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1	1166·1	1·1146	1046·20	47·83	·019412
2	1332·2	1·2177	1094·03	47·27	·018371
3	1498·3	1·3128	1141·30	49·09	·018288
4	1664·4	1·3982	1190·39	49·28	·017618
5	1830·5	1·4766	1239·67	49·87	·017129
6	1996·6	1·5483	1289·54	50·34	·016630
7	2162·7	1·6141	1339·88	50·53	·016077
8	2328·8	1·6749	1390·54		(3·143143)

Table No. 99.

RUBIDIUM IODIDE. RbI = 212·5.

T = 19·5° C.

m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/2	1106·25	1·0771	1026·99	27·46	·011459
1	1212·5	1·1498	1054·45	55·77	·022383
2	1425·0	1·2835	1110·22	57·23	·021829
3	1637·5	1·4026	1167·45	57·48	·020873
4	1850·0	1·5102	1224·93	57·59	·019952
5	2062·5	1·6081	1282·52	58·70	·019435
6	2275·0	1·6962	1341·22	63·72	·020157
7	2487·5	1·7705	1404·94		(3·147657)

Table No. 100.

CÆSIUM IODIDE. CsI = 260·0.

T = 23·1° C.

m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1	1260·0	1·1847	1063·55	65·41	·025920
2	1520·0	1·3463	1128·96	63·74	·023852
3	1780·0	1·4924	1192·70		(3·076531)
3·385	*1880·1*	*1·5426*	*1218·76*		

STRONG SOLUTIONS.

A. General Tables giving the Facts of Observation in the columns under *m*, W, and S.

Table No. 101.

RUBIDIUM NITRATE. $RbNO_3 = 147·5$.

T = 19·5° C.

	Weight of Solution. Grams.	Specific Gravity.	Displacement. G_T.	Differences of Displacements.	Differences of Logarithms of Displacements.
m.	W.	S.	W/S = Δ.	*d*Δ.	*d* log Δ.
1/2	1073·75	1·0505	1022·13	22·76	·009564
1	1147·5	1·0982	1044·89	46·92	·019077
2	1295·0	1·1861	1091·81	48·06	·018705
3	1442·5	1·2655	1139·87		(3·056853)

Table No. 102.

LITHIUM NITRATE. $LiNO_3 = 69·0$.

T = 19·5° C.

m.	W.	S.	Δ.	*d*Δ.	*d* log Δ.
1	1069·0	1·0389	1028·97	29·93	·012451
2	1138·0	1·0747	1058·90	30·35	·012273
3	1207·0	1·1081	1089·25	30·83	·012122
4	1276·0	1·1392	1120·08	30·89	·011880
5	1345·0	1·1684	1151·14	31·23	·011623
6	1414·0	1·1959	1182·37	31·81	·011528
7	1483·0	1·2214	1214·18	31·70	·011194
8	1552·0	1·2457	1245·88	32·10	·011048
9	1621·0	1·2684	1277·98	31·49	·010568
10	1690·0	1·2906	1309·47		(3·117095)

Table No. 103.

SODIUM NITRATE. $NaNO_3 = 85·0$.

T = 19·5° C.

m.	W.	S.	Δ.	*d*Δ.	*d* log Δ.
1	1085·0	1·0542	1029·18	31·54	·013109
2	1170·0	1·1030	1060·72	32·64	·013162
3	1255·0	1·1478	1093·36	31·28	·012250
4	1340·0	1·1887	1124·64	36·99	·014054
5	1425·0	1·2267	1161·63	35·48	·013066
6	1510·0	1·2613	1197·11	35·52	·012698
7	1595·0	1·2939	1232·63	34·94	·012139
8	1680·0	1·3253	1267·57	35·99	·012159
9	1765·0	1·3539	1303·56		(3·115131)

STRONG SOLUTIONS.

§ 32. Tables of the Classes C, D, and E, giving Summaries of their Specific Gravities (S), their Increments of Displacement, v, and their Mean Increments of Displacement per gram-molecule Salt, v/m, respectively.

R=	CHLORIDES.			BROMIDES.			IODIDES.		
M=	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	19·5° C.			19·5° C.		21·4° C.	19·5° C.		23·1° C.

Table No. 104. / Table No. 105. / Table No. 106.

C. Specific Gravity.

m.	S.	S.	S.	S.	S.	S.	S.	S.	S.
1/2		1·0426	1·0616		1·0613			1·0771	
1	1·0449	1·0832	1·1188	1·0808	1·1193	1·1590	1·1146	1·1498	1·1847
2	1·0853	1·1592	1·2270	1·1545	1·2281	1·3041	1·2177	1·2835	1·3463
3	1·1222	1·2283	1·3245	1·2220	1·3272	1·4326	1·3128	1·4026	1·4924
4	1·1562	1·2936	1·4113	1·2843	1·4178	1·5548	1·3982	1·5102	
5		1·3541	1·4956	1·3425	1·5009	1·6624	1·4766	1·6081	
6		1·4084	1·5670		1·5772		1·5483	1·6962	
7		1·4586	1·6374				1·6141	1·7705	
8			1·7025				1·6749		
9			1·7590						

Table No. 107. / Table No. 108. / Table No. 109.

D. Increment of Displacement.

m.	v.	v.	v.	v.	v.	v.	v.	v.	v.
1/2		17·17	21·37		20·21			26·99	
1	28·42	34·89	44·39	35·44	41·26	46·54	46·20	54·45	63·55
2	58·87	71·43	89·61	72·49	83·71	93·45	94·03	110·22	128·96
3	90·52	109·66	136·59	110·71	127·56	144·02	141·30	167·45	192·70
4	122·93	147·18	186·06	149·57	172·23	191·10	190·39	224·93	
5		185·29	231·88	188·16	217·56	242·16	239·67	282·52	
6		225·85	283·31		263·57		289·54	341·22	
7		266·28	331·03				339·88	404·94	
8			378·81				390·54		
9			430·59						

Table No. 110. / Table No. 111. / Table No. 112.

E. Mean Increment of Displacement per gram-molecule.

m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.	v/m.
1/2		34·34	42·74		40·42			53·98	
1	28·42	34·89	44·39	35·44	41·26	46·54	46·20	54·45	63·55
2	29·43	35·72	44·80	36·25	41·85	46·72	47·02	55·11	64·48
3	30·17	36·55	45·53	36·90	42·52	48·00	47·10	55·81	64·23
4	30·73	36·79	46·51	37·39	43·05	47·77	47·59	56·23	
5		37·06	46·37	37·63	43·51	48·43	47·93	56·50	
6		37·64	47·21		43·92		48·26	56·87	
7		38·04	47·29				48·55	57·84	
8			47·35				48·81		
9			47·84						

STRONG SOLUTIONS.

TABLES of the Classes C, D, and E, giving Summaries of their Specific Gravities (S), their Increments of Displacement, v, and their Mean Increments of Displacement per gram-molecule Salt, v/m, respectively.

POTASSIUM SALTS. KR.

R=	Cl.	Br.	I.
T=	19·5° C.		

TABLE No. 113. C. Specific Gravity.

m.	S.	S.	S.
1	1·0449	1·0808	1·1146
2	1·0853	1·1545	1·2177
3	1·1222	1·2220	1·3128
4	1·1562	1·2843	1·3982
5		1·3425	1·4766
6			1·5483
7			1·6141
8			1·6749

TABLE No. 116. D. Increment of Displacement.

m.	*v.*	*v.*	*v.*
1	28·42	35·44	46·20
2	58·87	72·49	94·03
3	90·52	110·71	141·30
4	122·93	149·57	190·39
5		188·16	239·67
6			289·54
7			339·88
8			390·54

TABLE No. 119. E. Mean Increment of Displacement per gram-molecule.

m.	*v/m.*	*v/m.*	*v/m.*
1	28·42	35·44	46·20
2	29·43	36·25	47·02
3	30·17	36·90	47·10
4	30·73	37·39	47·59
5		37·63	47·93
6			48·26
7			48·55
8			48·81

RUBIDIUM SALTS. RbR.

R=	Cl.	Br.	I.
T=	19·5° C.		

TABLE No. 114. C. Specific Gravity.

m.	S.	S.	S.
1/2	1·0426	1·0613	1·0771
1	1·0832	1·1193	1·1498
2	1·1592	1·2281	1·2835
3	1·2283	1·3272	1·4026
4	1·2936	1·4178	1·5102
5	1·3541	1·5009	1·6081
6	1·4084	1·5772	1·6962
7	1·4586		1·7705
7·5	1·4833		

TABLE No. 117. D. Increment of Displacement.

m.	*v.*	*v.*	*v.*
1/2	17·17	20·21	26·99
1	34·89	41·26	54·45
2	71·43	83·71	110·22
3	109·66	127·56	167·45
4	147·18	172·23	224·93
5	185·29	217·56	282·52
6	225·85	263·57	341·22
7	266·28		404·94
7·5	285·98		

TABLE No. 120. E. Mean Increment of Displacement per gram-molecule.

m.	*v/m.*	*v/m.*	*v/m.*
1/2	34·34	40·42	53·98
1	34·89	41·26	54·45
2	35·72	41·85	55·11
3	36·55	42·52	55·81
4	36·79	43·05	56·23
5	37·06	43·51	56·50
6	37·64	43·92	56·87
7	38·04		57·84
7·5	38·13		

CÆSIUM SALTS. CsR.

R=	Cl.	Br.	I.
T=	19·5° C.	21·4° C.	23·1° C.

TABLE No. 115. C. Specific Gravity.

m.	S.	S.	S.
1/2	1·0616		
1	1·1188	1·1590	1·1847
2	1·2270	1·3041	1·3463
3	1·3245	1·4326	1·4924
4	1·4113	1·5548	
5	1·4956	1·6624	
6	1·5670		
7	1·6374		
8	1·7025		
9	1·7590		

TABLE No. 118. D. Increment of Displacement.

m.	*v.*	*v.*	*v.*
1/2	21·37		
1	44·39	46·54	63·55
2	89·61	93·45	128·96
3	136·59	144·02	192·70
4	186·06	191·10	
5	231·88	242·16	
6	283·31		
7	331·03		
8	378·81		
9	430·59		

TABLE No. 121. E. Mean Increment of Displacement per gram-molecule.

m.	*v/m.*	*v/m.*	*v/m.*
1/2	42·74		
1	44·39	46·54	63·55
2	44·80	46·72	64·48
3	45·53	48·00	64·23
4	46·51	47·77	
5	46·37	48·43	
6	47·21		
7	47·29		
8	47·35		
9	47·84		

STRONG SOLUTIONS.

TABLES of the Classes C, D, and E, giving Summaries of their Specific Gravities (S), their Increments of Displacement, v, and their Mean Increments of Displacement per gram-molecule Salt, v/m, respectively.

R=	NITRATES.			NITRATES.			NITRATES.		
M=	Li.	Na.	Rb.	Li.	Na.	Rb.	Li.	Na.	Rb.
T=	19·5° C.			19·5° C.			19·5° C.		
	TABLE No. 122.			TABLE No. 123.			TABLE No. 124.		
	C. Specific Gravity.			D. Increment of Displacement.			E. Mean Increment of Displacement per gram-molecule.		
m.	S.	S.	S.	v.	v.	v.	v/m.	v/m.	v/m.
1/2			1·0505			22·13			44·26
1	1·0389	1·0542	1·0982	28·97	29·18	44·89	28·97	29·18	44·89
2	1·0747	1·1030	1·1861	58·90	60·72	91·81	29·45	30·36	45·90
3	1·1081	1·1478	1·2655	89·25	93·36	139·87	29·75	31·12	46·62
4	1·1392	1·1887		120·08	124·64		30·02	31·16	
5	1·1684	1·2267		151·14	161·63		30·23	32·33	
6	1·1959	1·2613		182·37	197·11		30·39	32·85	
7	1·2214	1·2939		214·18	232·63		30·59	33·23	
8	1·2457	1·3253		245·88	267·57		30·73	33·45	
9	1·2684	1·3539		277·98	303·56		30·89	33·73	
10	1·2906			309·47			30·95		

SECTION VI.—GENERAL DESCRIPTION OF TABLES.

§ 33. In the tables giving the results of the experiments made with solutions of a particular salt, the weights given are those which would have been used if the weighings had actually been made in a vacuum; and the standard temperature, T, at which all the operations have been made, is given at the top with the name of the salt, both being constants.

Of the variables, we have under m the quantity of the salt, expressed as the number, whole or fractional, of gram-molecules, which is dissolved in 1000 grams of water, under W the weight in grams of the solution so produced, and under S the specific gravity of the solution referred to that of distilled water as unity, both having the standard temperature T.

With the exception of the determination of the temperature, the result of every series of operations depends only on determinations of weight, and they are independent of the work of others. Even the tyro has no difficulty in being assured of the true weight of the hydrometer when floating up to the same mark in the solution and in distilled water respectively. The difficulty which requires manipulative skill, laboratory experience, and perseverance to overcome, is to satisfy the condition that the temperatures of the solution and of the distilled water respectively, and that of the hydrometer when immersed in them, are identical, and are really the temperature shown by the thermometer, and that this temperature is exactly that chosen as the standard for the series of experiments. It requires much study and practice in a suitable room before even an experienced chemist or physicist can feel confident that he can produce this combination of equalities when required.

It is to the failure to perceive the necessity of this preliminary education that, though the method has been the property of science for forty years, it has been used practically by none except myself and those whom I have personally instructed.

§ 34. In the tables of Class A all the facts of observation are to be found. In all the solutions the quantity of water is the same, namely, 1000 grams; the quantity of salt dissolved in this mass of water is specified for each solution of the same salt in the first column under m, in terms of the gram-molecule. In the second column, under W, we have the weight in grams of each solution; it is given by the sum $1000 + m.\mathrm{MR} = \mathrm{W}$; where MR represents the molecular weight of the salt. The symbol used to express the weight of salt dissolved in 1000 grams of water is w, whence $w = m.\mathrm{MR}$. In the third column, under S, we have the specific gravity of the solution. The experimental data on which it is founded are the weight, H′, in grams, of the hydrometer when it floats at a given division of the stem in the solution, and its weight, H, when it floats at the same division in distilled water, both of these liquids having the same temperature, T. The quotient H′/H is the specific gravity, S, of the solution, as entered in the third column of the tables. If we divide the weight of the solution, W, by its specific gravity, S, we obtain the displacement of the solution, which is entered in the fourth column under Δ. It is the expression of the proportion $\mathrm{H} : \mathrm{H}' :: \Delta : \mathrm{W}$, in which H and Δ are weights of distilled water, and H′ and W are weights of the solution. It may be expressed in words as follows:—The displacement of the solution, Δ, bears to its weight, W, the same relation as the weight H of distilled water displaced by the hydrometer bears to H′, that of the solution displaced by the same portion of the same hydrometer at the same temperature, T. Therefore, the unit of displacement used in the tables is the space occupied by 1 gram of distilled water having the temperature T.

Generally, our measure of displacement of a body having the temperature T is the weight of distilled water having the same temperature which the body displaces when totally immersed in the water. The body in question may be solid, liquid, or gaseous. The unit of displacement is then the unit of weight, gram or kilogram, of water having the particular temperature, T, which is chosen to suit the conditions of the experiment, and it must be the *common* temperature of the body and the water. Under this convention the unit of displacement is the space occupied by, say, 1 gram water at T, whatever value T may have. Thus, in our experiments the value of T is in some 15°, in some 19·5°, and in others 23°; but whichever temperature is used as that of the distilled water, it is also that of the salt or saline solution which is supposed to displace it when its specific gravity is being determined. The use of *displacement* instead of *volume* to specify the amount of space occupied by a body is advantageous only when there is a common temperature and it can be accepted as constant. In cases where the temperature is subject to variation, the specification of displacement must be by volume, because a weight is not affected by change of temperature.

It is convenient to have a symbol to place after a number in order to indicate that its unit is that of displacement as specified above. It is to be used in cases corresponding to those in which the symbol c.c. is used when we express volumes in cubic centimetres. A suitable symbol for the unit of displacement is G_T or G_t, in which G is the unit of weight and T or *t* is the common temperature of the body and of the water displaced by it.

In this research the unit of weight used is the gram, so that our unit of displacement expresses the space occupied by 1 gram of water at the temperature T. When the unit of weight used is the kilogram the symbol becomes K_T. In naval architecture the displacement of a ship is always expressed in *tons*, that is, tons of water of ordinary atmospheric temperature. In this research the units of displacement used are expressed by the symbols $G_{15°}$, $G_{19·5°}$, and $G_{23°}$. If the adopted value of T were 4° C., then the unit of displacement would be $G_{4°}$, and this is the gravimetric symbol for the standard cubic centimetre.

In the fifth column we have the values of $d\Delta$, the differences of consecutive values of Δ. The entries in this column have a peculiar interest owing to the fact that the values of *m* which indicate the concentration of the solutions form an ascending geometrical series with the common ratio 2. The quantity of water, 1000 grams, is the same in all the solutions. If we consider any two consecutive values of Δ, for instance, $\Delta_{1/8}$ and $\Delta_{1/4}$, the increment of displacement produced by dissolving 1/8 MR in 1000 grams of water is $\Delta_{1/8} - 1000$, and the increment produced by dissolving a further quantity of salt equal to 1/8 MR in the solution the displacement of which is $\Delta_{1/8}$, is $d\Delta = \Delta_{1/4} - \Delta_{1/8}$. These increments of displacement have been produced by equal quantities of the salt, which has been dissolved in the first case in 1000 grams of distilled water, and in the second case in (1/8 MR + 1000) grams of the solution so produced. If the corresponding values of $(\Delta_m - 1000)$ and $(\Delta_{2m} - \Delta_m)$ be studied, they will be found to be almost always different. Considering only the first nine tables relating to the salts of the ennead MR, we see that the difference of these increments $(\Delta_{2m} - \Delta_m) - (\Delta_m - 1000)$ is positive for values of $m = 1/16$ and higher. It changes sign for a value of *m* lying between 1/16 and 1/32 in the case of KBr, KI, and RbI; for *m* lying between 1/32 and 1/64 in the case of KCl, RbBr, and CsI; for *m* lying between 1/64 and 1/128 in the case of CsBr; for *m* lying between 1/128 and 1/256 in the case of RbCl; and for *m* lying between 1/256 and 1/512 in the case of CsCl.

The main object with which this experimental research was begun was to ascertain if such a change of sign occurs at any concentration. It was only by using the hydrometric method that the question could be answered. In the sixth column we have the values of $d \log \Delta$, the consecutive differences of the log-displacement. The space in this column corresponding to the highest value of *m* is occupied in brackets by the logarithm of the corresponding highest value of Δ. With this, and the corresponding values of $d \log \Delta$, the log-displacement of each solution can be at once obtained.

In Class A there are thirty-seven tables; of these, twenty-four relate to solutions of chlorides, bromides, iodides, and nitrates of the alkalies and alkaline earths, the specific gravities of which have been determined with two hydrometers. The values of S given for each value of m in each of these tables is the mean of two groups of series of nine observations each, each group being made with a different hydrometer. The hydrometers chiefly used were Nos. 17 and 21, and for each value of m either three or four series of observations were made with each of these hydrometers. The mean of each series is the mean of nine independent values of the specific gravity, so that the final mean, $\bar{S}$, is the mean of 72 independent observations when four series have been made with each hydrometer, and the mean of 54 independent observations when three series have been made with each hydrometer.

§ 35. In the tables of Class B, we have the particulars of the series of observations made with hydrometers 21 and 17 respectively from which the final mean value of $\bar{S}$ in the table is obtained. In the tables of this class m has the same signification as in those of Class A; S_{21} gives the mean specific gravity for the particular value of m derived from s_{21} series of observations made with hydrometer No. 21; S_{17} the mean specific gravity similarly obtained from s_{17} series made with hydrometer No. 17. The final mean derived from $\bar{s}$ $(= s_{21} + s_{17})$, the sum of these series, is found under $\bar{S}$. Under r_0 we have the probable error of $\bar{S}$ calculated by the method of least squares, and under d, the maximum departure of the mean of any individual series from the mean specific gravity, $\bar{S}$. Numbers under r_0 and d are expressed in units of the sixth decimal place.

For each table of Class A referring to specific gravities derived from observations with two hydrometers a corresponding table of Class B has been prepared. In the twenty-four tables of Class B there are 189 entries under $\bar{S}$, r_0, and d respectively. Summing those under $\bar{s}$, we find that the experimental material on which these tables are founded consists of 1227 series, whence the mean number of series of observations per solution is 6·49. Each series consists of nine individual observations, and when each of them is compared with the corresponding observation made under the same conditions in distilled water, they give a mean per solution of 58·4 independent observations of the ratio of the weight of a given bulk of the saline solution to the same bulk of distilled water, both liquids being at the same temperature. The 1227 series accounted for in the twenty-four tables correspond to 11,043 independent observations of the hydrometric displacement, from each of which the specific gravity of the solution in which the instrument floated is deducible.

The sum of the values of r_0 in the twenty-four tables is 548·7, which divided by 189 gives $\pm 2{\cdot}90$, in the sixth decimal place, as the mean probable error of the mean specific gravity found for any one of the solutions. We have seen that this depends on a mean of 6·49 series per solution; therefore, admitting that the probable error varies inversely as the square root of the number of observations, the mean probable error of the mean specific gravity derived from any number of series s is as follows :—

$s =$ 1	2	3	4	5	6	7	8	9
$\pm r_0 = 7{\cdot}39$	5·22	4·27	3·69	3·30	3·02	2·79	2·61	2·46

Further, the probable error of the mean of one series being $\pm 7{\cdot}39$, and each series consisting of nine individual observations, the probable error of a single observation must be $3 \times 7{\cdot}39 = \pm 22{\cdot}17$ in the sixth place, or $\pm 2{\cdot}22$ in the fifth place.

§ 36. Following the tables of Class B we have those of Class C, which give a summary of the specific gravities of the solutions of different salts at different temperatures. The salts included in each table have a common acid or a common base. Thus the first table of the class contains only chlorides, the second only bromides, and so on. These tables furnish the means of comparing the effect of concentration and of the specific nature of the salt dissolved on the specific gravity of the solution.

§ 37. The specific gravity, S, of one of our solutions expresses the weight in kilograms of the quantity of the solution having the composition m.MR grams of salt plus 1000 grams of water, which exactly displaces 1 kilogram of distilled water having the temperature T. When we compare the specific gravities of the different solutions, we are considering equal volumes of those solutions; but the proportion between the salt and the water present in this volume of solution is different for different solutions. Therefore the specific gravities of the solutions alone do not offer a simple theme for discussion.

The values of W, on the other hand, contain always the constant quantity of water in which the different salts are dissolved in quantities proportional to their molecular weights. It follows, therefore, that the values of $(W - 1000) = w$ are always exactly proportional to the molecular weights of the salts used.

If the increments of specific gravity $(S - 1)$ were also proportional to the molecular weight of the salt dissolved, the quotient $W/S = \Delta$ would be constant for all the solutions of the different salts having the same molecular concentration. This is not found to be the case. The increments of specific gravity do not follow the periodic law exactly, although in the nature of things they cannot depart very far from it.

But we may consider the specific gravity of a solution from another point of view. Let us consider a kilogram of water having the temperature T; it fills a certain space which we may call 1 litre (L_T). We propose to make the solution having the concentration 1/2 KCl + 1000 grams of water by dissolving portions of the salt KCl in the water, but removing so much pure water from the litre-flask as to keep the sum of the volumes of water and salt always equal to the litre. When we have in this way prepared our litre of (1/2 KCl + 1000 grams water), it weighs 1022·98 grams, and is composed of 36·78 grams KCl and 986·20 grams of water. As we started with 1000 grams of water, we have had to remove 13·8 grams of it in order to make room for the 36·78 grams KCl which have been dissolved. Consequently, in the construction of the solution we have replaced water by KCl in the proportion of 2·665 grams KCl per gram of water.

We have carried out the calculation for the volume of a litre of initial water and final solution. It is much simpler when we take the volume displaced by the weight W of the solution, or $\Delta = W/S$.

For the solution 1/2 KCl + 1000 grams water $\Delta = 1014{\cdot}001$ G_T. We then take 1014·001 grams of water, and we add KCl, removing at the same time pure water so as to preserve the constant displacement 1014·001 G_T. We may imagine that equal small portions of KCl added take possession of the amount of water required to form with it a solution of the concentration (1/2 KCl + 1000 grams water), and that the remainder of the water is uncontaminated. We proceed on this principle with the fractional dissolution of the salt and removal of water so as to keep the displacement constant. When we have removed 14·001 grams of water, we find that we have dissolved 37·3 grams or 1/2 KCl in the 1000 grams of water remaining.

But the operation so described is one of substitution. Consequently it is legitimate to regard solutions as products of substitution. In fact, the result of the operation is that we have replaced 14·001 grams of water by 1/2 KCl, so that the substitution has taken place at the rate of 28·002 grams or 1·555 gram-molecules of water per gram-molecule of KCl.

If we turn to Table No. 82, the first table in Class E, we find the first entry in the fourth column is 28·00 as the value of v/m, or the mean increment of displacement per gram-molecule of KCl in the solution (1/2 KCl + 1000 grams of water) at 19·5° C.

The tables of Class D give a summary of the Increments of Displacement, v, caused by the dissolution of m grm.-mol. of salt in 1000 grams of water at different temperatures. Here $v = \Delta - 1000$. The arrangement of the tables in this class is similar to that of Class C.

The tables in Class E enable us to see at a glance the comparative volumetric effect of dissolving different quantities of different salts in 1000 grams of water. Each entry in these tables is derived from the corresponding entry, v, in the corresponding table of Class D, by increasing it in the proportion $m : 1$, whence we obtain the values v/m.

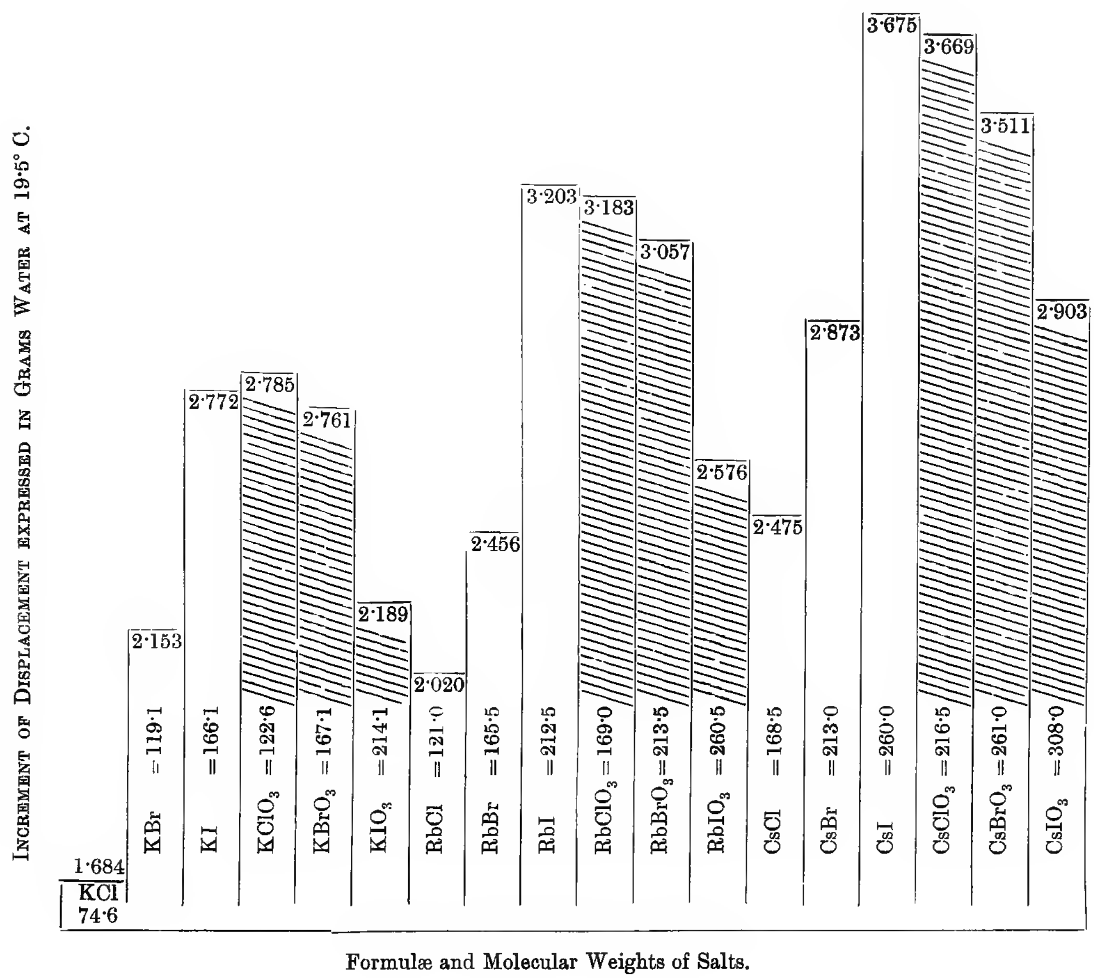
§ 38. In the following table we have solutions of the eighteen salts of the double ennead (MR, MRO_3) for which $m = 1/16$. It gives under w the weight of 1/16 gram-molecule of the salt dissolved in 1000 grams of water; under S, the specific gravity of this solution at 19·5° C., referred to that of distilled water at the same temperature as unity; and under v, the increment of displacement caused by dissolving 1/16 gram-molecule of the salt in 1000 grams of water, expressed in grams of distilled water having the temperature 19·5° C.

The solutions are arranged in three groups, each group containing six solutions of salts having the same metallic base (K, Rb, or Cs). These six solutions fall into two groups of three, or triads, the first three being the salts having the general formula MR, and the second triad those having the general formula MRO_3. Each triad is entered in the ascending order of the molecular weights of the salts which compose it, and each group of three triads forms the ennead MR or MRO_3 respectively.

Salt in Solution.	Molecular Weight of Salt.	Weight of 1/16 grm.-mol. Salt. *w*.	Specific Gravity of Solution of 1/16 grm.-mol. Salt in 1000 grams Water. S.	Increment of Displacement produced by the Dissolution of *w* grams Salt in 1000 grams Water. *v*.
KCl	74·6	4·6625	1·002973	1·684
KBr	119·1	7·4437	1·005279	2·153
KI	166·1	10·3812	1·007588	2·772
$KClO_3$	122·6	7·6625	1·004863	2·785
$KBrO_3$	167·1	10·4443	1·007662	2·761
KIO_3	214·1	13·3812	1·011169	2·189
RbCl	121·0	7·5625	1·005531	2·020
RbBr	165·5	10·3437	1·007868	2·456
RbI	212·5	13·2812	1·010046	3·203
$RbClO_3$	169·0	10·5593	1·007354	3·183
$RbBrO_3$	213·5	13·3412	1·010253	3·057
$RbIO_3$	260·5	16·2812	1·013673	2·576
CsCl	168·5	10·5312	1·008036	2·475
CsBr	213·0	13·3125	1·010409	2·873
CsI	260·0	16·2500	1·012529	3·675
$CsClO_3$	216·5	13·5312	1·009825	3·669
$CsBrO_3$	261·0	16·3125	1·012756	3·511
$CsIO_3$	308·0	19·2500	1·016299	2·903

If we consider the increments of displacement, *v*, produced by the dissolution of 1/16 gram-molecule of each of these salts in 1000 grams of water, we see that, for the salts of the same metal, the values increase from the chloride to the bromide, and from the bromide to the iodide; and that for the chlorates the values of *v* are almost identical with those for the iodides; they diminish from the chlorates to the bromates, and suffer a considerable fall from the bromates to the iodates. There is also a decided fall in the value of *v* from that of the iodate of potassium or rubidium to that of the chloride of rubidium or cæsium respectively.

The following diagram, illustrative of the above table, shows graphically, by the heights of the columns, the different increments of displacement produced by the dissolution in 1000 grams of water at 19·5° C. of 1/16 gram-molecule of each salt of the double ennead (MR, MRO_3). The columns representing the increment of displacement produced by salts of the ennead MRO_3 are shaded. It shows in a very striking manner the regular periodic variation of values of *v* from ennead to ennead. It is unfortunate that a complete series of solutions of higher concentration of all the salts of the double ennead cannot be obtained, on account of the sparing solubility of the oxyhalides, especially those of rubidium and cæsium.



Section VII.—The Displacement of the Solutions.

§ 39. When successive equal quantities of a salt are dissolved in a constant quantity of water, the successive increments of displacement of the solution, so produced, are generally unequal. They are usually the greater, the greater is the amount of salt which has already been dissolved.

In order usefully to discuss the change produced by any physical action, it is advisable to compare it with that which would be produced if it acted in accordance with some law which can be specified with precision. If the results observed agree with those calculated in terms of the law postulated, it is good evidence that the particular physical action takes place under the law. If no such agreement appears, then the observed results must be compared with those calculated in terms of the specification of some other law.

For the purpose of discussing the changes of displacement produced in a constant quantity of water by the dissolution of successive quantities of a salt in it, we compare them with those which would take place under one of two hypotheses.

§ 40. *First Hypothesis.*—It is assumed that, when a quantity of salt, insufficient for saturation, is dissolved in a quantity of water, it takes possession of the quantity of water which it requires in order to produce a saturated solution, and saturates it, after which the saturated solution disseminates itself through the remaining water, forming a simple mixture with it.

To take a particular case:—Let the constant quantity of water be 1 kilogram, and, when saturated with the particular salt used, let it take up 4 gram-molecules of it. When 1 gram-molecule of the salt has been dissolved in it, let the displacement of the solution so produced be 1·030 kilogram. We have then one-fourth of the water saturated by the first gram-molecule of salt added, producing an increment of displacement amounting to 30 grams. Let us now add a second gram-molecule of salt. There are 750 grams of free water remaining, and of these the second gram-molecule of salt takes possession of 250 grams, with which it forms a saturated solution; and this, with the 250 grams saturated water already present, disseminates itself through the remaining 500 grams of free water, and forms a homogeneous mixture of the two liquids.

If the increment of displacement produced by the dissolution of the first gram-molecule was 30 grams, that produced by the dissolution of the second must be the same, because it has been produced by an exact repetition of the first operation; and the total displacement after addition of the second gram-molecule salt must be 1·060 kilogram. Similarly, when the third gram-molecule has been dissolved, the total displacement will be 1·090 kilogram, and, when the fourth gram-molecule has been added, and saturation has been reached, the displacement must be 1·120 kilogram.

The numerical criterion, therefore, by which to decide if the aqueous solution of a particular salt follows this law is that, for equal additions of salt dissolved in a constant quantity of water, equal increments of displacement are produced.

If, by Δ, we represent the displacement of the solution produced by dissolving n parts of the salt in a constant quantity of water, then the above criterion finds expression in the equation:

$$\frac{d\Delta}{dn} = \text{Const.}$$

§ 41. *Second Hypothesis.*—It is assumed that, when a quantity of salt, insufficient to produce saturation, is dissolved in a quantity of water, it exercises no selection, but salinifies every particle of the water alike, producing a homogeneous solution of uniform concentration, and that, when a second quantity of salt, equal to the first, is dissolved in this solution, it intensifies its salinity uniformly and produces an increased displacement, which bears the same proportion to that of the first solution as the displacement of the first solution bore to that of the original quantity of water; further, that when a third, equal, quantity of salt is added to the solution of the second quantity, it intensifies its salinity uniformly and produces an increased displacement, which bears the same relation to that of the second solution as the displacement of the second solution bore to that of the first, and as that of the first bore to that of the original water; and so on.

As we may consider the equal quantities of salt successively added to the constant quantity of water to be as small as we please, our imagined process of solution becomes more and more nearly continuous.

If we represent by Δ_0 the displacement of the constant quantity of water, and by r the ratio of its displacement to Δ_1, that of the solution produced by the dissolution of the first of a series of n equal quantities of salt, then $r = \frac{\Delta_1}{\Delta_0}$ and is the common ratio of the displacement of each solution to that of the succeeding one in the series. Then the displacement of the solution after the n^{th} portion of salt has been dissolved is expressed by the equation:

$$\Delta_n = \Delta_0 r^n.$$

This equation expresses the fact that, when the quantities of salt dissolved in a constant quantity of water form an arithmetical series, the displacements of the respective solutions so produced form a geometrical series; consequently the logarithms of these displacements form an arithmetical series.

A convenient numerical criterion, therefore, by which to decide if the aqueous solution of a particular salt follows this law, is furnished by the degree of conformity of the observed displacements with those calculated on the basis of the equation:

$$\frac{d \log \Delta}{dn} = \text{Const.}$$

To take an example:—As before, let the constant quantity of water be 1 kilogram, which becomes saturated when 4 gram-molecules of the salt used are dissolved in it. When the first gram-molecule has been dissolved in it, let the displacement of the solution so produced be 1·030 kilogram. As the displacement of the water was 1·000 kilogram, the effect of the first operation has been to increase the total displacement in the proportion 1·000 : 1·030 = 1·030. When the second gram-molecule of salt is dissolved, it, by hypothesis, increases the displacement of the solution containing the first gram-molecule in the same proportion as the dissolution of the first gram-molecule increased that of the water, that is, in the proportion 1·000 : 1·030; consequently the displacement of the solution containing the first two gram-molecules of salt must be $(1{\cdot}030)^2$. Similarly, when the third gram-molecule has been dissolved, the total displacement must be $(1{\cdot}030)^3$; and, when the fourth gram-molecule has been dissolved and saturation has been produced, the displacement must be $(1{\cdot}030)^4$.

§ 42. The following table gives the means of comparing the effect produced by diluting or concentrating a given solution according as it takes place in terms of the first or the second of these hypotheses. The hypothetical salt MR has a molecular weight 160, which is nearly the mean of those of the ennead MR. The fundamental solution, from which all the others included in the table are derived, is that for which $m = 1/2$, and it is made by dissolving 80 grams of the salt in 1 kilogram of water. It is assumed that the dissolution of this mass of the salt in the kilogram of water at the standard temperature T, which may be 19·5° or any other, increases its displacement by 20 grams, so that the displacement of the fundamental solution is 1·020 kilogram. The concentration of this solution is then supposed to be altered by the addition to, or the withdrawal from, the kilogram of water, of salt so as to produce the series of solutions having the concentrations indicated under m in the first column of the table. From this the weight of each solution, W, given in the second column, follows as a matter of course.

Hypothetical Case.

Table giving the calculated Specific Gravities and Displacements of Solutions of m.MR + 1 kilogram of Water at T, where MR = 160 and the Displacement for $m = 1/2$ is 1·020 kilogram of Distilled Water at T.

m.	W.	S_A.	$1{\cdot}000 + 0{\cdot}04m = \Delta_A$.	$\text{Log } \Delta_{1/2} \times 2m = \log \Delta_L$.	Δ_L.	S_L.	$S_A - S_L$.
10	2·600000	1·857142	1·400000	0·1720034	1·485947	1·749725	0·107417
9	2·440000	1·794118	1·360000	0·1548030	1·428346	1·708390	85728
8	2·280000	1·727273	1·320000	0·1376027	1·372785	1·660856	56417
7	2·120000	1·656250	1·280000	0·1204023	1·319478	1·606695	49555
6	1·960000	1·580645	1·240000	0·1032020	1·268242	1·545447	35198
5	1·800000	1·500000	1·200000	0·0860017	1·218994	1·476627	23373
4	1·640000	1·413792	1·160000	0·0688013	1·171659	1·399724	14068
3	1·480000	1·321368	1·120000	0·0516010	1·126162	1·314197	7171
2	1·320000	1·222222	1·080000	0·0344006	1·082342	1·219473	2749
1	1·160000	1·115384	1·040000	0·0172003	1·040400	1·114955	329
1/2	1·080000	1·058823	1·020000	0·0086001	1·020000	1·058823	0·0
1/4	1·040000	1·029703	1·010000	0·0043000	1·009951	1·029753	− 50
1/8	1·020000	1·014925	1·005000	0·0021500	1·004963	1·014963	− 38
1/16	1·010000	1·007481	1·002500	0·0010750	1·002478	1·007503	− 21
1/32	1·005000	1·003745	1·001250	0·0005375	1·001238	1·003757	− 12
1/64	1·002500	1·001874	1·000625	0·0002687	1·000619	1·001880	− 6
1/128	1·001250	1·000937	1·000313	0·0001343	1·000309	1·000940	− 3
1/256	1·000625	1·000468	1·000156	0·0000671	1·000155	1·000470	− 2
1/512	1·000313	1·000234	1·000078	0·0000335	1·000077	1·000235	− 1
1	2	3	4	5	6	7	8

We now apply our first hypothesis (§ 40), and by means of it we obtain the displacement Δ_A of each of these solutions expressed in terms of the kilogram of distilled water of the standard temperature as unit, and recorded in column 4 of the table. Their specific gravities, the quotients obtained by dividing the weight of each solution (column 2) by that of the distilled water which it displaces (column 4), are entered in column 3 of the table under S_A.

Let us now apply the second hypothesis, as specified in § 41. The fundamental solution is that for which $m = 1/2$ and $w = 80$ grams, the displacement of which is taken as 1·020 kilogram; its logarithm is therefore 0·0086001.

The log-displacement of the solution for which $m = 1$ is then

$$\log \Delta_1 = 2 \times 0{\cdot}0086001 = 0{\cdot}0172002,$$

and the log-displacements entered in column 5 under Δ_L are obtained from the equation

$$\log \Delta_m = 0{\cdot}0172002 \times m.$$

The values of the displacements corresponding to these logarithms are given in column 6 under Δ_L. The quotient obtained by dividing the weight of the solution (column 2) by the corresponding displacement (column 6) gives the specific gravity of the solution, $\frac{W}{\Delta_L} = S_L$, the values of which are given in column 7.

For the value of $m = 1/2$ the numbers in columns 3 and 7, and in columns 4 and 6, are of course identical. For values of m greater than 1/2 the values of Δ_L are always greater than those of Δ_A, and for values of m less than 1/2 they are less; but, as the value of m increases the difference $\Delta_L - \Delta_A$ increases, so that when $m = 10$, $\Delta_L - \Delta_A =$ 0·085947. When the value of m diminishes, the difference $\Delta_A - \Delta_L$ diminishes also. If we turn to the specific gravities, we see that, in the case where $m = 1/32$, $S_A - S_L =$ $-$ 0·000012, so that, at this concentration, it is still possible to determine by observation whether the change of displacement with change of concentration has taken place according to the terms of the first or the second hypothesis. For greater dilutions the differences of specific gravity approach too nearly to the probable uncertainty of the observations to make this possible.

§ 43. If we study the tables in this memoir, we shall find that in the solutions of the majority of the salts the values of $d\Delta/dm$ and v/m reach a minimum for values of m in the vicinity of 1/32, and they increase whether the concentration of the solution is increased or diminished. At concentrations corresponding to $m > 2$ a number of the salts give solutions which conform nearly to the arithmetic law of the first hypothesis. The salts which furnish solutions which conform most closely to this law are those which contain at least one of the elements Li, Cs, or I. In the ennead MR, after the cæsium salts and the iodides come some of the rubidium salts and the bromides; the remainder of the bromides and nearly all the chlorides conform more nearly to the geometric law of the second hypothesis, and some of them may be said to conform exactly to it.

§ 44. From the equation $\log \Delta_m = 0{\cdot}0172002 \times m$ it follows that $\Delta_m = (1{\cdot}0404)^m$. Therefore, if the solutions of a salt follow strictly the law expressed by our second hypothesis, the general expression for the displacement of a solution containing m.MR in 1 kilogram of water, when the displacement for any particular value of m—for instance, for $m = 1$—is Δ_1, is $\Delta_m = \Delta_1^m$.

When the solution does not follow this law quite exactly, let the displacement for any particular value of m be $\Delta_m = \Delta_1^x$; then the degree in which the solution conforms to the law is indicated by the difference $x - m$.

In the table, the displacements given in column 6 are calculated on the basis of the second hypothesis. For them, therefore, the relation $\Delta_m = \Delta_1{}^m$ holds good, and the value of m (column 1) for any solution expresses not only its molecular concentration, but also the exponent of its displacement, that of Δ_1 being taken as unity.

The values of the displacement of the solutions in column 4 of the table are arrived at on the basis of our first hypothesis; consequently any value of Δ in this column is given by the equation $\Delta_m = 1{\cdot}000 + 0{\cdot}04m$. But none of the solutions dealt with in this memoir follow this law at all concentrations, though some of them approximate to it at high concentrations. It is therefore of use, in order to augment the illustrative value of the table, to determine the exponents of the values of Δ in column 4 when referred to that of $\Delta = 1{\cdot}020$ for $m = 1/2$ as $1/2$. This has been done, and the results are entered in the following table:—

m.	x.	$x - m$.	m.	x.	$1/m - 1/x$.
10	8·495	− 1·505	1/2	1/2	0·00
9	7·76	− 1·24	1/4	1/3·98	+ 0·02
8	7·00	− 1·00	1/8	1/7·943	+ 0·057
7	6·23	− 0·77	1/16	1/15·873	+ 0·127
6	5·43	− 0·57	1/32	1/31·706	+ 0·294
5	4·60	− 0·40	1/64	1/63·412	+ 0·588
4	3·75	− 0·25	1/128	1/127·52	+ 0·480
3	2·86	− 0·14	1/256	1/254·06	+ 1·940
2	1·94	− 0·06	1/512	1/508·90	+ 3·100
1	0·99	− 0·01			
1/2	0·50	0·00			

§ 45. In the following tables the displacements of most of the solutions have been treated along these lines. In the first four tables we have for the solutions of the salts of the ennead MR the values of x and of $x - m$ for the strong solutions, and those of x and $1/m - 1/x$ for the dilute solutions. For solutions of other salts the tables give only the values of $x - m$ or $1/m - 1/x$, as they are sufficient.

The numbers representing the values of x and m for the strong solutions are the numerators of vulgar fractions having unity for common denominator. The numbers representing the values of $1/x$ and $1/m$ for the weak solutions are the denominators of vulgar fractions having unity for common numerator. The measure of the departure of the displacements of solutions of a particular salt from the geometric law of the second hypothesis is found for the strong solutions in the column headed $(x - m)$. For the weak solutions the corresponding column is headed $(1/m - 1/x)$, so that the signs prefixed to the numbers in these columns mean the same thing in both tables:—the + sign means that $x > m$, the − sign that $x < m$. In the strong solutions $x = m$ when $m = 1$, and the remaining values of m increase; in the weak solutions $x = m$ when $m = 1/2$, and the remaining values of m diminish.

TABLE I.

Values of x for the Ennead MR. (Strong Solutions.)

	T=19·5° C.							T=21·4° C.	T=23·1° C.
m.	KCl.	KBr.*	KI.	RbCl.*	RbBr.	RbI.	CsCl.	CsBr.	CsI.
1	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00
2	2·04	2·01	1·98	2·01	1·97	1·97	1·97	1·96	1·96
3	3·09	3·01	2·92	3·03	2·96	2·92	2·94	2·95	2·86
4	4·13	4·00	3·86	4·00	3·93	3·82	3·92	3·84	3·76
5		4·95	4·76	4·95	4·86	4·69	4·80	4·81	
6			5·63	5·93	5·78	5·53	5·74		
7			6·48	6·88		6·41	6·58		
8			7·30				7·39		
9							8·24		

* Compare table, p. 173.

TABLE II.

Values of x − m for the Ennead MR. (Strong Solutions.)

	T=19·5° C.							T=21·4° C.	T=23·1° C.
m.	KCl.	KBr.	KI.	RbCl.	RbBr.	RbI.	CsCl.	CsBr.	CsI.
1	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00
2	0·04	0·01	−0·02	0·01	−0·03	−0·03	−0·03	−0·04	−0·04
3	0·09	0·01	−0·08	0·03	−0·04	−0·08	−0·06	−0·05	−0·14
4	0·13	0·00	−0·14	0·00	−0·07	−0·18	−0·08	−0·16	−0·24
5		−0·05	−0·24	−0·05	−0·14	−0·31	−0·20	−0·19	
6			−0·37	−0·07	−0·22	−0·47	−0·26		
7			−0·52	−0·12		−0·59	−0·42		
8			−0·70				−0·61		
9							−0·76		

TABLE III.

Values of x for the Ennead MR.

T = 19·5° C.

m.	KCl.	KBr.	KI.	RbCl.	RbBr.	RbI.	CsCl.	CsBr.	CsI.
1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2	1/2
1/4	1/4·04	1/4·02	1/4·00	1/4·04	1/4·04	1/3·98	1/3·60	1/4·04	1/3·92
1/8	1/8·14	1/8·08	1/8·08	1/8·14	1/8·16	1/7·94	1/8·12	1/8·14	1/7·98
1/16	1/16·54	1/16·16	1/16·26	1/16·34	1/16·34	1/15·92	1/16·32	1/16·36	1/13·88
1/32	1/33·06	1/32·18	1/32·30	1/32·80	1/32·84	1/31·82	1/32·96	1/32·86	1/32·28
1/64	1/65·60	1/62·82	1/64·82	1/67·40	1/63·98	1/62·70	1/66·80	1/64·00	1/62·32
1/128	1/127·52	1/124·90	1/129·70	1/136·22	1/130·12	1/120·76	1/138·82	1/130·28	1/120·88
1/256	1/282·24	1/249·02	1/266·98	1/268·58	1/224·98	1/233·80	1/279·86	1/212·28	1/211·20
1/512	1/434·42	1/466·90	1/505·40	1/450·84	1/444·92	1/356·20	1/506·40	1/440·20	1/248·94
1/1024			1/1120·46			1/835·36		1/489·28	1/382·38

Table IV.

Values of $1/m - 1/x$ *for the Ennead MR.*

T = 19·5° C.

m.	KCl.	KBr.	KI.	RbCl.	RbBr.	RbI.	CsCl.	CsBr.	CsI.
1/2	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00
1/4	− 0·04	− 0·02	0·00	− 0·04	− 0·04	+ 0·02	+ 0·40	− 0·04	+ 0·08
1/8	− 0·14	− 0·08	− 0·08	− 0·14	− 0·16	+ 0·06	− 0·12	− 0·14	+ 0·02
1/16	− 0·54	− 0·16	− 0·26	− 0·34	− 0·34	+ 0·08	− 0·32	− 0·36	+ 2·12
1/32	− 1·06	− 0·18	− 0·26	− 0·80	− 0·84	+ 0·18	− 0·96	− 0·86	− 0·28
1/64	− 1·60	+ 1·18	− 0·82	− 3·40	+ 0·02	+ 1·30	− 2·80	0·00	+ 1·68
1/128	+ 0·28	+ 3·10	− 1·70	− 8·22	− 2·12	+ 7·24	10·82	− 2·28	+ 7·12
1/256	− 26·24	+ 6·98	− 10·98	− 12·58	+ 31·02	+ 22·20	− 23·86	+ 33·72	+ 44·80
1/512	+ 77·58	+ 45·10	+ 6·60	+ 53·16	+ 67·08	+ 155·80	+ 5·60	+ 71·80	+ 263·06
1/1024			− 96·46			+ 188·64		+ 534·72	+ 641·62

Table V.

Values of $1/m - 1/x$ *for the Ennead* MRO_3.

T = 19·5° C.

m.	$KClO_3$.	$KBrO_3$.	KIO_3.	$RbClO_3$.	$RbBrO_3$.	$RbIO_3$.	$CsClO_3$.	$CsBrO_3$.	$CsIO_3$.
1/2									
1/4	0·00	0·00	0·00	0·00			0·00		
1/8	0·00	− 0·08	− 0·12	+ 0·02			+ 0·04		
1/16	− 0·20	− 0·28	− 0·08	+ 0·09	0·00	0·00	+ 0·28	0·00	0·00
1/32	− 1·76	− 0·76	− 0·08	+ 0·05	+ 0·32	− 0·16	+ 0·04	+ 0·32	+ 0·64
1/64	− 4·24	− 1·24	+ 3·76	− 1·24	+ 0·48	+ 0·32	+ 4·64	− 0·80	+ 5·12
1/128	11·12	− 1·36	− 2·56	+ 1·76	+ 8·00	+ 9·20	− 2·92	− 0·96	+ 26·72
1/256	− 28·28	+ 1·72	− 20·36	+ 4·16	+ 1·92	+ 30·16	− 59·16	+ 17·60	+ 86·08
1/512	− 274·92	+ 8·80	− 103·20	+ 70·08	+ 6·72	− 53·44	+ 240·96	+ 95·04	+ 206·88

Table VI.

Values of $1/m - 1/x$ *for Solutions of the Nitrates.*

M′ or M″ =	Na.	K.	Sr″	Ba″.	Li.	Na.	K.	Rb.	Cs.	Ba″	Pb″.	Rb.	Cs.
T =	15·0° C.				19·5° C.							23·0° C.	
1/2													
1/4					− 0·04	− 0·04	− 0·02	− 0·02					
1/8					− 0·06	− 0·16	− 0·14	− 0·08	0·00			− 0·24	
1/16		− 0·14			− 0·38	− 0·54	− 0·42	− 0·72	− 0·16			− 0·72	− 0·16
1/32	+ 0·48	+ 0·02			− 1·32	− 1·28	− 1·60	− 2·84	− 1·56	− 0·80	− 0 80	− 0·28	− 0·48
1/64	+ 0·72	+ 0·34	+ 0·32	− 1·60	− 4 04	− 2·44	− 2·90	− 14·70	+ 2·88	− 3·04	− 4·48	− 12·32	− 2·56
1/128	+ 0·80	+ 1·90	− 1·44	− 28·32	− 13·94		− 4·86	− 59·36	− 5·80	− 16·64	− 7·84	− 93·24	− 21·60
1/256			+ 4·48	+ 7·68				− 112·22	− 64·16	− 35·20	− 19·84	+ 2·00	− 20·96
1/512			+ 23·68	− 6·72				+ 233·04	− 317·72	− 109·12	− 139·84		− 253·76
1/1024				− 228·80					+ 161·56	+ 69·60	− 951·04		− 3258·32

TABLE VII.

The following Table gives Values of m and x − m for the Solutions of a number of Salts derived from Determinations of their Specific Gravity at 19·5° C. by Kremers. (Pogg, 1855, vols. xcv. and xcvi.)

	$NaNO_3$.	$NaClO_3$.	$NaBrO_3$.	LiCl.	LiBr.	LiI.	NaCl.	NaBr.	NaI.
Δ_1 for $m = 1$	1029·70	1037·025	1036·51	1018·52	1028·35	1036·09	1018·25	1025·225	1036·03
m.	Values of $x - m$.								
1	0	0	0	0	0	0	0	0	0
2	0·024	0·021	0·027	0·056	0·000	0·000	0·072	0·037	− 0·010
3	0·066	0·030		0·117	0·067	− 0·032	0·167	0·089	− 0·035
4	0·110	0·011		0·182	0·203	− 0·064	0·310	0·140	− 0·145
5	0·114	0·301		0·232	0·338	− 0·129	0·458	0·186	− 0·140
6	0·170			0·271	0·350	− 0·265	0·611	0·216	− 0·233
7	0·177			0·290	0·234	− 0·365		0·234	− 0·340
8	0·164			0·290	0·125	− 0·585		0·243	− 0·466
9	0·131			0·290	0·013	− 0·805		0·240	− 0·635
10	0·081			0·290	− 0·190	− 1·025			− 0·803

In the tables given above we have, under the values of x, the values of log Δ_m/log Δ_1, and they show, when compared with the values of m, the degree in which the displacement of the solutions follows the logarithmic law. But there is a certain disadvantage in taking as our unit of comparison the logarithm of the displacement for $m = 1$ when the difference between the values of m and 1 is considerable, inasmuch as the effects due to departures from the logarithmic law are cumulative. In the following tables we have the values of log Δ_m/log $\Delta_{\frac{m}{2}}$ for a number of pairs of solutions of salts belonging to the enneads MR and MRO_3, and of certain nitrates.

TABLE VIII.

Values of $\dfrac{\log \Delta_m}{\log \Delta_{\frac{m}{2}}}$ *for the Ennead MR.*

T = 19·5° C.

m.	KCl.	KBr.	KI.	RbCl.	RbBr.	RbI.	CsCl.	CsBr.	CsI.
8			1·891	1·966		1·894	1·883		
4	2·027	1·992	1·939	2·021	1·977	1·940	1·988	1·010	1·981
2	2·041	2·010	1·990	2·076	1·988	1·972	1·976	1·920	1·969
1	2·015	2·002	2·006	2·155	2·020	1·991	2·054	2·040	2·107
1/2	2·022	2·010	2·008	2·020	2·019	1·998	2·016	2·000	1·992
1/4	2·016	2·010	2·018	2·017	2·015	1·992	2·012	2·020	2·006
1/8	2·026	1·999	2·008	2·006	2·009	2·002	2·012	2·016	1·995
1/16	2·001	1·988	1·985	2·006	2·008	1·997	2·016	1·958	2·023
1/32	1·983	1·947	2·006	2·053	1·945	1·969	2·021	2·108	1·930
1/64	1·945	1·979	1·999	2·050	2·031	1·923	2·067	1·765	1·938
1/128	2·202	1·973	2·053	1·937	1·628	1·936	1·994	1·748	1·744
1/256	1·530	1·842	1·882	1·670	2·089	1·521	1·778	2·074	1·176
1/512			2·184		1·091	2·303		1·704	1·529

Table IX.

Values of $\dfrac{\log \Delta_m}{\log \Delta_{\frac{m}{2}}}$ *for the Ennead* MRO_3.

T = 19·5° C.

m.	$KClO_3$.	$KBrO_3$.	KIO_3.	$RbClO_3$.	$RbBrO_3$.	$RbIO_3$.	$CsClO_3$.	$CsBrO_3$.	$CsIO_3$.
1/4	2·010	2·019	2·031	1·997			1·999		
1/8	2·019	2·017	1·980	1·993			1·968		
1/16	2·081	2·013	1·996	2·007	1·983	2·018	2·029	1·985	2·028
1/32	2·021	1·990	1·875	1·982	2·008	1·931	1·859	2·044	1·918
1/64	2·037	1·983	2·165	1·882	1·884	1·924	2·039	2·047	1·788
1/128	2·039	1·965	1·112	1·887	2·122	1·814	1·624	1·792	1·795
1/256	2·746	1·977	1·211	1·653	1·974	2·635	1·376	1·746	2·080
1/512									

Table X.

Values of $\dfrac{\log \Delta_m}{\log \Delta_{\frac{m}{2}}}$ *for some Nitrates.*

m.	Li.	Na.	K.	Rb.	Cs.	Ba″.	Pb″.	Rb.	Cs.
T =	19·5° C.							23·0° C.	
8	1·939	2·019							
4	1·982	1·993							
2	2·003	2·049		2·000					
1	1·984	2·027		2·006					
1/2	2·023	2·026	2·014	2·008					
1/4	1·995	2·016	2·025	2·015	2·006			2·069	
1/8	2·030	2·020	2·015	2·068	2·017			2·019	2·019
1/16	2·034	2·013	2·046	2·082	2·075	2·048	2·056	2·091	2·013
1/32	2·040	1·995	1·983	2·258	1·880	2·046	2·086	2·182	2·048
1/64	2·083		2·050	2·379	2·117	2·159	1·981	2·901	2·245
1/128				1·961	2·388	2·010	2·031	1·146	1·851
1/256						2·129	2·359		
1/512						1·528	3·010		

Section VIII.—Comments on the Changes in the Values of $d\Delta - v$ for Different Values of m in the Case of Solutions of Individual Salts of the Type MR and MRO_3.

§ 46. The earlier introductory notes and discussion of the values of the specific gravity and displacement explain the precise meaning of these terms. A comparison of the values of the increment of displacement (v) caused by the dissolution of m gram-molecules of a salt in 1000 grams of water with the difference ($d\Delta$) of these consecutive increments of displacement possesses considerable interest.

The increment of displacement (v) due to the dissolution of m gram-molecules of a salt in 1000 grams of water may be looked on as being the result of two operations, namely: (*a*) the dissolution of $\frac{m}{2}$ gram-molecules of the salt in 1000 grams of water, which produces the first increment of displacement; and (*b*) the further dissolution of $\frac{m}{2}$ gram-molecules of the salt in the solution formed, which produces the second increment of displacement. These increments of displacement are very seldom found to be alike; the second portion of salt dissolved generally produces a greater increment of displacement than the first. In the case of solutions of such concentration that their specific gravity can be ascertained by the use of the pyknometer without too great probability of error, this feature is found to be general, and has often been held to be universal. *One of the principal motives for making this research was to find out, by the use of the more refined hydrometric method, if there is any point in the dilution of a saline solution at which further dilution is accompanied by expansion, in place of contraction. The general result of the work is to show that in solutions having the concentrations here used, where* $m < 1/16$, *cases of expansion on dilution are not uncommon.*

The following table gives the values of m for which the value of $(d\Delta - v)$ is positive and becomes negative for the next lower value of m. That is, the value $(d\Delta - v)$ changes sign at some concentration lower than that indicated by m and higher than that indicated by $1/2m$.

T = 19·5° C.	MR	KCl	RbCl	CsCl	KBr	RbBr	CsBr	KI	RbI	CsI
	m	1/32	1/128	1/256	1/16	1/32	1/64	1/16	1/16	1/32
	MRO_3	$KClO_3$	$RbClO_3$	$CsClO_3$	$KBrO_3$	$RbBrO_3$	$CsBrO_3$	KIO_3	$RbIO_3$	$CsIO_3$
	m	1/512	1/64	1/32	1/32	1/64	1/128	1/16	1/32	1/16
	MNO_3	$LiNO_3$	$NaNO_3$	KNO_3	$RbNO_3$	$CsNO_3$	$Ba(NO_3)_2$	$Pb(NO_3)_2$		
	m	1/2	1/32	1/32	1/128	1/32	1/512	1/64		
T = 23·0° C.	MR	RbBr	CsBr	KI	RbI	CsI				
	m	1/32	1/8	1/16	1/16	1/32				
	MNO_3	$RbNO_3$	$CsNO_3$							
	m	1/128	1/128							
T = 15·0° C.	MR	KCl	NaCl							
	m	1/32	1/16							
	MNO_3	$NaNO_3$	KNO_3	$Sr(NO_3)_2$	$Ba(NO_3)_2$					
	m	1/16	1/16	1/128	1/128					

§ 47. If we consider Table No. 1, Class A, § 26, in which are recorded the results of experiment on solutions of chloride of potassium having the concentrations specified by the general expression mKCl + 1000 grams of water, we have, besides the weight of the solution and its specific gravity, columns containing its displacement, Δ, and the difference, $d\Delta$, between consecutive increasing values of Δ. If we consider the pair of solutions for which $m = 1/8$ and $1/16$, we find that the dissolution of the quantity 1/16 KCl in 1000 grams of water produces an increment of displacement $v = 1{\cdot}684$ $G_{19{\cdot}5^\circ}$. When we dissolve a further 1/16 KCl in the solution, the displacement is increased by $d\Delta = 1{\cdot}732$ $G_{19{\cdot}5^\circ}$. The difference $1{\cdot}732 - 1{\cdot}684 = 0{\cdot}048$ is quite genuine. If we now consider the pair of solutions for which $m = 1/256$ and $1/512$, we have the displacements $\Delta = 1000{\cdot}098$ and $1000{\cdot}064$ respectively. Here the dissolution of the first 1/512 KCl causes an increment of displacement of $v = 0{\cdot}064$ $G_{19{\cdot}5^\circ}$, while the dissolution of the second 1/512 increases the displacement by only $0{\cdot}034$ $G_{19{\cdot}5^\circ}$, showing that contraction has taken place when the concentration of the solution has been increased from 1/512 to 2/512 gram-molecule per 1000 grams of water. From this it follows necessarily that the dilution of 1/256 KCl + 1000 grams of water to 1/512 KCl + 1000 grams of water is accompanied by expansion. The specimen table for KCl in § 50 is constructed so as to show the character of the change of displacement of a solution with change of its concentration. The first line (n) gives the ordinal number of each column; in the second line (m) we have the quantity of salt, expressed in terms of the gram-molecule, dissolved in 1000 grams of water; and in the third line (v) we have the increment of displacement in terms of grams of water at the temperature of observation, G_T. Then follow three sub-tables (a, b, c). In sub-table a consecutive values of m, Δ, etc., are considered; in sub-table b alternate values of m, Δ, etc., are considered; and in sub-table c values are given which represent the mean of those given in a and b. The last line in each sub-table contains the values of the differences $d\Delta - v$, $d\Delta' - v'$, and $d\Delta'' - v''$, arrived at in sub-tables a, b, and c respectively.

§ 48. Before entering upon a detailed examination of the tables giving values of $d\Delta$ and v and their differences $d\Delta - v$, we will consider the influence of change in specific gravity on the value of the displacement, from which it will be possible to ascertain how far the differences $d\Delta - v$ are to be accepted as independent of experimental error.

For this purpose we will consider the effect of change of specific gravity on the least concentrated solution of the salt KCl, the value of m being 1/512, and the molecular weight of the salt being 74·6.

The mean specific gravity is given in the table as 1·000082; the weight of solution composed of 1000 grams of water and 1/512 gram-molecule KCl is 1000·1457 grams; the displacement is therefore 1000·064. The difference of displacement between the 1/256 and 1/512 gram-molecule solution is 0·034.

We have seen (§ 35) that the mean probable error of the specific gravity of any of the solutions entered in the tables, Class A, is ± 3 in the sixth decimal place; and this is independent of the concentration of the solution. When the values of the displacement are obtained by the use of any of these specific gravities, the probable error is only increased by that due to the preparation of the solution, which may be neglected (see § 49). The values of displacement so arrived at are affected by a probable error which is also independent of the concentration.

When, however, we consider the values of v, $(=\Delta-1000)$, or those of $d\Delta$, the probable error has a close relation to the concentration. Thus, when the value for Δ is 1000·064, this effect of an error of ± 3 in the third decimal place, which corresponds to the sixth place of decimals in the value of the specific gravity, is insensible; but when we consider the value of $v = 0{\cdot}064$, a difference of $\pm 0{\cdot}003$ gives the values 0·061 and 0·067 as possible values, and this is equivalent to an uncertainty in the value of v having a range of 0·006, which is 9·4 per cent. of 0·064, the mean value of v.

If we consider a solution of greater concentration, *e.g.* that of 1/128 KCl in 1000 grams of water, the value of v is 0·217, which, when affected by a probable error of $\pm 0{\cdot}003$, gives a possible range of uncertainty of 0·006, and this is only 2·75 per cent. of the mean value of v, which is 0·217.

We have considered the case of KCl, which has a low molecular weight, and its solutions have a comparatively low specific gravity. Salts of higher molecular weight, such as KI or the salts of rubidium and cæsium, form solutions which have higher specific gravities for an equivalent concentration. In these cases the range of uncertainty of the values of v and $d\Delta$ is relatively less considerable than that in the case of KCl.

§ 49. The following table furnishes evidence of the possible uncertainty of the value of the displacement of a solution which is due to the accumulation of the errors affecting the preparation of the solution and those affecting the determination of its specific gravity when practised by different experimenters.

POTASSIUM CHLORIDE. KCl = 74·6.

T = 19·5° C.

m.	Weight of Solution.	Specific Gravity obtained by		Corresponding Displacement.	
		D.	B.	D.	B.
1/8	1009·325	1·005889	1·005911	1003·416	1003·394
1/16	1004·662	1·002973	1·002972	1001·684	1001·685
1/32	1002·331	1·001489	1·001473	1000·841	1000·857
1/64	1001·166	1·000741	1·000740	1000·423	1000·424
1/128	1000·582	1·000365	1·000376	1000·217	1000·206
1/256	1000·291	1·000193	1·000195	1000·098	1000·096
1/512	1000·146	1·000082	1·000073	1000·064	1000·073

The numbers in the columns headed D are abstracted from Table No. 1, Class A. The experiments were made by Mr H. Royal-Dawson in May 1904, using hydrometers Nos. 17 and 21. The scheme for the preparation of the various concentrations of the solutions in this series was that of diluting a known quantity of the stronger solution with the quantity of water necessary to produce the solution whose strength was one-half that of the solution from which it was made; thus providing a series of solutions whose strengths diminish in a geometric succession.

The numbers in the columns headed B are abstracted from a series of experiments made by Mr S. M. Bosworth in March 1911, using hydrometers Nos. 17 and 3. The series from which these values have been extracted was an arithmetical one with the common difference 1/64 gram-molecule, with the exception of the last three, which formed a geometric series. The method of preparation of these solutions was as follows:—

The first solution was made by dissolving 1/64 gram-molecule KCl in 1000 grams of water. From this the 2/64 gram-molecule solution was obtained by the addition of 1/64 gram-molecule KCl to the quantity of the 1/64 gram-molecule solution containing 1000 grams of water.

Thus in each case the more concentrated solution was prepared from the more dilute by the addition of the requisite quantity of salt, while in Mr Royal-Dawson's practice the more dilute solution was prepared from the more concentrated by the addition of the requisite quantity of water.

Mr Royal-Dawson's results obtained with solutions of chloride of potassium, quoted in the table, were among the first which were accepted as final and admitted into this memoir. Before May 1904 Mr Royal-Dawson's work consisted in learning the art of the exact use of the hydrometer and in assisting me in the elaboration of the plan of making and recording the experiments, which has since been adhered to. Mr Bosworth's results were obtained after he had perfected himself in the use of the instrument and after the system had been in continuous operation for seven years. When it is further taken into account that the two experimenters prepared their solutions in almost opposite ways, it will be admitted that the juxtaposition of the two sets of experiments subjects the method to a very stringent test, and that their agreement affords the best evidence of the trustworthiness of the experimental method used in this research.

§ 50. *Specimen Table indicating the Stages in the Calculations used in the Discussion of the Values of v and dΔ in the Case of Solutions of Different Concentrations of KCl.*

a	n	9	8	7	6	5	4	3	2	1
b	m	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024
c	v	6·899	3·416	1·684	0·841	0·423	0·217	0·098	0·064	
d	SUB-TABLE *a*.									
e	$v_{n+1} - v_n = d\Delta$	7·102	3·483	1·732	0·843	0·418	0·206	0·119	0·034	
f	$\Delta - 1000 = v$	6·899	3·416	1·684	0·841	0·423	0·217	0·098	0·064	
g	$d\Delta - v$	+0·203	+0·067	+0·048	+0·002	−0·005	−0·011	+0·021	−0·030	
h	SUB-TABLE *b*.									
j	$v_{n+1} - v_{n-1}$		10·585	5·215	2·575	1·261	0·624	0·325	0·153	
k	$\frac{1}{3}(v_{n+1} - v_{n-1})$		3·528	1·738	0·858	0·420	0·208	0·108	0·051	
l	$v'_{n+1} - v'_n = d\Delta'$		3·522	1·723	0·856	0·418	0·219	0·091	0·051	
m	$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	6·944	3·422	1·699	0·843	0·425	0·206	0·115	0·064	
n	$d\Delta' - v'$		+0·100	+0·024	+0·013	−0·007	+0·013	−0·024	−0·013	
o	SUB-TABLE *c*.									
p	$v''_{n+1} - v''_n = d\Delta''$	7·080	3·502	1·728	0·849	0·416	0·213	0·105	0·042	
q	$\frac{v' + v}{2} = v''$	6·921	3·419	1·691	0·842	0·424	0·211	0·106	0·064	
r	$d\Delta'' - v''$	+0·159	+0·083	+0·037	+0·007	−0·008	+0·002	−0·001	−0·022	

Note.—The numbers in line n designate the columns of the table; m is the concentration of the solution expressed in gram-molecules of salt dissolved in 1000 grams of water.

The significance of the symbols m, $d\Delta$, v, and $d\Delta - v$ has been already explained.

The differences $(d\Delta - v)$ are accounted positive when $d\Delta$ is greater than v, and negative when the reverse condition holds.

From an inspection of the values of $d\Delta$ and v in sub-table a, as derived from consecutive values of Δ, it will be seen that there is a decrease in the positive values of $d\Delta - v$ for diminishing values of m from 1/4 to 1/32, when change of sign occurs and negative values are obtained for $m = 1/64$ and 1/128, then a reversal to a positive quantity of +0·021 for 1/256, and again a negative value for 1/512.

There is thus a steady decrease in the positive values for $d\Delta - v$ from $m = 1/4$ to $m = 1/32$. Between $m = 1/32$ and $m = 1/64$ there is change of sign, and from $m = 1/64$ to $m = 1/512$ there are increasing negative values, with exception of +0·021 at $m = 1/256$.

The value +0·021 for $d\Delta - v$ at $m = 1/256$ is of appreciable magnitude, but the concentration of the solution is low, and, as shown in the example for calculating the effect of specific gravity changes on the displacement, its relative value would be considerably affected by slight changes in the value of the specific gravity.

If we now consider alternate values v_{n+1} and v_{n-1} instead of consecutive values

v_n and v_{n+1}, the difference of these quantities as given in sub-table b, line j, includes a value for $d\Delta$ which is approximately three times greater than that obtained by taking consecutive values v_{n+1} and v_n as in sub-table a.

Hence one-third of the difference of displacement (line k, sub-table b), added to the lower value v_{n-1} employed for the purpose of obtaining the difference, gives a new interpolated value v' for the solution of m (see line m, sub-table b).

From this series of interpolated values of v' a corresponding set of values, $d\Delta'$ (see line l, sub-table b), is obtained by taking the difference of consecutive values of v', and the differences of these values, $d\Delta'$ and v', give a new series of values, $d\Delta' - v'$, set out in line n, sub-table b.

A comparison of the values of $d\Delta' - v'$ (sub-table b) with those of $d\Delta - v$ (in sub-table a) shows the same general character of the change of values and of sign from $m = 1/8$ to $m = 1/512$, only that the reversal of sign takes place at $m = 1/128$ instead of at $m = 1/256$ as in sub-table a.

A third series of values is derived by using the mean of corresponding values of v and v', namely, $\frac{1}{2}(v + v') = v''$, and obtaining the difference $d\Delta'' - v''$ in the same way as the corresponding values of $d\Delta - v$ and $d\Delta' - v'$ were obtained.

An inspection of these new values shows that the character of the decrease of the positive value is the same as in the other two cases, but the reversal of sign remains at $m = 1/128$ as in sub-table b. The magnitudes of the values $d\Delta'' - v''$ when $m < 1/32$ are, however, so small that they would indicate no comparative change of volume when successive small increments of salt were added to these dilute solutions.

These calculations serve to indicate the character of the relative changes of volume occurring when successive dilutions are effected.

The results of the treatment of the values of v in the case of KCl justifies a similar treatment of the values of v in the solutions of the other salts included in the series of tables, Class A, and the results for the salts of the types MR and MRO_3 are given in § 51 in a series of tables of Class F for each salt, similar to the specimen table given above, and arranged in triads of the salts having the same acid radical, R or RO_3 as the case may be.

§ 51. Tables of Class F, illustrating the Method of arriving at the Volumetric Effect produced by changing the Concentration of a Solution.

TRIAD OF CHLORIDES.

Table I. POTASSIUM CHLORIDE. KCl = 74·6. T = 19·5° C.

n	9	8	7	6	5	4	3	2	1
m	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024
v	6·899	3·416	1·684	0·841	0·423	0·217	0·098	0·064	
Sub-table *a*.									
$v_{n+1} - v_n = d\Delta$	7·102	3·483	1·732	0·843	0·418	0·206	0·119	0·034	
$\Delta - 1000 = v$	6·899	3·416	1·684	0·841	0·423	0·217	0·098	0·064	
$d\Delta - v$	+0·203	+0·067	+0·048	+0·002	−0·005	−0·011	+0·021	−0·030	
Sub-table *b*.									
$v_{n+1} - v_{n-1}$		10·585	5·215	2·575	1·261	0·624	0·325	0·153	
$\frac{1}{3}(v_{n+1} - v_{n-1})$		3·528	1·738	0·858	0·420	0·208	0·108	0·051	
$v'_{n+1} - v'_n = d\Delta'$		3·522	1·723	0·856	0·418	0·219	0·091	0·051	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	6·944	3·422	1·699	0·843	0·425	0·206	0·115	0·064	
$d\Delta' - v'$		+0·100	+0·024	+0·013	−0·007	+0·013	−0·024	−0·013	
Sub-table *c*.									
$v''_{n+1} - v''_n = d\Delta''$	7·080	3·502	1·728	0·849	0·416	0·213	0·105	0·042	
$\frac{v'+v}{2} = v''$	6·921	3·419	1·691	0·842	0·424	0·211	0·106	0·064	
$d\Delta'' - v''$	+0·159	+0·083	+0·037	+0·007	−0·008	+0·002	−0·001	−0·022	

Table II. RUBIDIUM CHLORIDE. RbCl = 121·0. T = 19·5° C.

	9	8	7	6	5	4	3	2	1
Sub-table *a*.									
$v_{n+1} - v_n = d\Delta$	8·435	4·145	2·037	1·014	0·517	0·251	0·116	0·049	
$\Delta - 1000 = v$	8·202	4·057	2·020	1·006	0·489	0·238	0·122	0·073	
$d\Delta - v$	+0·233	+0·088	+0·017	+0·008	+0·028	+0·013	−0·006	−0·024	
Sub-table *b*.									
$v_{n+1} - v_{n-1}$		12·580	6·182	3·051	1·531	0·768	0·367	0·165	
$\frac{1}{3}(v_{n+1} - v_{n-1})$		4·193	2·060	1·017	0·510	0·256	0·122	0·082	
$v'_{n+1} - v'_n = d\Delta'$		4·170	2·057	1·024	0·505	0·250	0·089	0·082	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	8·250	4·080	2·023	0·999	0·494	0·244	0·155	0·073	
$d\Delta' - v'$		+0·090	+0·034	+0·025	+0·011	+0·006	−0·066	+0·009	
Sub-table *c*.									
$v''_{n+1} - v''_n = d\Delta''$	8·411	4·158	2·047	1·019	0·511	0·250	0·103	0·065	
$\frac{v'+v}{2} = v''$	8·226	4·068	2·021	1·002	0·491	0·241	0·138	0·073	
$d\Delta'' - v''$	+0·185	+0·090	+0·026	+0·017	+0·020	+0·009	−0·035	−0·008	

Table III. CÆSIUM CHLORIDE. CsCl = 168·5. T = 19·5° C.

	9	8	7	6	5	4	3	2	1
Sub-table *a*.									
$v_{n+1} - v_n = d\Delta$	10·335	5·077	2·514	1·250	0·621	0·313	0·147	0·065	
$\Delta - 1000 = v$	10·066	4·989	2·475	1·225	0·604	0·291	0·144	0·079	
$d\Delta - v$	+0·269	+0·088	+0·039	+0·025	+0·017	+0·022	+0·003	−0·014	
Sub-table *b*.									
$v_{n+1} - v_{n-1}$		15·412	7·591	3·764	1·871	0·934	0·460	0·216	
$\frac{1}{3}(v_{n+1} - v_{n-1})$		5·137	2·530	1·255	0·624	0·311	0·153	0·072	
$v'_{n+1} - v'_n = d\Delta'$		5·121	2·525	1·252	0·626	0·305	0·150	0·068	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	10·126	5·005	2·480	1·228	0·620	0·297	0·147	0·079	
$d\Delta' - v'$		+0·116	+0·045	+0·024	+0·024	+0·008	+0·003	−0·011	
Sub-table *c*.									
$v''_{n+1} - v''_n = d\Delta''$	10·305	5·099	2·520	1·251	0·623	0·309	0·149	0·070	
$\frac{v'+v}{2} = v''$	10·096	4·997	2·477	1·226	0·603	0·294	0·145	0·075	
$d\Delta'' - v''$	+0·209	+0·102	+0·044	+0·025	+0·020	+0·015	+0·004	−0·005	

Tables of Class F, illustrating the Method of arriving at the Volumetric Effect produced by changing the Concentration of a Solution.

TRIAD OF BROMIDES.

Table IV. POTASSIUM BROMIDE. KBr = 119·1. T = 19·5° C.

n	9	8	7	6	5	4	3	2	1
m	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024
v	8·690	4·314	2·153	1·081	0·554	0·278	0·139	0·074	
Sub-table *a*.									
$v_{n+1} - v_n = d\Delta$	8·857	4·376	2·161	1·072	0·527	0·276	0·139	0·065	
$\Delta - 1000 = v$	8·690	4·314	2·153	1·081	0·554	0·278	0·139	0·074	
$d\Delta - v$	+0·167	+0·062	+0·008	−0·009	−0·027	−0·002	0·000	−0·009	
Sub-table *b*.									
$v_{n+1} - v_{n-1}$		13·233	6·537	3·233	1·599	0·803	0·415	0·204	
$\frac{1}{3}(v_{n+1} - v_{n-1})$		4·411	2·179	1·078	0·533	0·268	0·138	0·068	
$v'_{n+1} - v'_n = d\Delta'$		4·393	2·173	1·072	0·541	0·269	0·135	0·068	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	8·725	4·332	2·159	1·087	0·546	0·277	0·142	0·074	
$d\Delta' - v'$		+0·061	+0·014	−0·015	−0·005	−0·008	−0·007	−0·006	
Sub-table *c*.									
$v''_{n+1} - v''_n = d\Delta''$	8·840	4·384	2·167	1·072	0·534	0·273	0·137	0·066	
$\frac{v' + v}{2} = v''$	8·707	4·323	2·156	1·084	0·550	0·277	0·140	0·074	
$d\Delta'' - v''$	+0·133	+0·061	+0·011	−0·012	−0·016	−0·004	−0·003	−0·008	

Table V. RUBIDIUM BROMIDE. RbBr = 165·5. T = 19·5° C.

	9	8	7	6	5	4	3	2	1
Sub-table *a*.									
$v_{n+1} - v_n = d\Delta$	10·279	5·042	2·485	1·234	0·595	0·319	0·119	0·099	0·008
$\Delta - 1000 = v$	9·983	4·941	2·456	1·222	0·627	0·308	0·189	0·090	0·082
$d\Delta - v$	+0·296	+0·101	+0·029	+0·012	−0·032	+0·011	−0·070	+0·009	−0·074
Sub-table *b*.									
$v_{n+1} - v_{n-1}$		15·321	7·527	3·719	1·829	0·914	0·438	0·218	0·107
$\frac{1}{3}(v_{n+1} - v_{n-1})$		5·107	2·509	1·240	0·610	0·305	0·146	0·073	0·036
$v'_{n+1} - v'_n = d\Delta'$		5·083	2·503	1·225	0·624	0·278	0·172	0·045	0·036
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	10·048	4·965	2·462	1·237	0·613	0·335	0·163	0·118	0·082
$d\Delta' - v'$		+0·118	+0·041	−0·012	+0·011	−0·057	+0·009	−0·073	−0·046
Sub-table *c*.									
$v''_{n+1} - v''_n = d\Delta''$	10·247	5·062	2·494	1·230	0·609	0·299	0·145	0·072	0·022
$\frac{v' + v}{2} = v''$	10·015	4·953	2·459	1·229	0·620	0·321	0·176	0·104	0·082
$d\Delta'' - v''$	+0·232	+0·109	+0·035	+0·001	−0·011	−0·022	−0·031	−0·032	−0·060

Table VI. CÆSIUM BROMIDE. CsBr = 213·0. T = 19·5° C.

	9	8	7	6	5	4	3	2	1
Sub-table *a*.									
$v_{n+1} - v_n = d\Delta$	11·894	5·954	2·929	1·407	0·771	0·302	0·169	0·116	0·045
$\Delta - 1000 = v$	11·756	5·802	2·873	1·466	0·695	0·393	0·224	0·108	0·063
$d\Delta - v$	+0·138	+0·152	+0·056	−0·059	+0·076	−0·091	−0·055	+0·008	−0·018
Sub-table *b*.									
$v_{n+1} - v_{n-1}$		17·848	8·883	4·336	2·178	1·073	0·471	0·285	0·161
$\frac{1}{3}(v_{n+1} - v_{n-1})$		5·949	2·961	1·445	0·726	0·358	0·157	0·095	0·054
$v'_{n+1} - v'_n = d\Delta'$		5·917	2·923	1·490	0·670	0·370	0·178	0·086	0·054
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	11·751	5·834	2·911	1·421	0·751	0·381	0·203	0·117	0·063
$d\Delta' - v'$		+0·083	+0·012	+0·069	−0·081	−0·011	−0·025	−0·031	−0·009
Sub-table *c*.									
$v''_{n+1} - v''_n = d\Delta''$	11·897	5·935	2·926	1·449	0·720	0·336	0·174	0·101	0·049
$\frac{v' + v}{2} = v''$	11·753	5·818	2·892	1·443	0·723	0·387	0·213	0·112	0·063
$d\Delta'' - v''$	+0·144	+0·117	+0·034	+0·006	−0·003	−0·051	−0·039	−0·011	−0·014

Tables of Class F, illustrating the Method of arriving at the Volumetric Effect produced by changing the Concentration of a Solution.

TRIAD OF IODIDES.

Table VII. POTASSIUM IODIDE. KI = 166·1. T = 19·5° C.

n	9	8	7	6	5	4	3	2	1
m	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024
v	11·281	5·574	2·772	1·395	0·695	0·347	0·168	0·089	0·040
Sub-table a.									
$v_{n+1}-v_n=d\Delta$	11·497	5·707	2·802	1·377	0·700	0·348	0·179	0·079	0·049
$\Delta-1000=v$	11·281	5·574	2·772	1·395	0·695	0·347	0·168	0·089	0·040
$d\Delta-v$	+0·216	+0·133	+0·030	−0·018	+0·005	+0·001	+0·011	−0·010	+0·009
Sub-table b.									
$v_{n+1}-v_{n-1}$		17·204	8·509	4·179	2·077	1·048	0·527	0·258	0·128
$\frac{1}{3}(v_{n+1}-v_{n-1})$		5·735	2·836	1·393	0·692	0·349	0·176	0·086	0·043
$v'_{n+1}-v'_n=d\Delta'$		5·701	2·820	1·401	0·691	0·352	0·169	0·092	0·043
$v_{n-1}+\frac{1}{3}(v_{n+1}-v_{n-1})=v'$	11·309	5·608	2·788	1·387	0·696	0·344	0·175	0·083	0·040
$d\Delta'-v'$		+0·093	+0·032	+0·014	−0·005	+0·008	−0·006	+0·009	+0·003
Sub-table c.									
$v''_{n+1}-v''_n=d\Delta''$	11·483	5·704	2·811	1·389	0·696	0·350	0·174	0·085	0·043
$\frac{v'+v}{2}=v''$	11·295	5·591	2·780	1·391	0·695	0·345	0·171	0·086	0·043
$d\Delta''-v''$	+0·183	+0·113	+0·031	−0·002	+0·001	+0·005	+0·003	−0·001	0·000

Table VIII. RUBIDIUM IODIDE. RbI = 212·5. T = 19·5° C.

Sub-table a.									
$v_{n+1}-v_n=d\Delta$	12·969	6·412	3·221	1·601	0·789	0·391	0·204	0·075	0·082
$\Delta-1000=v$	12·836	6·424	3·203	1·602	0·813	0·422	0·218	0·143	0·061
$d\Delta-v$	+0·133	−0·012	+0·018	−0·001	−0·024	−0·031	−0·014	−0·068	+0·021
Sub-table b.									
$v_{n+1}-v_{n-1}$		19·381	9·633	4·822	2·390	1·180	0·595	0·279	0·157
$\frac{1}{3}(v_{n+1}-v_{n-1})$		6·460	3·211	1·607	0·797	0·393	0·198	0·093	0·052
$v'_{n+1}-v'_n=d\Delta'$		6·470	3·205	1·599	0·795	0·399	0·180	0·123	0·052
$v_{n-1}+\frac{1}{3}(v_{n+1}-v_{n-1})=v'$	12·884	6·414	3·209	1·610	0·815	0·416	0·236	0·113	0·061
$d\Delta'-v'$		+0·056	−0·004	−0·011	−0·020	−0·017	−0·056	+0·010	−0·009
Sub-table c.									
$v''_{n+1}-v''_n=d\Delta''$	12·945	6·441	3·213	1·600	0·792	0·395	0·192	0·099	0·067
$\frac{v'+v}{2}=v''$	12·860	6·419	3·206	1·606	0·814	0·419	0·227	0·128	0·061
$d\Delta''-v''$	+0·085	+0·022	+0·007	−0·006	−0·022	−0·024	−0·035	−0·029	+0·006

Table IX. CÆSIUM IODIDE. CsI = 260·0. T = 19·5° C.

Sub-table a.									
$v_{n+1}-v_n=d\Delta$	14·893	7·445	3·668	1·861	0·875	0·455	0·207	0·042	0·082
$\Delta-1000=v$	14·788	7·343	3·675	1·814	0·939	0·484	0·277	0·235	0·153
$d\Delta-v$	+0·105	+0·102	−0·007	+0·047	−0·064	−0·029	−0·070	−0·193	−0·071
Sub-table b.									
$v_{n+1}-v_{n-1}$		22·338	11·113	5·529	2·736	1·330	0·662	0·249	0·124
$\frac{1}{3}(v_{n+1}-v_{n-1})$		7·446	3·704	1·843	0·912	0·443	0·221	0·083	0·041
$v'_{n+1}-v'_n=d\Delta'$		7·410	3·722	1·806	0·924	0·429	0·180	0·124	0·041
$v_{n-1}+\frac{1}{3}(v_{n+1}-v_{n-1})=v'$	14·789	7·379	3·657	1·851	0·927	0·498	0·318	0·194	0·153
$d\Delta'-v'$		+0·031	+0·065	−0·045	−0·003	−0·069	−0·138	−0·070	−0·112
Sub-table c.									
$v''_{n+1}-v''_n=d\Delta''$	14·893	7·427	3·695	1·834	0·899	0·442	0·194	0·083	0·061
$\frac{v'+v}{2}=v''$	14·788	7·361	3·666	1·832	0·933	0·491	0·297	0·214	0·153
$d\Delta''-v''$	+0·105	+0·066	+0·029	+0·002	−0·034	−0·049	−0·103	−0·131	−0·092

§ 52. Tables of Class F, illustrating the Method of arriving at the Volumetric Effect produced by changing the Concentration of a Solution.

TRIAD OF CHLORATES.

Table X. POTASSIUM CHLORATE. $KClO_3 = 122·6$. T = 19·5° C.

n	9	8	7	6	5	4	3	2	1
m	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024
v	11·352	5·632	2·785	1·337	0·661	0·324	0·158	0·057	

Sub-table *a*.

$v_{n+1} - v_n = d\Delta$		5·720	2·847	1·448	0·676	0·337	0·166	0·101	
$\Delta - 1000 = v$	11·352	5·632	2·785	1·337	0·661	0·324	0·158	0·057	
$d\Delta - v$		+0·088	+0·062	+0·111	+0·015	+0·013	+0·008	+0·044	

Sub-table *b*.

$v_{n+1} - v_{n-1}$			8·567	4·295	2·124	1·013	0·503	0·267	
$\frac{1}{3}(v_{n+1} - v_{n-1})$			2·856	1·432	0·708	0·338	0·168	0·089	
$v'_{n+1} - v'_n = d\Delta'$		5·711	2·872	1·400	0·707	0·336	0·180	0·089	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	11·352	5·641	2·769	1·369	0·662	0·326	0·146	0·057	
$d\Delta' - v'$		+0·070	+0·103	+0·031	+0·045	+0·010	+0·034	+0·032	

Sub-table *c*.

$v''_{n+1} - v''_n = d\Delta''$		5·716	2·859	1·424	0·692	0·336	0·173	0·095	
$\frac{v' + v}{2} = v''$	11·352	5·636	2·777	1·353	0·661	0·325	0·152	0·057	
$d\Delta'' - v''$		+0·080	+0·082	+0·071	+0·031	+0·011	+0·021	+0·038	

Table XI. RUBIDIUM CHLORATE. $RbClO_3 = 169·0$. T = 19·5° C.

Sub-table *a*.

$v_{n+1} - v_n = d\Delta$		6·373	3·170	1·599	0·809	0·375	0·200	0·089	
$\Delta - 1000 = v$	12·726	6·353	3·183	1·584	0·775	0·400	0·200	0·111	
$d\Delta - v$		+0·020	−0·013	+0·015	+0·034	−0·025	0·000	−0·022	

Sub-table *b*.

$v_{n+1} - v_{n-1}$			9·543	4·769	2·408	1·184	0·575	0·289	
$\frac{1}{3}(v_{n+1} - v_{n-1})$			3·181	1·590	0·803	0·395	0·192	0·096	
$v'_{n+1} - v'_n = d\Delta'$		6·362	3·190	1·596	0·783	0·403	0·185	0·096	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	12·726	6·364	3·174	1·578	0·795	0·392	0·207	0·111	
$d\Delta' - v'$		−0·002	+0·016	+0·018	−0·012	+0·011	−0·022	−0·015	

Sub-table *c*.

$v''_{n+1} - v''_n = d\Delta''$		6·368	3·180	1·597	0·796	0·389	0·193	0·092	
$\frac{v' + v}{2} = v''$	12·726	6·358	3·178	1·581	0·785	0·396	0·203	0·111	
$d\Delta'' - v''$		+0·010	+0·002	+0·016	+0·011	−0·007	−0·010	−0·019	

Table XII. CÆSIUM CHLORATE. $CsClO_3 = 216·5$. T = 19·5° C.

Sub-table *a*.

$v_{n+1} - v_n = d\Delta$		7·282	3·564	1·865	0·833	0·495	0·184	0·080	
$\Delta - 1000 = v$	14·515	7·233	3·669	1·804	0·971	0·476	0·292	0·212	
$d\Delta - v$		+0·049	−0·105	+0·061	−0·138	+0·019	−0·108	−0·132	

Sub-table *b*.

$v_{n+1} - v_{n-1}$			10·846	5·429	2·698	1·328	0·679	0·264	
$\frac{1}{3}(v_{n+1} - v_{n-1})$			3·615	1·810	0·899	0·443	0·226	0·088	
$v'_{n+1} - v'_n = d\Delta'$		7·231	3·670	1·744	0·951	0·401	0·218	0·088	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	14·515	7·284	3·614	1·870	0·919	0·518	0·300	0·212	
$d\Delta' - v'$		−0·053	+0·056	−0·126	+0·032	−0·117	−0·082	−0·124	

Sub-table *c*.

$v''_{n+1} - v''_n = d\Delta''$		7·257	3·617	1·804	0·892	0·448	0·201	0·084	
$\frac{v' + v}{2} = v''$	14·515	7·258	3·641	1·837	0·945	0·497	0·296	0·212	
$d\Delta'' - v''$		−0·001	−0·024	−0·033	−0·053	−0·049	−0·095	−0·128	

TABLES of Class F, illustrating the Method of arriving at the Volumetric Effect produced by changing the Concentration of a Solution.

TRIAD OF BROMATES.

TABLE XIII. POTASSIUM BROMATE. $KBrO_3$ = 167·1. T = 19·5° C.

n	9	8	7	6	5	4	3	2	1
m	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024
v	11·290	5·576	2·761	1·370	0·688	0·347	0·176	0·089	
SUB-TABLE *a*.									
$v_{n+1} - v_n = d\Delta$		5·714	2·815	1·391	0·682	0·341	0·171	0·087	
$\Delta - 1000 = v$	11·290	5·576	2·761	1·370	0·688	0·347	0·176	0·089	
$d\Delta - v$		+0·138	+0·054	+0·021	−0·006	−0·006	−0·005	−0·002	
SUB-TABLE *b*.									
$v_{n+1} - v_{n-1}$			8·529	4·206	2·073	1·023	0·512	0·258	
$\frac{1}{3}(v_{n+1} - v_{n-1})$			2·843	1·402	0·691	0·341	0·171	0·086	
$v'_{n+1} - v'_n = d\Delta'$		5·686	2·832	1.393	0·691	0·341	0·172	0·086	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	11·290	5·604	2·772	1·379	0·688	0·347	0·175	0·089	
$d\Delta' - v'$		+0·082	+0·060	+0·014	+0·003	−0·006	−0·003	−0·003	
SUB-TABLE *c*.									
$v''_{n+1} - v''_n = d\Delta''$		5·700	2·824	1·392	0·686	0·341	0·172	0·086	
$\frac{v' + v}{2} = v''$	11·290	5·590	2·766	1·374	0·688	0·347	0·175	0·089	
$d\Delta'' - v''$		+0·110	+0·058	+0·018	−0·002	−0·006	−0·003	−0·003	

TABLE XIV. RUBIDIUM BROMATE. $RbBrO_3$ = 213·5. T = 19·5° C.

	9	8	7	6	5	4	3	2	1
SUB-TABLE *a*.									
$v_{n+1} - v_n = d\Delta$				1·516	0·774	0·360	0·216	0·095	
$\Delta - 1000 = v$			3·057	1·541	0·767	0·407	0·191	0·096	
$d\Delta - v$				−0·025	+0·007	−0·047	+0·025	−0·001	
SUB-TABLE *b*.									
$v_{n+1} - v_{n-1}$					2·290	1·134	0·576	0·311	
$\frac{1}{3}(v_{n+1} - v_{n-1})$					0·763	0·378	0·192	0·104	
$v'_{n+1} - v'_n = d\Delta'$				1·527	0·745	0·402	0·183	0·104	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$			3·057	1·530	0·785	0·383	0·200	0·096	
$d\Delta' - v'$				−0·003	−0·040	+0·019	−0·017	+0·008	
SUB-TABLE *c*.									
$v''_{n+1} - v''_n = d\Delta''$				1·522	0·759	0·381	0·200	0·099	
$\frac{v' + v}{2} = v''$			3·057	1·535	0·776	0·395	0·195	0·096	
$d\Delta'' - v''$				−0·013	−0·017	−0·014	+0·005	+0·003	

TABLE XV. CÆSIUM BROMATE. $CsBrO_3$ = 261·0. T = 19·5° C.

	9	8	7	6	5	4	3	2	1
SUB-TABLE *a*.									
$v_{n+1} - v_n = d\Delta$				1·744	0·903	0·443	0·186	0·101	
$\Delta - 1000 = v$			3·511	1·767	0·864	0·421	0·235	0·134	
$d\Delta - v$				−0·023	+0·039	+0·022	−0·049	−0·033	
SUB-TABLE *b*.									
$v_{n+1} - v_{n-1}$					2·647	1·346	0·629	0·287	
$\frac{1}{3}(v_{n+1} - v_{n-1})$					0·882	0·449	0·210	0·096	
$v'_{n+1} - v'_n = d\Delta'$				1·765	0·876	0·425	0·215	0·096	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$			3·511	1·746	0·870	0·445	0·230	0·134	
$d\Delta' - v'$				+0·019	+0·006	−0·020	−0·015	−0·038	
SUB-TABLE *c*.									
$v''_{n+1} - v''_n = d\Delta''$				1·755	0·889	0·434	0·201	0·098	
$\frac{v' + v}{2} = v''$			3·511	1·756	0·867	0·433	0·232	0·134	
$d\Delta'' - v''$				−0·001	+0·022	+0·001	−0·031	−0·036	

Tables of Class F, illustrating the Method of arriving at the Volumetric Effect produced by changing the Concentration of a Solution.

TRIAD OF IODATES.

Table XVI. POTASSIUM IODATE. $KIO_3 = 214·1$. T = 19·5° C.

n	9	8	7	6	5	4	3	2	1
m	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024
v	8·832	4·339	2·189	1·096	0·584	0·269	0·127	0·057	
Sub-table a.									
$v_{n+1} - v_n = d\Delta$		4·493	2·150	1·093	0·512	0·315	0·142	0·070	
$\Delta - 1000 = v$	8·832	4·339	2·189	1·096	0·584	0·269	0·127	0·057	
$d\Delta - v$		+0·154	−0·039	−0·003	−0·072	+0·046	+0·015	+0·013	
Sub-table b.									
$v_{n+1} - v_{n-1}$			6·643	3·243	1·605	0·827	0·457	0·212	
$\frac{1}{3}(v_{n+1} - v_{n-1})$			2·214	1·081	0·535	0·276	0·152	0·071	
$v'_{n+1} - v'_n = d\Delta'$		4·429	2·226	1·058	0·574	0·266	0·151	0·071	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$	8·832	4·403	2·177	1·119	0·545	0·279	0·128	0·057	
$d\Delta' - v'$		+0·026	+0·049	−0·061	+0·029	−0·013	+0·023	+0·014	
Sub-table c.									
$v''_{n+1} - v''_n = d\Delta''$		4·461	2·188	1·076	0·543	0·290	0·147	0·070	
$\frac{v' + v}{2} = v''$	8·832	4·371	2·183	1·107	0·564	0·274	0·127	0·057	
$d\Delta'' - v''$		+0·090	+0·005	−0·031	−0·021	+0·016	+0·020	+0·013	

Table XVII. RUBIDIUM IODATE. $RbIO_3 = 260·5$. T = 19·5° C.

Sub-table a.									
$v_{n+1} - v_n = d\Delta$				1·300	0·615	0·317	0·154	0·118	
$\Delta - 1000 = v$			2·576	1·276	0·661	0·344	0·190	0·072	
$d\Delta - v$				+0·024	−0·046	−0·027	−0·036	+0·046	
Sub-table b.									
$v_{n+1} - v_{n-1}$					1·915	0·932	0·471	0·272	
$\frac{1}{3}(v_{n+1} - v_{n-1})$					0·638	0·311	0·157	0·091	
$v'_{n+1} - v'_n = d\Delta'$				1·277	0·644	0·308	0·184	0·091	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$			2·576	1·299	0·655	0·347	0·163	0·072	
$d\Delta' - v'$				−0·022	−0·011	−0·039	+0·021	+0·019	
Sub-table c.									
$v''_{n+1} - v''_n = d\Delta''$				1·289	0·629	0·313	0·169	0·104	
$\frac{v' + v}{2} = v''$			2·576	1·287	0·658	0·345	0·176	0·072	
$d\Delta'' - v''$				+0·002	−0·029	−0·032	−0·007	+0·032	

Table XVIII. CÆSIUM IODATE. $CsIO_3 = 308·0$. T = 19·5° C.

Sub-table a.									
$v_{n+1} - v_n = d\Delta$				1·432	0·685	0·329	0·185	0·120	
$\Delta - 1000 = v$			2·903	1·471	0·786	0·457	0·272	0·152	
$d\Delta - v$				−0·039	−0·101	−0·128	−0·087	−0·032	
Sub-table b.									
$v_{n+1} - v_{n-1}$					2·117	1·014	0·514	0·305	
$\frac{1}{3}(v_{n+1} - v_{n-1})$					0·706	0·338	0·171	0·102	
$v'_{n+1} - v'_n = d\Delta'$				1·411	0·697	0·352	0·189	0·102	
$v_{n-1} + \frac{1}{3}(v_{n+1} - v_{n-1}) = v'$			2·903	1·492	0·795	0·443	0·254	0·152	
$d\Delta' - v'$				−0·081	−0·098	−0·091	−0·065	−0·050	
Sub-table c.									
$v''_{n+1} - v''_n = d\Delta''$				1·422	0·691	0·340	0·187	0·111	
$\frac{v' + v}{2} = v''$			2·903	1·481	0·790	0·450	0·263	0·152	
$d\Delta'' - v''$				−0·059	−0·099	−0·110	−0·076	−0·041	

Summary.

§ 53. *Volumetric Effect produced on changing the Concentration of a Solution.*

n / m	9 / 1/4	8 / 1/8	7 / 1/16	6 / 1/32	5 / 1/64	4 / 1/128	3 / 1/256	2 / 1/512	1 / 1/1024
MR					$d\Delta'' - v''$.				
KCl	+0·159	+0·083	+0·037	+0·007	−0·008	+0·002	−0·001	−0·022	
RbCl	+0·185	+0·090	+0·026	+0·017	+0·020	+0·009	−0·035	−0·008	
CsCl	+0·209	+0·102	+0·044	+0·025	+0·020	+0·015	+0·004	−0·005	
KBr	+0·133	+0·061	+0·011	−0·012	−0·016	−0·004	−0·003	−0·008	
RbBr	+0·232	+0·109	+0·035	+0·001	−0·011	−0·022	−0·031	−0·032	−0·060
CsBr	+0·144	+0·117	+0·034	+0 006	−0·003	−0·051	−0·039	−0·011	−0·014
KI	+0·183	+0·113	+0·031	−0·002	+0·001	+0·005	+0·003	−0·001	0·000
RbI	+0·085	+0·022	+0·007	−0·006	−0·022	−0·024	−0·035	−0·029	+0·006
CsI	+0·105	+0·066	+0·029	+0·002	−0·034	−0·049	−0·103	−0·131	−0·092
MRO_3					$d\Delta'' - v''$.				
$KClO_3$		+0·080	+0·082	+0·071	+0·031	+0·011	+0·021	+0·038	
$RbClO_3$		+0·010	+0·002	+0·016	+0·011	−0·007	−0·010	−0·019	
$CsClO_3$		−0·001	−0·024	−0·033	−0·053	−0·049	−0·095	−0·128	
$KBrO_3$		+0·110	+0·058	+0·018	−0·002	−0·006	−0·003	−0·003	
$RbBrO_3$				−0·013	−0·017	−0·014	+0·005	+0·003	
$CsBrO_3$				−0·001	+0·022	+0·001	−0·031	−0·036	
KIO_3		+0·090	+0·005	−0·031	−0·021	+0·016	+0·020	+0·013	
$RbIO_3$				+0·002	−0·029	−0·032	−0·007	+0·032	
$CsIO_3$				−0·059	−0·099	−0·110	−0·076	−0·041	

§ 54. *Solutions of the Salts of the Ennead MR.*—The notes deal principally with the character of the change in the value of $d\Delta - v$ with changes of the value of m in the different solutions.

Table I. :—*KCl.*—This salt is the subject of the specimen table, § 50, and it has been commented on in connection with that table.

Table II. :—*RbCl.*—In all three sub-tables the change of sign occurs between $m = 1/128$ and $m = 1/256$. There is a steady decrease in the positive values from $m = 1/4$ to $m = 1/128$ in all three cases, excepting sub-table a at $m = 1/64$, when an increase over the previous value occurs.

The solutions of this salt show a regular decrease from high positive values of $d\Delta - v$ to small negative values at low concentrations.

Table III. :— *CsCl.*—The series of values of $d\Delta - v$ in the three sub-tables shows a very regular decrease in magnitude from $m = 1/4$ to $m = 1/256$; change of sign occurs between $m = 1/256$ and $1/512$ with a small negative value.

Table IV. :—*KBr.*—There is a rapid decrease in the value of $d\Delta - v$ from $m = 1/4$ to $m = 1/16$, and between $m = 1/16$ and $m = 1/32$ there is change of sign to a small negative value, which persists right on to $m = 1/512$, and which is the characteristic feature of the three sub-tables.

It would seem to indicate that in the solutions of any concentration less than $m = 1/64$ the dissolution of a small quantity of salt produces the same increment of displacement.

Table V. :—*RbBr.*—In sub-table *a* the value of $d\Delta - v$ decreases very rapidly from 0·296 at $m = 1/4$, and changes to a negative value between $m = 1/32$ and $m = 1/64$, and then oscillates between alternate high negative values and low positive values. In sub-table *c* there is a regular increase in the negative values from $m = 1/64$ to $m = 1/512$. *We have here definite evidence of expansion on dilution.*

Table VI. :—*CsBr.*—The character of the decline in positive values for $d\Delta - v$ is the same as that observed in the case of KBr and RbBr, and change of sign occurs between $m = 1/16$ and $m = 1/32$. After this there is an irregular oscillation between high positive and high negative values in sub-table *a*.

In sub-table *c* the positive values persist to $m = 1/32$, after which change of sign occurs, and the negative sign remains for all values down to $m = 1/512$. The negative values reach a maximum at $m = 1/128$ and then decline. *Here again we have definite evidence of expansion.*

Table VII. :—*KI.*—A rapid decrease in positive values occurs between $m = 1/4$ and $m = 1/16$. Then, with the exception of the small negative values of 0·018 and 0·010 at $m = 1/32$ and $m = 1/512$ respectively, there are small positive values for $d\Delta - v$. This holds also in sub-table *c*, only that the positive and negative values for values of *m* below 1/32 are less.

The final note for KBr applies with greater force in the case of KI.

Table VIII. :—*RbI.*—In sub-table *a* there is a sudden fall from a high positive value of 0·133 for $m = 1/4$ to a negative value of 0·012 at $m = 1/8$, and then a reversion to positive at 1/16 ; after this, the values are negative, except where $m = 1/1024$.

In sub-table *c* there is a more progressive decrease in positive values from $m = 1/4$ to $m = 1/16$, and between this and $m = 1/32$ there is change of sign. The negative values increase gradually to a maximum of 0·035 at 1/256 and then decrease, and we have a positive value of 0·006 at 1/1024.

The evidence of expansion is conclusive. The character of these values is comparable with those of CsBr.

Table IX. :—*CsI.*—In sub-table *a*, with the exception of the sudden fall to a negative value of 0·007 at $m = 1/16$, there is a progressive decrease to $m = 1/32$, where change of sign occurs between $m = 1/32$ and $m = 1/64$, and an increase in magnitude of negative values to a maximum of 0·193 at $m = 1/256$, and then a slight fall.

In sub-table *c* there is a progressive decrease in positive values to $m = 1/32$; then change of sign, and progressive increase in negative values to 0·131 where $m = 1/512$, a slight decrease occurring at $m = 1/1024$.

The character of values for $d\Delta - v$ in all three sub-tables is the same. *This is the most marked instance where expansion occurs.*

§ 55. *Solutions of the Salts of the Ennead* MRO_3.—The reason for the small number of results in the cases of $RbBrO_3$, $RbIO_3$, $CsBrO_3$, and $CsIO_3$ is the very sparing solubility of these salts in water.

Table X. :—$KClO_3$.—The values of $d\Delta - v$ in all the sub-tables are positive.

In sub-table a they are oscillatory in character, the maximum value being 0·111 at $m = 1/32$; the minimum of 0·008 at $m = 1/256$ rises to 0·044 at $m = 1/512$.

In sub-table c the maximum occurs at $m = 1/16$, while the minimum is at $m = 1/128$. The nature of the oscillation is the same, but less pronounced.

Table XI. :—$RbClO_3$.—There are somewhat irregular alternations between positive and negative values of $d\Delta - v$ in the sub-table a. They are also of appreciable magnitude, and in sub-table c the oscillations are still apparent, although a definite change of sign occurs at $m = 1/128$, when the values of $d\Delta - v$ for higher values of m are positive, and the lower values of m are negative.

In the table for MR = RbCl the same character is observed as here in the sub-table c; see note above.

Table XII. :—$CsClO_3$.—There are exhibited here regular alternations between comparatively high positive values and high negative values for $d\Delta - v$ in sub-table a, except at $m = 1/256$ and $m = 1/512$, both of which are negative.

The maximum positive value is at $m = 1/32$, while the maximum negative value is at $m = 1/64$.

Owing to the greater magnitude of the negative values than the positive ones in the sub-table a, we obtain a complete series of negative values of comparatively high magnitude in sub-table c, and with the exception of a slight diminution in the negative value at $m = 1/128$ below that where $m = 1/64$, there is a progressive increase of the negative values from the beginning to the end, giving distinct evidence of expansion on dilution.

Table XIII. :—$KBrO_3$.—The character of changes in the values for $d\Delta - v$ in all three sub-tables is the same: a rapid fall in the positive values from 0·138 at $m = 1/8$ to 0·021 where $m = 1/32$, the negative values from $m = 1/64$ onwards being negligible.

This is observed in all three tables, although in sub-tables b and c the high magnitude of the positive values for $d\Delta - v$ is slightly modified.

This feature of the steady fall in positive values to a series of negligible negative values is also to be seen in the series for MR = KBr, with the exception that there are two negative quantities of appreciable magnitude in the values for $d\Delta - v$, where $m = 1/32$ and $m = 1/64$ in sub-table c for KBr.

Table XIV. :—$RbBrO_3$.—In sub-table a there is a regular alternation between rather high negative values and positive values, while in sub-table c there is a regular transition from negative to positive values, the change of sign occurring at $m = 1/256$, the general character being that of commencing with a negative value and rising to a positive quantity.

This is the only instance in these tables of the occurrence of contraction from the highest concentration, $m = 1/32$, to the lowest, $m = 1/512$.

Table XV. :—$CsBrO_3$.—Here an irregular feature is observed in that there is a negative value of 0·023 at $m = 1/32$, which changes to a positive value of 0·039 at $m = 1/64$, then a diminution in a positive value at $m = 1/128$ to the maximum negative value of the series of 0·049 at $m = 1/256$, with a slight diminution at $m = 1/512$.

In sub-table c a similar series of values for $d\Delta - v$ is seen, except that they are more regular.

Table XVI. :—KIO_3.—In sub-table a the high positive value of 0·154 for $d\Delta - v$ at $m = 1/8$ is changed to a negative value of 0·039 at $m = 1/16$, and with a diminution of the negative value to 0·003 at 1/32 the maximum negative value of 0·072 is reached at $m = 1/64$; then a transition occurs at $m = 1/128$ to a positive value of 0·046. Afterwards the positive value falls away.

There is thus a change from a high positive value to a high negative value, and then reversion to moderately high positive value.

The same feature is observed in sub-table c, but it is of a more undulatory character.

Table XVII. :—$RbIO_3$.—The positive value of 0·024 at $m = 1/32$ gives place to the maximum negative value of 0·046 at $m = 1/64$; then there is a diminution in negative values leading to a positive value of 0·046 at 1/512 in sub-table a.

The same character is observed in sub-table c, only more regular, the maximum negative value of 0·032 occurring at $m = 1/128$.

Table XVIII. :—$CsIO_3$.—The values for $d\Delta - v$ in each of the three sub-tables constitute the most regular of all the series. All the values are negative and reach a maximum value at $m = 1/128$ in all three sub-tables, and fall away regularly for higher and lower values of m.

The character of the values for $CsIO_3$ somewhat resembles those for $RbIO_3$.

Section IX.—Notes on the Values of v for the Enneads MR and MRO_3.

§ 56. The increment of displacement produced in 1000 grams of water at 19·5° C., when 1/2 gram-molecule of potassium chloride is dissolved in it, is 14·001 G_T; when a molecularly equivalent amount of potassium bromide is dissolved in the same quantity of water, the increase in the displacement is 17·547 G_T; when the salt in solution is potassium iodide, the number is 22·778 G_T. Replacing, therefore, the chlorine by bromine increases the displacement by 3·546 G_T; and if the bromine be now replaced by iodine, there is a further increase of 5·231 G_T in the displacement; or, replacing the chlorine by iodine causes an increment of displacement of 8·777 G_T. Proceeding in a similar manner with the other salts of the ennead MR and tabulating the results, we obtain Table I. An inspection of the table shows us that the differences for Br—Cl when equivalent quantities of the salts of K, Rb, and Cs are dissolved in the same quantity of water are of the same order of magnitude till $m = 1/32$. The same characteristic is observed between the same limits of m when bromine is replaced by iodine, but the differences for the same gram-molecular weight of the salt are in these three series greater than those observed when chlorine is replaced by bromine. In the third section of the table, when chlorine is replaced by iodine, the numbers expressing the differences of the increments of displacement for the same value of m are the sum of the differences for Br—Cl, I—Br.

Turning now to Table II., we have the increase in the increment of displacement produced when the metal in combination with the same acid is varied. Here again it will be found that the numbers on the same line in each section are, *inter se*, of the same order of magnitude till $m = 1/32$, after which they vary more or less irregularly.

TABLE I., giving the corresponding Differences between the Values of the Increments of Displacement, v, caused by the Dissolution of m gram-molecule of MBr and MCl; MI and MBr; MI and MCl (where M = K, Rb, or Cs) in 1000 grams of Water at 19·5° C.

ENNEAD MR.

R =	BROMIDE—CHLORIDE.			IODIDE—BROMIDE.			IODIDE—CHLORIDE.		
M =	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
m.									
1/2	3·546	3·625	3·249	5·231	5·543	6·031	8·777	9·168	9·280
1/4	1·791	1·781	1·690	2·591	2·853	3 032	4·382	4·634	4·722
1/8	0·898	0·884	0·813	1·260	1·483	1·541	2·158	2·367	2·354
1/16	0·469	0·436	0·398	0·619	0·747	0·802	1·088	1·183	1·200
1/32	0·240	0·216	0·241	0·314	0·380	0·348	0·554	0·596	0·589
1/64	0·131	0·138	0·091	0·141	0·186	0·244	0·272	0·324	0·355
1/128	0·061	0·070	0·102	0·069	0·114	0·091	0·130	0·184	0·193
1/256	0·041	0·067	0·080	0·029	0·029	0·053	0·070	0·096	0·133
1/512	0·010	0·017	0·029	0·015	0·053	0·127	0·025	0·070	0·156
1/1024					− 0·021	0·090			

TABLE II., giving the corresponding Differences between the Values of the Increments of Displacement, v, caused by the Dissolution of m gram-molecule of RbR and KR; CsR and RbR; CsR and KR (where R = Cl, Br, or I) in 1000 grams of Water at 19·5° C.

ENNEAD MR.

M =	RUBIDIUM—POTASSIUM.			CÆSIUM—RUBIDIUM.			CÆSIUM—POTASSIUM.		
R =	Cl.	Br.	I.	Cl.	Br.	I.	Cl.	Br.	I.
m.									
1/2	2 636	2·715	3·027	3·764	3·388	3·876	6·400	6·103	6·903
1/4	1·303	1·293	1·555	1·864	1·773	1·952	3·167	3·066	3·507
1/8	0·641	0·627	0·850	0·932	0·861	0·919	1·573	1·488	1·769
1/16	0·336	0·303	0·431	0·455	0·417	0·472	0·791	0·720	0·903
1/32	0·155	0·140	0·207	0·219	0·244	0·212	0·374	0·384	0·419
1/64	0·066	0·073	0·118	0·115	0·068	0·126	0·181	0·141	0·244
1/128	0 021	0·030	0·075	0·053	0·085	0·062	0 074	0·115	0·137
1/256	0·024	0·050	0·050	0·022	0·035	0·059	0·046	0 085	0·109
1/512	0·009	0·016	0·054	0·006	0·018	0·092	0·015	0·024	0·146
1/1024			0·021		− 0·019	0·092			0·113

§ 57. Tables III. and IV. deal with the salts of the ennead MRO_3, and correspond in arrangement with Tables I. and II. respectively; and what has been said of the two previous tables holds good, in general, with these two tables, but the agreement of the values is not so close, nor is the number of values tabulated so great. In Table III. the differences of the increments are nearly all negative quantities; in Table IV. they are all positive.

Table V. gives the differences of v between the oxyhalides and the halides of the same metal for the two enneads, MRO_3 and MR, and here also the same characteristic agreement is noticed between the numbers on the same line in each section of the table. Having thus briefly explained the contents of the tables and the chief characteristics of the numbers in them, we will proceed to more fully discuss the effects produced on the displacement of 1000 grams of water at 19·5° C. when the constituents of the salts dissolved in it are changed; and in order to compare the results of the halides with those of the oxyhalides, we shall confine our attention to the numbers for $m = 1/16$ and less.

TABLE III., giving the corresponding Differences between the Values of the Increments of Displacement, v, caused by the Dissolution of m gram-molecule of $MBrO_3$ and $MClO_3$; MIO_3 and $MBrO_3$; MIO_3 and $MClO_3$ (where M = K, Rb, or Cs) in 1000 grams of Water at 19·5° C.

ENNEAD MRO_3.

RO_3=	BROMATE—CHLORATE.			IODATE—BROMATE.			IODATE—CHLORATE.		
M=	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
m.									
1/2									
1/4	−0·062			−2·458			−2·520		
1/8	−0·056			−1·137			−1·193		
1/16	−0·024	−0·126	−0·158	−0·572	−0·481	−0·608	−0·596	−0·607	−0·766
1/32	+0·033	−0·043	−0·037	−0·274	−0·265	−0·296	−0·241	−0·308	−0·333
1/64	+0·027	−0·008	−0·107	−0·104	−0·106	−0·078	−0 077	−0·114	−0·185
1/128	+0·023	+0·007	−0·055	−0·078	−0·063	+0·036	−0·055	−0·056	−0·019
1/256	+0·018	−0 009	−0·057	−0·049	−0·001	+0·037	−0·031	−0·010	−0·020
1/512	+0·032	−0·015	−0·078	−0 032	−0·024	+0·028	0·000	−0·039	−0·050

TABLE IV., giving the corresponding Differences between the Values of the Increments of Displacement, v, caused by the Dissolution of m gram-molecule of $RbRO_3$ and KRO_3; $CsRO_3$ and $RbRO_3$; $CsRO_3$ and KRO_3 (where $RO_3 = ClO_3$, BrO_3, or IO_3) in 1000 grams of Water at 19·5° C.

ENNEAD MRO_3.

M=	RUBIDIUM—POTASSIUM.			CÆSIUM—RUBIDIUM.			CÆSIUM—POTASSIUM.		
RO_3=	ClO_3.	BrO_3.	IO_3.	ClO_3.	BrO_3.	IO_3.	ClO_3.	BrO_3.	IO_3.
m.									
1/2									
1/4	1·374			1·789			3·163		
1/8	0·720			0·880			1·600		
1/16	0·398	0·296	0·387	0·486	0·454	0·327	0·884	0·750	0·714
1/32	0·247	0·171	0·180	0·220	0·226	0·195	0·467	0·397	0·375
1/64	0·114	0·079	0·077	0·196	0·097	0·125	0·210	0·176	0·202
1/128	0·076	0·060	0·075	0·076	0·014	0·113	0·152	0·074	0·188
1/256	0·042	0·015	0·063	0·092	0·044	0·082	0·134	0·059	0·145
1/512	0·054	0·007	0·015	0·101	0·038	0·080	0·155	0·045	0·095

TABLE V., giving the corresponding Differences between the Values of the Increments of Displacement, v, caused by the Dissolution of m gram-molecule of $MClO_3$ and MCl; $MBrO_3$ and MBr; MIO_3 and MI (where M = K, Rb, or Cs) in 1000 grams of Water at 19·5° C.

MRO_3—MR.

RO_3— R=	CHLORATE—CHLORIDE.			BROMATE—BROMIDE.			IODATE—IODIDE.		
M=	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
m.									
1/2									
1/4	4·453	4·524	4·449	2·600			− 2·449		
1/8	2·216	2·296	2·244	1·262			− 1·235		
1/16	1·101	1·163	1·194	0·608	0·601	0·638	− 0·583	− 0·727	− 0·772
1/32	0·496	0·578	0·579	0·289	0·319	0·301	− 0·299	− 0·326	− 0·343
1/64	0·238	0·286	0·367	0·134	0·140	0·169	− 0·111	− 0·152	− 0·253
1/128	0·107	0·162	0·185	0·069	0·099	0·028	− 0·078	− 0·078	− 0·027
1/256	0·060	0·038	0·148	0·037	0·002	0·011	− 0·041	− 0·028	− 0·005
1/512	− 0·007	0·038	0·133	0·015	0·006	0·026	− 0·032	− 0·071	− 0·083

§ 58. If, in a solution containing 1/16 grm.-mol. KCl in 1000 grams of water at 19·5° C., we imagine the chlorine to be replaced by bromine, the process is accompanied by an increase of 0·469 G_T in the displacement of the original solution. If the operation be performed upon an equivalent solution of RbCl, the increase in the displacement is 0·436 G_T; if the salt in solution be 1/16 grm.-mol. CsCl, the increase is 0·398 G_T. Therefore, it appears that, when we replace Cl by Br in 1/16 grm.-mol. solutions of the chlorides of K, Rb, and Cs, approximately the same increment of displacement is produced. Commencing again with our solution of KCl, and replacing the potassium by rubidium, causes an increase in the displacement of 0·336 G_T; when rubidium is replaced by cæsium, the increase is 0·455 G_T; replacing potassium by cæsium produces an increase of 0·791 G_T. If we now consider a solution of 1/16 grm.-mol. of KBr, and replace the potassium by rubidium, there is an increase in the displacement of 0·303 G_T; replacing the rubidium by cæsium causes a further increase of 0·417 G_T; or if potassium be replaced by cæsium, the increase is 0·720 G_T. Setting out the numbers in the manner shown below gives a clearer view of the various changes that take place when one element in a compound is replaced by another:—

	K.	Diff.		Rb.	Diff.		Cs.
	v.			v.			v.
Cl .	1·684	+0·336	=	2·020	+0·455	=	2·475
Br . .	2·153	+0·303	=	2·456	+0·417	=	2·873
Diff.	0·469			0·436			0·398

It will be seen that the difference between RbCl and RbBr is nearly the same as that between KCl and KBr, because the increase in the increment of displacement produced when K is replaced by Rb in KCl is about the same as that produced when Rb takes the place of K in KBr. Therefore, when in solutions containing 1/16 grm.-mol. of KCl or KBr the potassium is replaced by rubidium, approximately the same increase in the increment of displacement is produced; further, when Cl is replaced by Br in KCl and RbCl, the increase is nearly the same in each case, but of a higher value than when the change is made in the metals. The difference between the atomic weights of Cl and Br is 44·5; between K and Rb it is 46·4. The atomic weight of Cl is less than that of K; so, also, is the atomic weight of Br less than that of Rb; yet there is a greater difference of displacement produced by changing the acid than by changing the base.

Turning our attention now to the chlorides and bromides of rubidium and cæsium, we find that replacing Cl by Br in RbCl causes an increase in the displacement of RbCl which is the mean of the increases produced when Cs replaces Rb in RbCl and RbBr, and is greater than when Br replaces Cl in CsCl. There is a greater effect produced by changing the metals when Cl is the acid than when Br is the acid; also, when the acid united with the same base is changed, the variation is greatest for the lightest metal, and least for the heaviest.

Proceeding on the lines set forth above, we will next consider the changes caused by replacing bromine by iodine, when combined with the same three metals:—

	K.	Diff.		Rb.	Diff.		Cs.
	v.			v.			v.
Br . .	2·153	+0·303	=	2·456	+0·417	=	2·873
I . .	2·772	+0·431	=	3·203	+0·472	=	3·675
Diff.	0·619			0·747			0·802

Replacing Br by I in KBr, RbBr, or CsBr produces increases in the displacements which become greater as the atomic weight of the metal increases.

There is a greater increase produced by replacing K by Rb in KI than that produced when Rb takes the place of K in KBr; a similar effect is seen with the corresponding salts of rubidium and cæsium, but in both instances the replacing of one metal by another causes a smaller change than when iodine takes the place of bromine in the bromide of the metal. The exchange of iodine for bromine produces increases in the values of v which rise with the increase of the atomic weight of the metal in combination; as was previously observed, the replacement of chlorine by bromine caused changes in the increase of v which diminished with the increase in the atomic weight of the metal. We may summarise the foregoing observations by saying that the replacement of the acid in a salt of the general formula MR (where M = K, Rb, or Cs, and R = Cl, Br, or I) by another acid causes a greater change in the value of the displacement of a solution containing 1/16 grm.-mol. of the salt in 1000 grams of water at 19·5° C. than the replacement of the metal by another metal. The only exceptions to this are when Cs takes the place of Rb in RbCl, and when Br replaces Cl in CsCl, this latter being the smallest change produced when one acid is replaced by another. The values of the differences of the displacements for 1/16 grm.-mol. solutions of the salts of the halides are given in the table below.

	K.	Diff.	Rb.	Diff.	Cs.	Diff. (K by Cs).
Cl		0·336		0·455		0·791
Diff.	0·469		0·436		0·398	
Br		0·303		0·417		0·720
Diff.	0·619		0·747		0·802	
I		0·431		0·472		0·903
Diff. Cl by I	1·088		1·183		1·200	

In the table, differences on the same line are, *inter se*, comparable; the differences in the same column headed "Diff." are also comparable.

§ 59. The salts of the oxyhalides will now be dealt with in a manner similar to that of the halides.

	K. v.	Diff.		Rb. v.	Diff.		Cs. v.
ClO_3 . .	2·785	+ 0·398	=	3·183	+ 0·486	=	3·669
BrO_3 . .	2·761	+ 0·296	=	3·057	+ 0·454	=	3·511
Diff.	− 0·024			− 0·126			− 0·158

If, in a solution containing 1/16 grm.-mol. $KClO_3$ per 1000 grams of water at 19·5° C., we replace the chlorine by bromine, the process is accompanied by a *decrease* in the displacement of the solution by an amount equal to − 0·024 G_T; if the operation be performed on an equivalent solution of $RbClO_3$, the decrease is $-0·126\ G_T$; if the salt in solution be $CsClO_3$, the decrease is $-0·158\ G_T$. Therefore, when we replace the chlorine by bromine in 1/16 grm.-mol. solutions of the chlorates of K, Rb, and Cs we observe that the change produces a decrease in the displacement of the solution, and this decrease becomes greater as the atomic weight of the metal increases.

Confining our attention next to the changes produced in the displacement by varying the metal combined with the same acid, we find that when K is replaced by Rb in $KClO_3$ the displacement of the solution increases, as it also does when Rb is replaced by Cs in $RbClO_3$. There is a similar increase when we consider the bromates of the same three metals; but replacing K by Rb in $KBrO_3$ produces a smaller increase than when the same change is made in $KClO_3$. Similarly, the replacement of Rb by Cs in $RbBrO_3$ is less than when Cs replaces Rb in $RbClO_3$. In the four cases where an exchange of metals takes place the corresponding changes in the displacements are positive; with the three changes obtained by replacing Cl by Br the changes in the displacement are negative, and numerically less than when the metals are changed.

	K. *v.*	Diff.		Rb. *v.*	Diff.		Cs. *v.*
BrO_3 . .	2·761	+0·296	=	3·057	+0·454	=	3·511
IO_3 . .	2·189	+0·387	=	2·576	+0·327	=	2·903
Diff.	−0·572			−0·481			−0·608

With the bromates and iodates of the same three metals we find that replacing the bromine by iodine causes a reduction in the displacement of $-0·572\ G_T$ in the case of $KBrO_3$; with $RbBrO_3$ the change in the displacement is $-0·481\ G_T$; and with $CsBrO_3$ it is $-0·608\ G_T$. When we compare the results obtained by changing the metal combined with the same acid, we find that the iodates have positive values for the change in the displacement, just as the bromates and chlorates had, but that, whereas the replacement of Rb by Cs in $RbClO_3$ and $RbBrO_3$ gave an increase in the displacement which was greater than that produced by the exchange of Rb for K in $KClO_3$ and $KBrO_3$, the replacement of Rb by Cs in $RbIO_3$ causes an increase in the displacement which is less than that caused by replacing K by Rb in KIO_3. The results of the changes produced in the displacement by the replacement of one constituent by

	K.	Diff.	Rb.	Diff.	Cs.	Diff. (K by Cs).
ClO_3		0·398		0·486		0·884
Diff.	−0·024		−0·126		−0·158	
BrO_3		0·296		0·454		0·750
Diff.	−0·572		−0·481		−0·608	
IO_3		0·387		0·327		0·714
Diff. ClO_3 by IO_3	−0·596		−0·607		−0·766	

another in the salt dissolved is given in the preceding table. We do not here find so close an agreement between the numbers in a line, nor between those in the same column, but the agreement is still near enough to prevent any ambiguity as to which line or column a series belongs. Moreover, the columnar differences are all positive, while the line differences are all negative. We may further note that with the replacement of Rb by Cs and K by Cs the columnar differences decrease with an increase in the molecular weight of RO_3, while replacing K by Rb causes irregular changes in the displacement. With the line differences the replacement of ClO_3 by BrO_3 and ClO_3 by IO_3 causes an increase with an increase of the atomic weight of the metal; but with the replacement of BrO_3 by IO_3 the changes in the displacement are irregular.

§ 60. The next effect to consider is that produced by the addition of the three oxygen atoms to the salts of the halides to form the corresponding salts of the oxyhalides.

	K.	Rb.	Cs.
ClO_3—Cl . .	1·101	1·163	1·194
BrO_3—Br . .	0·608	0·601	0·638
IO_3—I . .	−0·583	−0·727	−0·772

In order to do this the above table has been constructed, in which the differences between the corresponding salts of the halides and oxyhalides for the same metal are entered in vertical columns. If we imagine that in a solution of 1/16 KCl + 1000 grams of water at 19·5° C. we add sufficient oxygen to the chloride and so produce $KClO_3$ in solution, the operation is accompanied by an addition to the displacement of 1·101 G_T; if the same operation be performed on a 1/16 grm.-mol. solution of the bromide of the same metal, the increase in the displacement is only 0·608 G_T; and if we treat a solution of the iodide in the same way it produces a diminution in the displacement of −0·583 G_T. An inspection of the changes occurring when the three corresponding salts of rubidium and cæsium are similarly treated shows us that they behave not only in an analogous manner, but that the amount of the change in each case is almost the same as that observed with the potassium salts, increasing slightly with the atomic weight of the metal.

This action of the three atoms of oxygen upon the displacements of solutions of 1/16 grm.-mol. of the halides in 1000 grms. of water at 19·5° C. is peculiar, since in each case we have added the same weight of oxygen, namely, 3 grams, and the effects produced by it are similar in the salts with the same acid but different bases, but differ when the acid in combination with the same base is varied.

§ 61. *A General Comparison and Summary of the Variation in the Values of the Mean Increment of Displacement for Dilute Solutions of Salts of the two Enneads MR and MRO_3 (where M may be K, Rb, or Cs, and R may be Cl, Br, or I).*—This comparison includes:—

(*a*) The variation produced by successive dilutions of a solution of an individual salt.

(*b*) The character of the variation in the case of the whole series of solutions of salts of the two enneads.

(*c*) The variation with the molecular weight.

The first point has been adequately dealt with in the immediately preceding section, and the two remaining ones will now be considered. The diagram on next page clearly shows the relations pointed out above, and the following are the more distinctive features which are illustrated.

All the halide salts, with the possible exception of KI, have the property of causing expansion with dilution of their respective solutions, this expansion, in the case of the chlorides, being nearly proportional to the rise in the atomic weights of the base, as shown by the almost parallel march of the curves. In the case of the bromides the march is not so regular, the solutions of the rubidium and cæsium salts inducing a greater relative expansion on dilution than is the case with the potassium salt, the change being greatest in the case of the rubidium salt.

With the iodides, this increased effect of expansion which occurs on dilution of solutions of the salts of rubidium and cæsium over that of potassium is considerably enhanced, the solutions of potassium iodide showing practically no expansion.

Thus, summarising the effects, the mutual relations of halogen and base in the cases of halide salts of potassium and the chlorine compounds of rubidium and cæsium produce normal effects, as shown by only slight changes in the values of v/m as the solutions decrease in concentration, while the remaining salts show expansion on dilution of solutions of them, which increases in magnitude with increase in molecular weight, reaching a maximum with cæsium iodide. This is interesting when it is considered that cæsium is the most electro-positive element, and seems to point to the expansive effect produced by both cæsium and iodine independently, while mutual interference occurs in the other cases.

The oxyhalides are not comparable in any sense with their respective halide compounds, which have been treated above.

The most obvious feature of the incorporation of the oxygen atoms is, that the values for v/m decrease with the increase in molecular weight when triads of the salts having common base and the same concentration are compared, the only exception being the case of potassium bromate; and this feature is the reverse of that in the case of the halide salts. Also in the case of $KClO_3$ and KIO_3 contraction occurs on dilution of their respective solutions; and where expansion occurs on dilution, the general order is that of proceeding from the iodates to the chlorates, where the greatest expansive effect is seen in the case of cæsium chlorate. This is the reverse of the order which is seen in the case of the halides. The chlorates show the greatest variations in the values of v/m with dilution of solutions of the salts, and least with the bromates, the iodates being intermediate.

Thus the effect of the inclusion of the oxygen in the molecule of the halides is to greatly increase the expansion effect when solutions of the chlorates of rubidium and cæsium are diluted, to exert very little if any effect in the case of the bromides, and to diminish the effect of expansion in the case of the iodides, the general effect in the

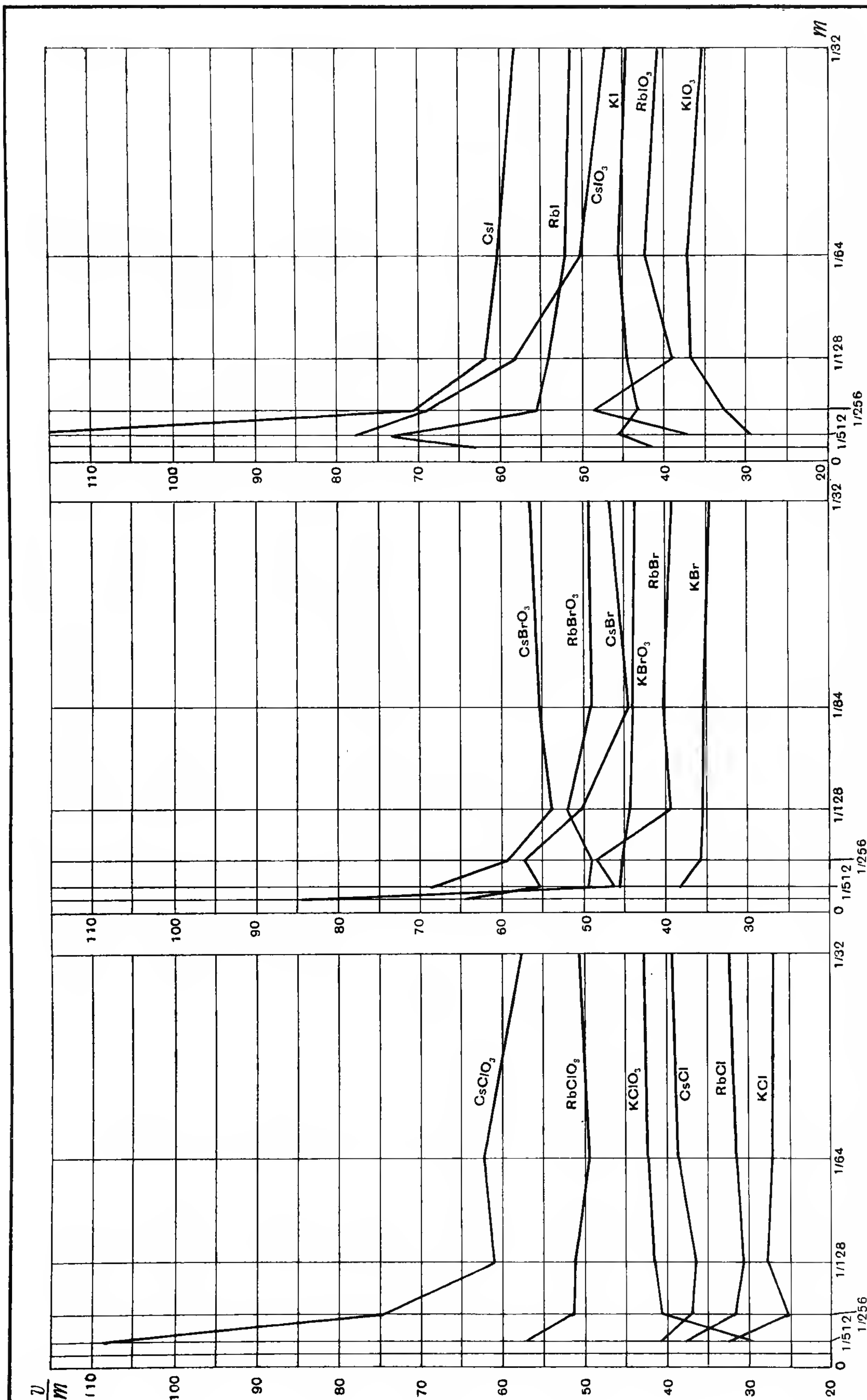


Diagram showing relation between the mean increment of the displacement produced by the dissolution of m gram-molecule of salt in 1000 grams of water at 19·5° C., v/m, and the amount of salt, m, in gram-molecules.

cases of potassium and rubidium iodates being that of contraction on dilution, while the iodate of cæsium simulates the character of the iodide of the same base, though to a modified degree; as though the dominating influence were the expansive effect of the basic radical.

§ 62. If we consider the solutions of the salts of the double ennead MR, MRO_3, we have eighteen solutions for each value of *m*. Owing to the sparing solubility of some of the salts of the ennead MRO_3, the highest value of *m* available for all the salts is 1/16. The eighteen salts can be divided into three hexads, the members of each hexad containing a common metallic element, K, Rb, or Cs, and into three other hexads having a common metalloidal element, Cl, Br, or I. The values of v ($=\Delta - 1000$) for the solution of 1/16 gram-molecule of each of the salts in the three hexads having the common elements K, Rb, Cs are arranged in the tables, and the graphic effect is illustrated in the diagram of § 38. When we wish to compare the solutions having different values of *m*, it is convenient to use the values of v/m, that is, the increment of displacement ($\Delta - 1000$) reduced to the value which it would have if $m = 1$. This is found in the general tables of Class E (§ 30); and for the solutions of 1/32 gram-molecule salt and under, with nucleus Cl, Br, or I, the values of v/m are represented graphically in the diagram § 61, in which the ordinates are values of v/m, and the abscissæ values of *m*.

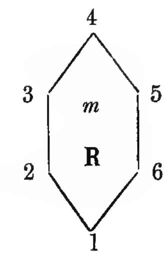
When this diagram is studied, it is seen that the arrangement of the curves is different in each of the three compartments which correspond to the solutions of salts having as common elements the metalloids Cl, Br, I respectively, and that their differences are not altogether irregular. In the first diagram, the common element being Cl, the values of v/m follow the same order as that of the arrangement for $m = 1/16$ in the tables, namely, KCl, RbCl, CsCl, $KClO_3$, $RbClO_3$, $CsClO_3$. This order is maintained for $m = 1/64$, 1/128, 1/256. When the common element is Br or I, the arrangement of the salts with respect to the values of v/m is different.

The values of v/m recorded in the general tables of Class E, with the curves in the above diagram, furnish the means of appreciating the changing characters of the different solutions with change of concentration, having regard to the numerical values of the constant v/m for the different salts for the different values of *m*.

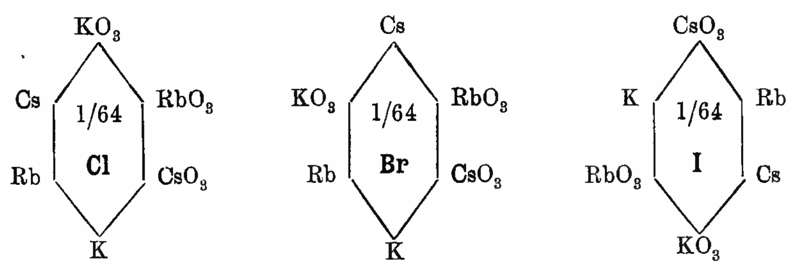
§ 63. It is instructive to consider the *order* in which the salts of each hexad follow each other when arranged in ascending order of values of v/m, without paying particular attention to their actual numerical values.

For this purpose it is convenient to represent each hexad of salts by a hexagon, the centre of which is occupied by the common element, metal or metalloid, as *nucleus*. The angles of the hexagon are then supposed to be occupied by the *residues* of the respective salts after abstraction of the common element, arranged in ascending order of magnitude of v/m, the lowest value occupying the lowest angle on the paper, and the other values of v/m occupying the other angles *seriatim* in ascending order of magnitude, and going round from left to right.

In the figure we have a hexagon the corners of which are numbered on this plan from 1 to 6. Inside the hexagon we have the common element M or R, and above it the value of m for the particular solution. The residue corresponding to the lowest value of v/m is entered at the corner numbered 1, the next higher at 2, the next at 3, and so on, the residue corresponding to the highest value of v/m occupying place No. 6. For concentrations higher than $m = 1/64$, the arrangement of residues is the same as that given for $m = 1/64$ in the six hexagons corresponding to the common elements Cl, Br, I, K, Rb, Cs.



In the accompanying figures we have the three hexagons 1/64 **[Cl]**, 1/64 **[Br]**, 1/64 **[I]** corresponding to the nuclei **Cl**, **Br**, **I**, and to the value of $m = 1/64$. The salts corresponding to the first hexagon are ClM and $ClMO_3$, and their residues after abstraction of Cl are K, Rb, Cs, KO_3, RbO_3, CsO_3. Entering these at the corners, on the plan above explained, in ascending order of magnitude of v/m, the residue corresponding to the lowest value of v/m being entered at place 1, we find that the order in which the residues follow each other is that of the salts of the two triads ClM and



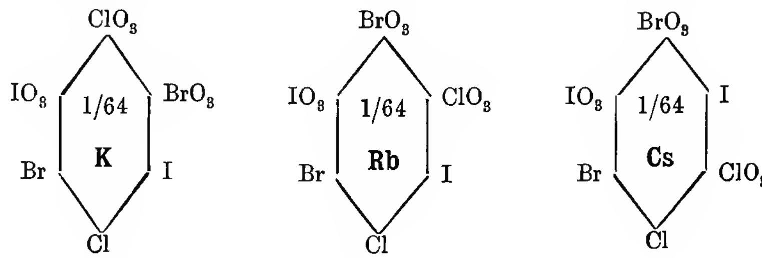
$ClMO_3$, and following the ascending order of molecular weight in each triad. When the nucleus is **Br**, the order differs from that corresponding to **Cl** in that the neighbouring residues Cs and KO_3 change places. When the nucleus is **I**, the arrangement seems to be quite different, but it is derived from that with the nucleus **Br** by replacing at each corner the residues M by MO_3 and MO_3 by M.

If we arrange the residues in parallel lines we have :—

Hexagon.	Residues in ascending order of magnitude of v/m.					
1/64 **[Cl]**	K	Rb	Cs	KO_3	RbO_3	CsO_3
1/64 **[Br]**	K	Rb	KO_3	Cs	RbO_3	CsO_3
1/64 **[I]**	KO_3	RbO_3	K	CsO_3	Rb	Cs

When the metals act as nucleus, we have the hexagons in the figures on p. 144. In each of these hexagons Cl occupies place 1, Br place 2, and IO_3 place 3. The 4th place is occupied in the consecutive hexagons by ClO_3, BrO_3, BrO_3; the 5th by BrO_3, ClO_3, I; and the 6th by I, I, ClO_3. The hexagon 1/64 **[K]** is derived from 1/64 **[Cl]** by an exchange of place between I and IO_3, which correspond to Cs and CsO_3; when

ClO_3 and BrO_3 change places, we get 1/64 **[Rb]**; and when ClO_3 further changes places with I, we get 1/64 **[Cs]**.



The expressions 1/64 **[Cs]**, 1/64 **[Cl]**, etc., are used as abbreviations to mean the hexagons corresponding to the nuclei **Cs**, **Cl**, etc., and the solutions containing 1/64 grm.-mol. salt per thousand grams of water. The general expressions m **[R]** and m **[M]** indicate the hexagons corresponding to salts with a metalloidal or a metallic nucleus respectively, the solutions of which contain m grm.-mol. of the salt per thousand grams of water.

§ 64. It has already been pointed out that the hexagonal arrangement of residues in ascending order of magnitude of v/m is the same for each nucleus at all concentrations for which $m > 1/64$, and we have shown how the arrangements expressed by 1/64 **[R]** and 1/64 **[M]** are derived from that corresponding to 1/64 **[Cl]**. We now proceed to consider the case of m **[R]** and m **[M]** when $m \lesseqgtr 1/64$. It is only in solutions of such low concentration that the phenomenon of expansion on further dilution with water shows itself.

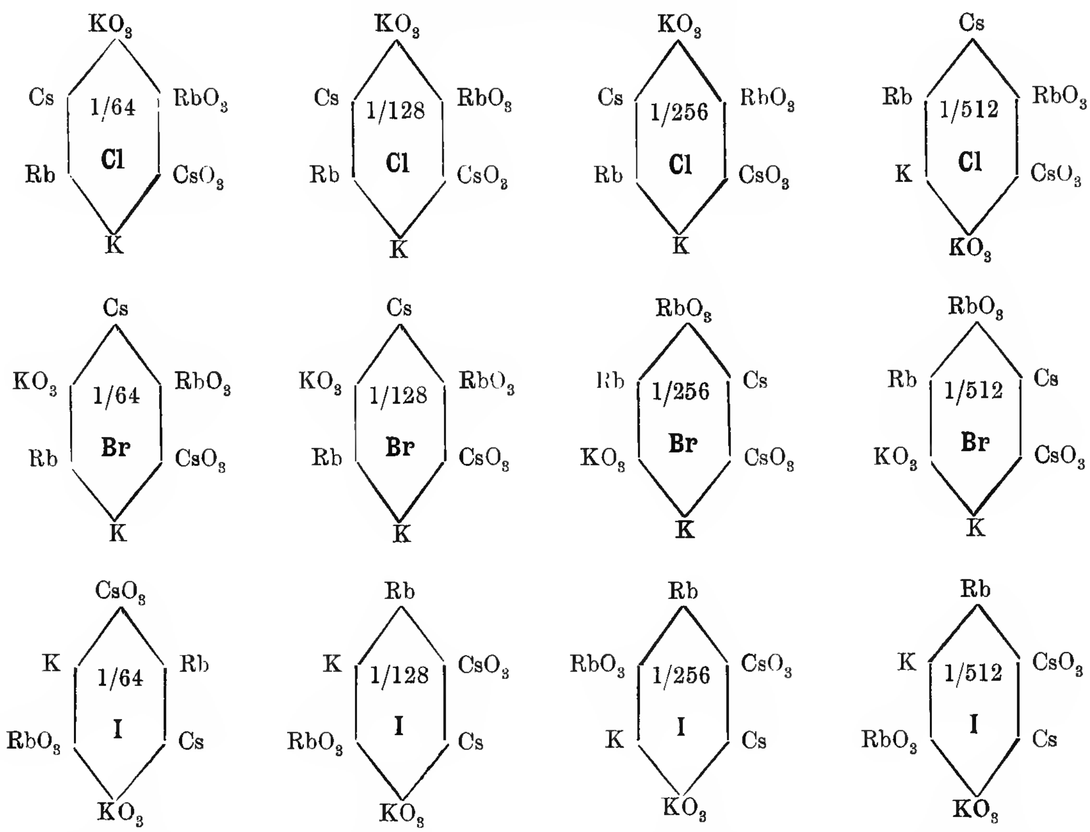
Beginning with the solutions of salts having a metalloidal nucleus, **R**, we arrange the hexagons in three lines of four hexagons each. For the top lines **R** = **Cl**, for the middle line **Br**, and for the lowest line **I**. In each line the concentrations are defined by $m = 1/64$, 1/128, 1/256, and 1/512 in consecutive order. This arrangement is exhibited in the group of hexagons on next page.

Considering the hexagons with nucleus **Cl**, we see that, for $m \geqq 1/256$, the residues follow in the orders of the triads M, MO_3, and in each triad in the ascending order of atomic weight, K, Rb, Cs, KO_3, RbO_3, CsO_3. This may be called the *regular system*. For $m = 1/512$ this order is preserved, with the exception that place 1 is taken by KO_3 and we have KO_3, K, Rb, Cs, RbO_3, CsO_3.

When the nucleus is **Br** and $m = 1/64$, 1/128, the order of residues is that of consecutive triads, with transposition of KO_3 and Cs. For $m = 1/256$ and 1/512, Rb and KO_3 are transposed, and also RbO_3 and Cs, so that the residues come to be arranged alternately after the type M and MO_3 respectively round the hexagons— K, KO_3, Rb, RbO_3, Cs, CsO_3. In the eight hexagons m **[Cl]**, m **[Br]**, place 6 is occupied by CsO_3, and in seven out of the eight hexagons place 1 is occupied by K.

When the nucleus is **I**, we find that place 1 is occupied in all cases by KO_3, and place 6 by Cs; that is, the initial residue of the second triad takes the lowest place, and the final residue of the first triad takes the highest place. Places 2 and 3 are occupied by RbO_3 and K respectively for $m = 1/64$, $1/128$, $1/512$, and by K, RbO_3 for $m = 1/256$; places 4 and 5 are occupied by Rb and CsO_3 respectively for $m = 1/128$, $1/256$, $1/512$, and by CsO_3, Rb for $m = 1/64$. For nucleus **I** and $m = 1/256$, the order of residues is the exact counterpart of that for nucleus **Br** and $m = 1/256$, $1/512$. In

Hexagons of the Type m **[R]**.

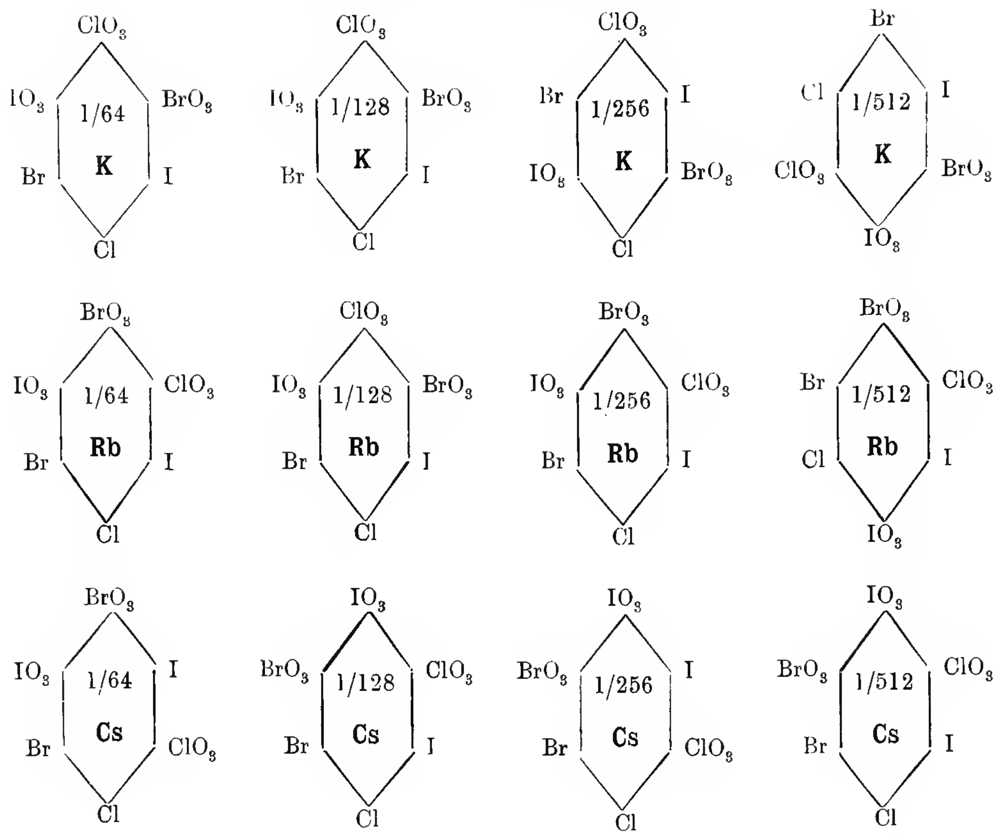


the latter, the residues of type MO_3 occupy the places with even numbers, and those of type M occupy those with odd numbers; in the former, the opposite is the case, so that we have the following arrangement of residues in—

Places numbered . . .	1	2	3	4	5	6
1/256 and 1/512 **[Br]** 1/256 **[I]**	K KO_3	KO_3 K	Rb RbO_3	RbO_3 Rb	Cs CsO_3	CsO_3 Cs

§ 65. Turning now to solutions of salts having a metallic nucleus, **M**, we arrange the hexagons in three lines of four hexagons each, as on the following page. For the top line the nucleus is **K**, for the middle line **Rb**, and for the lowest line **Cs**. In each line the concentrations of the solutions are defined by $m = 1/64$, $1/128$, $1/256$, and $1/512$, in this order.

Hexagons of the Type m **[M]**.



The outstanding feature of the m **[M]** hexagons is the position occupied by IO_3, which corresponds to CsO_3 in the m **[R]** hexagons. Whereas CsO_3 in these solutions occupies place 6 in eight out of twelve solutions, IO_3 does not occupy place 6 in any of the m **[M]** solutions. In two cases it occupies place 1, in one case place 2, and in six cases place 3; thus, in nine cases out of twelve it is found in the first three places of the hexagon, and in the three remaining cases it occupies place 4. It is most frequently found at place 3, which in the regular system is the place for I. But I is never found elsewhere than in places 5 and 6. It occupies place 6 eight times, and place 5 four times. Therefore, the feature of the m **[M]** hexagons is that I and IO_3 have exchanged their regular places. Indeed, if we change them back again in the m **[K]** solutions for which $m = 1/64$ and $1/128$, we get the regular arrangement corresponding to that of 1/64 **[Cl]**, namely, Cl, Br, I, ClO_3, BrO_3, IO_3. This preference of IO_3 for the final place in the first triad, in place of I, which takes that in the second triad, is quite comparable with the interchange of functions between K and KO_3 as regards the initial positions in the triads of salts of nucleus **R**.

The other four residues confine themselves very closely to their own triads; thus Cl does not leave it at all; Br leaves it only once; ClO_3 also leaves it once, and BrO_3 three times.

§ 66. In the following tables we collect the different schemes of ordinal sequence of residues for the different solutions of the hexads having metalloidal nuclei. It will be seen that the regularity of these sequences and that of their progressive development is remarkable.

Hexagon.	Concentration (m) and Remarks.	Ordinal Sequence of Residues.					
m [Cl]	The *regular system* of sequence is found in 1/64, 1/128, 1/256.	K	Rb	Cs	KO_3	RbO_3	CsO_3
	By shifting KO_3 to the first place and closing up, we get 5/6 of the regular system.	KO_3	K	Rb	Cs	RbO_3	CsO_3
m [Br]	When KO_3 and Cs exchange places in the regular system, we have 1/64, 1/128.	K	Rb	KO_3	Cs	RbO_3	CsO_3
	The arrangement for 1/256 and 1/512 is also a very regular one.	K	KO_3	Rb	RbO_3	Cs	CsO_3
m [I]	Equally regular but opposed to the last is 1/256 .	KO_3	K	RbO_3	Rb	CsO_3	Cs
	When K and RbO_3 exchange places, we have 1/128 and 1/512.	KO_3	RbO_3	K	Rb	CsO_3	Cs
	And when Rb and CsO_3 now exchange places, we have 1/64.	KO_3	RbO_3	K	CsO_3	Rb	Cs

When we consider the hexads with metallic nucleus, we find less regularity in the ordinal sequence of residues than we did in the hexads with metalloidal nucleus. The greatest regularity is shown by the hexads which exhibit the common initial sequence Cl, Br, IO_3, and we take them first.

Hexagon.	Concentration (m) and Remarks.		Ordinal Sequence of Residues.					
m [K]	1/64, 1/128	This arrangement is derived from the *regular system* by transposition of I and IO_3.						
m [Rb]	1/128		Cl	Br	IO_3	ClO_3	BrO_3	I
m [Rb]	1/64, 1/256	Transposing BrO_3 and ClO_3	Cl	Br	IO_3	BrO_3	ClO_3	I
m [Cs]	1/64	Transposing ClO_3 and I . . .	Cl	Br	IO_3	BrO_3	I	ClO_3
m [Cs]	1/256	Transposing IO_3 and BrO_3 . . .	Cl	Br	BrO_3	IO_3	I	ClO_3
m [Cs]	1/128, 1/512	Transposing ClO_3 and I	Cl	Br	BrO_3	IO_3	ClO_3	I
m [Rb]	1/512	Shifting IO_3 to the beginning . . .	IO_3	Cl	Br	BrO_3	ClO_3	I
m [K]	1/512	These two are not derived by any simple transpositions from others.	IO_3	ClO_3	Cl	Br	I	BrO_3
m [K]	1/256		Cl	IO_3	Br	ClO_3	I	BrO_3

In the following table the ordinal sequence of residues is shown in another form. It consists of six sub-tables corresponding to the six nuclei respectively. The middle column (m) gives the concentration of the solution in each line. The numbers in the body of each sub-table are the ordinal numbers of the places occupied by the residues at the top of each column in the solutions specified by the symbol of the hexagon at the top of the sub-table and the value of m on the same line in the column in the middle of the table. The numerals in the sub-tables are printed, some in black type and some in ordinary type. The black type is used only when the particular residue occurs three times out of four in its particular column; the ordinary type is used when it occurs twice or once in the same column.

Residues.						Grm.-mol. Salt per 1000 grms. Water.	Residues.					
K.	Rb.	Cs.	KO_3.	RbO_3.	CsO_3.		Cl.	Br.	I.	ClO_3.	BrO_3.	IO_3.
Places occupied by Residues.						m.	Places occupied by Residues.					
Hexagon m [Cl].							Hexagon m [K].					
1	2	3	4	5	6	1/64	1	2	6	4	5	3
1	2	3	4	5	6	1/128	1	2	6	4	5	3
1	2	3	4	5	6	1/256	1	3	5	4	6	2
2	3	4	1	5	6	1/512	3	4	5	2	6	1
Hexagon m [Br].							Hexagon m [Rb].					
1	2	4	3	5	6	1/64	1	2	6	5	4	3
1	2	4	3	5	6	1/128	1	2	6	4	5	3
1	4	2	5	3	6	1/256	1	2	6	5	4	3
1	4	2	5	3	6	1/512	2	3	6	5	4	1
Hexagon m [I].							Hexagon m [Cs].					
3	5	6	1	2	4	1/64	1	2	5	6	4	3
3	4	6	1	2	5	1/128	1	2	6	5	3	4
2	4	6	1	3	5	1/256	1	2	5	6	3	4
3	4	6	1	2	5	1/512	1	2	6	5	3	4

Section X.—Experimental Observations on the Displacement of Solutions of Sodium Chloride.

§ 67. Although chloride of sodium is not a member of either of the enneads which form the principal material of this research, its importance in nature justifies its inclusion in it.

The observations made at 15·0° C. and recorded in Table No. 26, Class A, were not sufficient. A complete series of observations of specific gravity in which the values of m formed a geometric series, descending from 1 to 1/512, was made at 19·50° C.

Besides this, two arithmetic series were included, in one of which the common difference in values of m was 1/128, and in the other 1/64.

In the first of these two series there were eight solutions, in which m = 1/128, 2/128, 3/128, 4/128, 5/128, 6/128, 7/128, 8/128, respectively.

The second series proceeded by a common difference of 1/64 from 1/64 to 8/64, the first four of this series being included in the first arithmetic series, namely, 2/128, 4/128, 6/128, 8/128.

Table of Results of Experiments made on Solutions of Sodium Chloride varying in Concentration from 1 *gram-molecule to* 1/512 *gram-molecule per* 1000 *grams of Water by the Hydrometric Method.*

SODIUM CHLORIDE. NaCl = 58·5.

T = 19·5° C.

m.	W.	S.	log Δ.	$\frac{d \log \Delta}{dm}$.	Δ.	*d*Δ.	$\frac{d\Delta}{dm}$.	$\frac{v}{m}$.	$\frac{\log \Delta_1 - 3}{\log \Delta_m - 3} = \frac{1}{x}$.	$\frac{1}{m} - \frac{1}{x}$.
1·000	1058·500	1·039683	3·0077899		1018·099			18·099	1·000	0·000
1/2	1029·250	1·020283	3·0038002	0·0079794	1008·789	9·310	18·620	17·578	2·049	– 0·049
1/4	1014·625	1·010300	3·0018552	0·0077800	1004·281	4·508	18·032	17·124	4·199	– 0·199
1/8	1007·312	1·005166	3·0009262	0·0074320	1002·135	2·146	17·168	17·080	8·410	– 0·410
7/64 = $\frac{1}{9·143}$	1006·398	1·004579	3·0007857	0·0089920	1001·811	0·324	20·736	16·558	9·914	– 0·771
6/64 = $\frac{1}{10·666}$	1005·484	1·003920	3·0006761	0·0070144	1001·558	0·253	16·192	16·619	11·522	– 0·856
5/64 = $\frac{1}{12·800}$	1004·570	1·003265	3·0005645	0·0071424	1001·301	0·257	16·448	16·653	13·800	– 1·000
1/16	1003·656	1·002636	3·0004411	0·0078976	1001·017	0·284	18·176	16·272	17·660	– 1·660
7/128 = $\frac{1}{18·286}$	1003·199	1·002294	3·0003919	0·0062976	1000·903	0·114	14·592	16·512	19·877	– 1·591
6/128 = $\frac{1}{21·333}$	1002·742	1·001947	3·0003444	0·0060800	1000·793	0·110	14·080	16·917	22·618	– 1·285
5/128 = $\frac{1}{25·600}$	1002·285	1·001615	3·0002904	0·0069120	1000·669	0·124	15·872	17·126	26·824	– 1·224
1/32	1001·828	1·001295	3·0002312	0·0075776	1000·530	0·139	17·792	16·960	33·693	– 1·693
3/128 = $\frac{1}{42·666}$	1001·371	1·001007	3·0001574	0·0094464	1000·364	0·166	21·248	15·531	49·491	– 6 825
1/64	1000·914	1·000652	3·0001137	0·0055936	1000·262	0·102	13·056	16·768	68·512	– 4·512
1/128	1000·457	1·000325	3·0000573	0·0072192	1000·132	0·130	16·640	16·897	135·949	– 7·949
1/256	1000·228	1·000131	3·0000421	0·0038912	1000·097	0·035	8·960	24·832	185·033	+ 70·967
1/512	1000·114	1·000058	3·0000243	0·0091136	1000·056	0·041	20·992	28·672	320·531	+ 191·469

§ 68. *Preparation of Solutions.*—Instead of diluting a stronger solution in order to obtain the solution of requisite concentration in these arithmetic series, the method was adopted of adding a quantity of solid sodium chloride to a weighed quantity of a solution whose concentration was included in the series so as to produce the next solution of higher concentration in the series.

The following schedule was drawn up from calculations made as to quantities of solution and salt required in the preparation of an arithmetic series of solutions from 1/128 to 8/128 gram-molecule of sodium chloride in 1000 grams of water, each solution being prepared from the more dilute one immediately preceding it.

a.	*b.*	*c.*	*d.*	*e.*	*f.*
n.	*m.*	Quantity of Water in Solution whose Concentration is *m* gram-molecules per 1000 grams Water.	Quantity of Salt in the Solution whose Concentration is *m* gram-molecules per 1000 grams of Water.	Quantity of *n* – 1 Solution required, whose Concentration is *m* – 1.	Extra Quantity of Salt to be added to Quantity of Solution given in Column *e*.
1	1/128	750 grams	0·3427 grams		
2	2/128	730 „	0·6673 „	730·3335 grams	0·3338 gram
3	3/128	710 „	0·9735 „	710·6490 „	0·3245 „
4	4/128	690 „	1·2614 „	690·9461 „	0·3153 „
5	5/128	670 „	1·5310 „	671·2248 „	0·3062 „
6	6/128	650 „	1·7824 „	651·4852 „	0·2972 „
7	7/128	630 „	2·0155 „	631·7275 „	0·2880 „
8	8/128	610 „	2·2303 „	611·9515 „	0·2788 „

Note.—The weights entered in this Table are the true weights, *in vacuo*, required for the production of the required solutions. The actual weights placed on the pan of the balance were those weights adjusted for the meteorological conditions at the time of experiment.

The second series of experiments was made on a series of solutions having a common difference of 1/64 gram-molecule. In this series it was necessary only to make experiments on solutions where $m = 5/64$, $6/64$, $7/64$, $8/64$, as the first four solutions were included in the first arithmetic series.

The remaining solutions necessary to complete the geometric series from 1 to 1/512 gram-molecule were prepared in each case by the direct dissolution of salt in water.

§ 69. *Experimental Results.*—The following table, abstracted from the full table, § 67, gives the experimental results for the geometric series from 1 to 1/512 gram-molecule solutions:—

NaCl = 58·5. T = 19·5° C.

m.	W.	S.	Δ.	$d\Delta$.	$d \log \Delta$.	$\frac{d \log \Delta}{dm}$.	$\frac{d\Delta}{dm}$.	$\frac{v}{m}$.
1	1058·500	1·039683	1018·099					18·099
1/2	1029·250	1·020283	1008·789	9·310	0·0039897	0·0079794	18·620	17·578
1/4	1014·625	1·010300	1004·281	4·508	0·0019450	0·0077800	18·032	17·124
1/8	1007·312	1·005166	1002·135	2·146	0·0009290	0·0074320	17·168	17·080
1/16	1003·656	1·002636	1001·017	1·118	0·0004851	0·0077616	17·888	16·272
1/32	1001·828	1·001295	1000·530	0·487	0·0002099	0·0067168	15·584	16·960
1/64	1000·914	1·000652	1000·262	0·268	0·0001175	0·0075200	17·152	16·768
1/128	1000·457	1·000325	1000·132	0·130	0·0000564	0·0072192	16·640	16·897
1/256	1000·228	1·000131	1000·097	0·035	0·0000152	0·0038192	8·960	24·832
1/512	1000·114	1·000058	1000·056	0·041	0·0000178	0·0091136	20·992	28·672

§ 70. *Specific Gravity.*—The observations were made with the two hydrometers 3 and 17, and as a rule three series of observations were made with each hydrometer, so that each entry in column S is the mean of six series. The greatest departure of any one value from this mean was 2·2 in the fifth decimal place.

From $m = 1/2$ to $m = 1/8$ the rate of decrease of the increment of the specific gravity $(S - 1)$ is less than that of the concentration. Between $m = 1/16$ and $m = 1/128$ there is an approach to a proportion between the rate of decrease of specific gravity and that of concentration, while at $m = 1/256$ and $1/512$ this decrease is out of all proportion to that of concentration.

§ 71. *Displacement*, Δ.—This value reflects the nature of changes in specific gravity. If we compare the values of v with those of $d\Delta$, we see that, between $m = 1/2$ and $m = 1/8$, $d\Delta$ is greater than v; between $m = 1/16$ and $m = 1/128$ their values are nearly identical, and for $m = 1/256$ and $m = 1/512$ there is considerable disparity.

If we take the specific gravity of NaCl in crystal to be 2·15, the sum of the separate displacements of 1 gram-molecule of salt and 1000 grams of water is 1027·209. The displacement after dissolution has been effected is 1018·099, showing a contraction of 8·510. When 1/16 NaCl is dissolved in 1000 grams of water the increment of displacement per gram-molecule, v/m, is 16·272, showing a contraction of 10·937. Here the compression accompanying the act of dissolution reaches a maximum. When the concentration of the solution is less or greater than 1/16 NaCl + 1000 grams of water the compression accompanying the act is less in both cases. When 1/256 NaCl has been dissolved in 1000 grams of water the increment of displacement per gram-molecule, v/m, is 24·832, and when 1/512 NaCl has been dissolved in 1000 grams of water the value of v/m is 28·672, which is greater than that of the salt in crystal, namely, 27·209.

§ 72. *Experiments on Solutions forming Arithmetic Series.*—Two sets of such experiments were made, the first series having the common difference 1/128 gram-molecule, while the other had a common difference 1/64.

The method of preparation of the solutions has been referred to in § 68, and in this connection it is only necessary to say that, owing to the interesting nature of the results obtained, it was considered advisable to repeat the experiments on certain solutions in order to check the results obtained.

In each of these cases the solution was prepared by the direct dissolution of the salt in water.

The result of these repetitions was in all cases to confirm the results obtained in the original experiments, and will be referred to later.

§ 73. *Series of Experiments on Solutions having the Common Difference* $dm = 1/128$.

NaCl = 58·5. T = 19·50° C.

m.	W.	S.	Δ.	$d\Delta$.	$d \log \Delta$.	$\frac{d \log \Delta}{dm}$.	$\frac{d\Delta}{dm}$.	$\frac{v}{m}$.
8/128	1003·656	1·002636	1001·017					16·272
7/128	1003·199	1·002294	1000·903	0·114	0·0000492	0·0062976	14·592	16·512
6/128	1002·742	1·001947	1000·793	0·110	0·0000475	0·0060800	14·080	16·917
5/128	1002·285	1·001615	1000·669	0·124	0·0000540	0·0069120	15·872	17·126
4/128	1001·828	1·001295	1000·530	0·139	0·0000592	0·0075776	17·792	16·960
3/128	1001·371	1·001007	1000·364	0·166	0·0000738	0·0094464	21·248	15·531
2/128	1000·914	1·000652	1000·262	0·102	0·0000437	0·0055936	13·056	16·768
1/128	1000·457	1·000325	1000·132	0·130	0·0000564	0·0072192	16·640	16·897

§ 74. *Difference of Displacement,* $d\Delta$.—The numbers in the column $d\Delta$ show a characteristic rise in value from 7/128 to 3/128 gram-molecule concentration, and then a fall at 2/128 gram-molecule.

Under $\frac{d\Delta}{dm}$ these differences are referred to the constant value $m = 1$ and reproduce in an exaggerated degree the rise from the 6/128 to 3/128 gram-molecule.

The values for v/m show a similar rise, but the maximum occurs at the value for m immediately preceding that for the maximum value for $\frac{d\Delta}{dm}$, but the character of the change is the same.

The following table, which gives the maximum, mean, and minimum specific gravities for each value of m, together with the corresponding values for displacement and the difference of displacement for successive maximum, mean, and minimum displacements, confirms the reality of the character of change in the values for $d\Delta$:—

NaCl = 58·5. T = 19·5° C.

m.	Weight of Solution (mMR + 1000).	Maximum, Mean, and Minimum Specific Gravity.	Displacement corresponding to each Value of Specific Gravity.	Difference of Displacement for successive Maximum, Mean, and Minimum Values of Displacement.
m.	W grams.	S.	Δ.	*d*Δ.
1/128	1000·457	1·000344 1·000325 1·000320	1000·123 ·132 ·137	
2/128	1000·914	1·000662 1·000652 1·000646	1000·252 ·262 ·268	0·129 ·130 ·131
3/128	1001·371	1·001022 1·001007 1·001000	1000·349 ·364 ·371	0·097 ·102 ·103
4/128	1001·828	1·001305 1·001295 1·001286	1000·522 ·532 ·542	0·173 ·168 ·171
5/128	1002·282	1·001621 1·001615 1·001610	1000·660 ·667 ·671	0·138 ·134 ·128
6/128	1002·742	1·001953 1·001947 1·001936	1000·788 ·793 ·804	0·128 ·127 ·133
7/128	1003·199	1·002306 1·002294 1·002282	1000·891 ·903 ·915	0·103 ·110 ·111
8/128	1003·656	1·002645 1·002636 1·002623	1001·007 ·017 ·029	0·116 ·114 ·114

The confirmation of the reality of the nature of the changes in displacement with change of concentration which is given by the preceding table is due to the close agreement of the specific gravity values in any series for a particular value of *m*, and the character of change in the difference of displacement is the same whether the maximum or minimum values for difference of displacement Δ for consecutive values of *m*, or maximum value of Δ_m with minimum value of Δ_{m+1}, are taken.

§ 75. Although the above table shows that no further confirmation was required, the repetitions intimated in § 72 were made, and gave the following results.

The concentrations selected for this purpose were 2/128, 3/128, and 7/128 gram-molecules.

The solutions were made by the direct dissolution of the requisite amount of salt in the appropriate quantity of water, and the results obtained in these later experiments are compared in the following table, side by side with those obtained in the first series of experiments :—

Concentration of the Solution in gram-molecules per 1000 grams of Water.	Results of Original Experiments.			Results of Repeated Experiments.			Difference between the Mean Specific Gravities in the two Series.
	Highest Value of Specific Gravity.	Lowest Value of Specific Gravity.	Mean Specific Gravity.	Highest Value of Specific Gravity.	Lowest Value of Specific Gravity.	Mean Specific Gravity.	
2/128	1·000662	1·000646	1·000652	1·000672	1·000647	1·000655	0·000003
3/128	1·001022	1·001000	1 001007	1·001019	1·000992	1·001007	0·000000
7/128	1·002306	1·002282	1·002294	1·002309	1·002292	1·002298	0·000004

Note.—The comparison of the figures given in the above table, where the greatest difference of the mean specific gravities is 4 in the sixth decimal place, shows that the changes in displacement of the solutions at these concentrations are real, and confirm the character of the changes in the values of $d\Delta$ and v, as shown in the table of original results.

§ 76. *The Arithmetic Series of Solutions having the Common Difference of Concentration* 1/64 *gram-molecule.*—Observations were made on the 5/64, 6/64, 7/64, and 8/64 gram-molecule concentrations, as the 1/64, 2/64, 3/64, and 4/64 gram-molecule concentrations were dealt with in the series having the common difference 1/128 (*vide supra*).

The solutions were prepared according to a scheme similar to the one described in § 68.

The following table gives all observed and derived data :—

NaCl = 58·5. T = 19·5° C.

m.	Weight of Solution.	Specific Gravity.	Displacement.	Difference of Displacements.	Difference of Logarithms of Displacement.	Mean Increment of Displacement per gram-molecule of Salt.	Difference of Displacement per gram-molecule.
m.	W.	S.	W/S = Δ.	$d\Delta$.	d log Δ.	$\frac{v}{m}$.	$\frac{d\Delta}{dm}$.
1/64	1000·914	1·000652	1000·262			16·768	
				0·268	·00011772		17·152
2/64	1001·828	1·001295	1000·530			16·960	
				0·263	·00011303		16·832
3/64	1002·742	1·001947	1000·793			16·917	
				0·224	·00009713		14·336
4/64	1003·656	1·002636	1001·017			16·272	
				0·284	·00012295		18·176
5/64	1004·570	1·003265	1001·301			16·653	
				0·257	·00011153		16·448
6/64	1005·484	1·003920	1001·558			16·619	
				0·253	·00010962		16·192
7/64	1006·398	1·004579	1001·811			16·558	
				0·324	·00014053		20·736
8/64	1007·312	1·005166	1002·135			17·080	

§ 77. *Displacement* (Δ) *and Difference of Displacement* ($d\Delta$).—Variations in the values of $d\Delta$ are obtained here as in the last series.

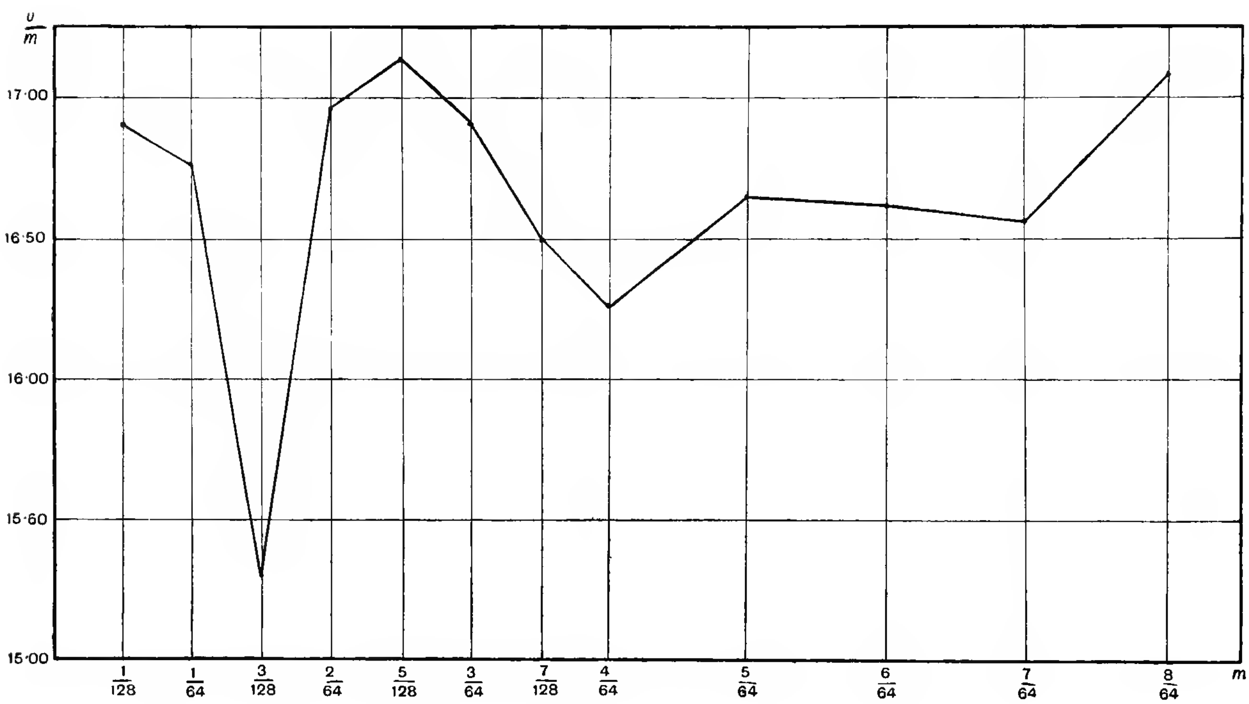
The maximum value is 0·324 between $m = 7/64$ and $m = 8/64$ gram-molecule concentration, and the minimum is 0·224 between $m = 3/64$ and $m = 4/64$, comprising, as it does, the interval represented in the former arithmetic series between $m = 6/128$ and $m = 8/128$, including the two low values of 0·110 and 0·114.

There is a sudden rise to the high value 0·284 between $m = 4/64$ and $m = 5/64$, comparable with the change between $m = 2/128$ and $m = 3/128$ in the earlier arithmetic series.

The variations are more strongly marked in the values of $\frac{d\Delta}{dm}$, which, in an exaggerated manner, indicate the nature of the changes from high to low values.

The values of v/m show an undulatory variation in consecutive values as contrasted with the values of $\frac{d\Delta}{dm}$.

§ 78. In the following diagram the values of v/m are represented by the ordinates, and those of m by the abscissæ :—



The changes in the value of v/m with change of m, as shown by the contour of the curve, indicate marked phases at concentrations where $m = 3/128$, $7/128$, $4/64$, and $7/64$ gram-molecules, and a very characteristic sequence of changes takes place between $m = 4/128$ and $m = 8/128$.

The advantage of introducing the arithmetical series is that these remarkable variations of displacement are clearly displayed, whereas they would be apt to be masked if we had only the geometric series.

The reality of the remarkable features of solutions of low concentration which have for the first time been demonstrated in this research is firmly established by the care with which the experiments have been made, and by the agreement between the results of independent series of observations.

Section XI.—The Principle and Construction of the Open Hydrometer.

§ 79. When the hydrometer is closed its mass cannot be diminished, and it can be increased only by external additions, which in practice must be immersed either in air or in the liquid. The use of submerged weights is attended by so much inconvenience that it has to be avoided; consequently, when the instrument is closed, its mass is increased only by adding weights at the top of the stem. The extent to which such additions can be made depends on the stability of the instrument when floating in the experimental liquid. The instrument (No. 0) which I used exclusively during the voyage of the *Challenger* weighed, in round numbers, 160 grams, and the greatest weight which had to be added to it was 4·071 grams, which produced no disturbing effect whatever. But I had the curiosity to find out what was the limiting weight which could be used without altering the "trim" of the instrument, and it turned out that it could be used in solutions of chloride of sodium of all concentrations. Taking the specific gravity of the saturated solution as 1·2, 160 cubic centimetres of it must weigh 192 grams, so that the weight to be added was 32 grams. The instrument bore it; but in a solution of chloride of calcium of slightly greater density it took a "list." These experiments showed that the efficiency of the hydrometer did not diminish when the concentration of the solution in which it was used was increased, but its handiness was affected when such heavy weights had to be attached to the stem.

In order to be able to use a method of such high precision for the determination of the density of solutions of all concentrations, I determined to construct hydrometers which should be left open at the top, so that their internal load, or ballast, could be altered, and they could then be used in exactly the same way as the closed instrument, by adding series of moderate weights to the top of the stem to produce corresponding series of immersions or displacements.

§ 80. The glass instrument is made after the ordinary pattern, fig. 4, consisting of a spherical bulb at the lower extremity to hold the ballast, a cylindrical body having a diameter not less than that of the ballast bulb, and above it the cylindrical stem of relatively small calibre; it is left open instead of being hermetically sealed as in the ordinary hydrometer. Instruments of this pattern may be ballasted either with mercury or shot. The latter is the material which has been generally used, because it is more easily handled than mercury, and, being confined in a spherical bulb of relatively small size, it cannot shift and thereby disturb the trim of the floating instrument. Moreover, when shot of a given and uniform size—for instance, No. 10—is used, the load may be altered and adjusted by counting pellets.

When we have to deal with concentrated solutions of salts which are at once very soluble and very expensive, the lower ballast bulb is suppressed, and the ballast is accommodated in the cylindrical body of the instrument, as in fig. 5. With this not uncommon form of instrument, and a cylinder of no greater diameter than that which is absolutely necessary to secure free flotation of the hydrometer, the specific gravity of a solution can be determined with a minimum quantity of liquid. With instruments of this form it is necessary to use mercury as ballast, not only in order to keep the

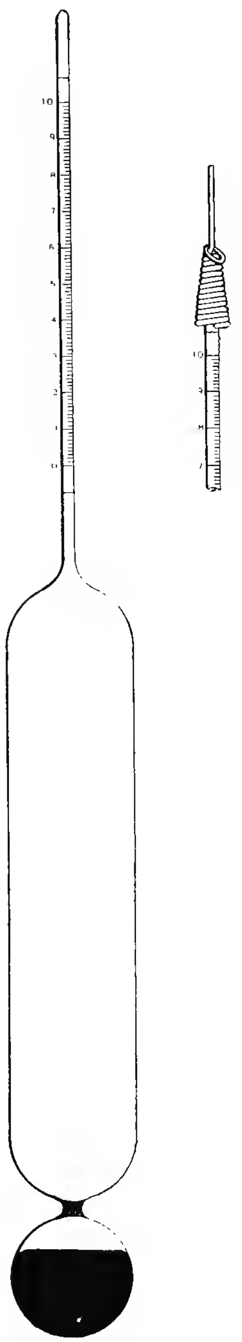


Fig. 3.

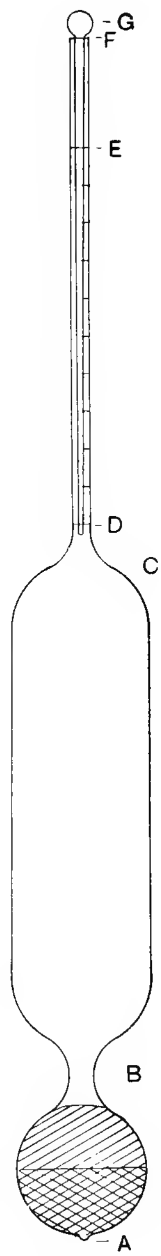


Fig. 4.

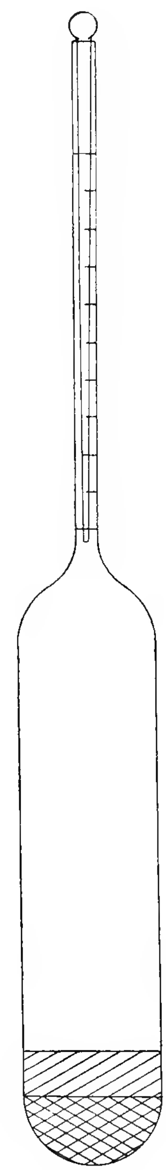


Fig. 5.

centre of gravity as low as possible, but also because, when free in a wide cylinder and not confined in a small sphere, a quantity of shot does not necessarily rest with its upper surface quite perpendicular to the direction of gravity.

The paper scale, so convenient in the closed hydrometer, is inadmissible when the internal load has to be shifted. Consequently the scale (millimetres) is etched on the outside of the stem. When prepared for use the top of the stem is loosely closed by a thin rod of white enamel glass which passes down the stem so far as the engraved scale extends, and it is kept suspended in this position by being thickened to a button at the top. This addition was made in order to provide an opaque white surface behind the scale. I was unable at the time to procure suitable tubing with white slip let into it. It is an integral part of the instrument, to the mass of which it contributes its share.

The mass of the hydrometer is equal to the sum of the masses of its parts, namely, the glass, the ballast, and the air respectively.

§ 81. In order fully to appreciate the importance of each of these masses in furnishing the effective weight of the instrument, let us imagine that we are actually working in a vacuum, and at sea-level in latitude 45° ; thus the air both outside and inside falls away, and we have only the glass and the ballast remaining. Let us assume that the load is of shot and has been so adjusted that the instrument, when immersed in distilled water, of the temperature which is to be maintained uniform during the experiment, floats with only a small portion of the stem immersed.

The first operation performed in the vacuum is to weigh the instrument; let its weight be W grams (true). *Experiment No.* 1.—Let it then be floated in distilled water of the fixed constant temperature, and let the surface of the water cut the stem in a line at C, fig. 6, next page. Then the weight of the water displaced by the instrument below the line C is W grams (true). If we replace the water in the cylinder by a liquid of greater density, such as a saline solution, and float the instrument in it, we find that less of the instrument is immersed, and we are obliged to add weights at the top of the stem in order to immerse it until the surface of the liquid cuts the stem in the line C. Let the weight so added be w grams (true). The weight of the liquid displaced by the hydrometer when immersed in it up to the line C is then $(\mathrm{W}+w)$ grams (true). From this it follows that the weights of equal volumes of distilled water and of the experimental liquid, at the particular fixed temperature, are in the proportion of $\mathrm{W} : \mathrm{W}+w$, and the specific gravity of the liquid is $\frac{\mathrm{W}+w}{\mathrm{W}}$, referred to that of distilled water of the same temperature as unity.

Experiment No. 2.—Let us now replace the experimental liquid in the cylinder by distilled water and float the hydrometer in it. As before, the surface of the water will cut the stem at C. Let us add a small weight to the stem, so that the instrument is depressed until the surface of the water cuts the stem at D. Let the weight of this small weight be u_1 : then the weight of water so displaced up to D is $(\mathrm{W}+u_1)$ grams (true); let us now replace the water in the cylinder by the same experimental liquid as before; and let small weights be added to the top of the stem until the surface of the liquid cuts it at D. Let the weight of these small weights be w_1. Then the weight of the liquid displaced by the hydrometer is $(W+w_1)$ grams (true), and the specific gravity of the liquid at the fixed temperature is $\frac{W+w_1}{W+u_1}$. We may repeat this operation, with different added weights, as often as we please, and it is evident that at each operation we obtain a perfectly independent determination of the specific gravity of the liquid referred to that of distilled water of the same temperature as unity.

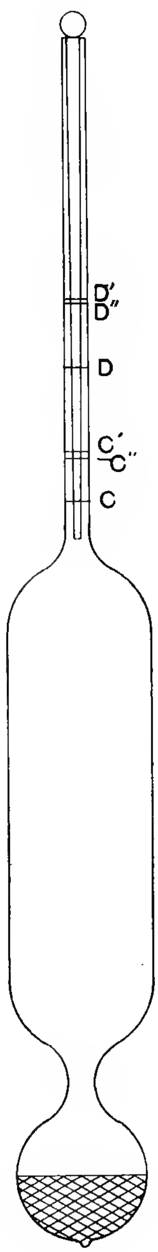


Fig. 6.

Experiment No. 3.—While continuing to work in the vacuum, let us allow air to enter and fill the hydrometer, after which the top of the stem is closed air-tight by a cover without weight. We now weigh the hydrometer, and find its weight to be $(W+a)$ grams (true).

Let it be immersed in distilled water of the fixed temperature. Being heavier than before, by a, the weight of air which it contains, it will sink in the water until its surface cuts the stem in a line a little above C, say C′; therefore $(W+a)$ grams (true) is the weight of the water displaced by the part of the hydrometer below C′. Let the water now be replaced by the same experimental liquid as before, and let the hydrometer be immersed in it, and let weights be added to the top until the surface of the liquid cuts the stem at C′; let this added weight be w' grams (true). w' will be a little greater than was w in the first experiment by the difference between the weights of the small cylinder CC′ of water and of liquid respectively, and the specific gravity of the liquid will be given by the ratio $\frac{W+a+w'}{W+a}$, which must be equal to $\frac{W+w}{W}$, as in the first experiment.

Experiment No. 4.—Similarly, if the hydrometer be now immersed in the distilled water with weight u_1, then the surface of the water will cut the stem at D′; if the water is now replaced in the cylinder by the experimental liquid, then we shall have to add a weight w_1', a little greater than w_1 in experiment 2, and the specific gravity of the liquid will be given by $\frac{W+a+w_1'}{W+a+u_1}$, and it will be the same as in the former experiments.

Let us now return to the experimental conditions in which the hydrometer full of air floats *in vacuo* at C′ in water, and, with the added weight w', at C′ in the liquid.

Experiment No. 5.—Let air be admitted generally, and let it be of the same density as that in the hydrometer, so that we are experimenting in air instead of in a vacuum. The hydrometer is still closed by the cover without weight; and, as the density of the air within and without the instrument is the same, there is no cause for disturbance of equilibrium between the two masses of air, and they will not interfere with each other.

The external air, admitted generally, reaches only to the surface of the water or liquid and cannot interfere with the immersed portion of the hydrometer. The stem, however, is now surrounded by a medium of given density, whereas, before, it was surrounded by one of insensible density. The exposed stem displaces its own volume of the air, and the downward vertical pressure which it exerts on the immersed portion of the hydrometer is diminished by the weight of this volume of air. The whole of its vertical pressure is exerted on the immersed part of the hydrometer below the line C′. As this pressure is diminished by the weight of the air displaced by the stem, the hydrometer will rise and will float a little higher; let it cut the stem at the line C″, which is situated a little lower than C′ but higher than C.

We have then the proportion :—

The volume of the cylinder C′C″ : that of the cylinder C′C :: the volume of the exposed stem : the volume of the air in the hydrometer.

The total vertical pressure exerted by the hydrometer when floating in the distilled water is now $W + a - s$, and the stem cuts the water at C″. s is the weight of the air displaced by the exposed portion of the stem.

Let the distilled water be now replaced by the same experimental liquid as before, and let the hydrometer be immersed in it, and let weights be added to the top of the stem until the surface of the liquid cuts the stem at C″; let this added weight be w'' grams (true). w'' will be a little less than was w' in Experiment No. 3, and the specific gravity of the liquid is given by the ratio

$$\frac{W + a + w'' - s}{W + a - s},$$

which must be equal to

$$\frac{W + a + w'}{W + a},$$

as in the third experiment, and also equal to $\frac{W + w}{W}$, as in the first experiment.

Experiment No. 6.—Similarly, if the hydrometer be now immersed in the distilled water with weight u_1, then the surface of the water will cut the stem at D″, a little lower than D′, but higher than D. If the water in the cylinder is now replaced by the same experimental liquid, then we shall have to add a weight w''_1, a little less than w'_1 in Experiment No. 4, and the specific gravity of the liquid will be given by the ratio

$$\frac{W + a + w''_1 - s}{W + a + u_1 - s},$$

and it will be the same as in the former experiments.

§ 82. The above suggested experiments will be best understood by reference to a specific instance of the use of the instrument.

The selected example is that of hydrometer A when immersed in distilled water, and afterwards in a 4·4 gram-molecule solution of potassium chloride.

Referring to Experiment No. 1, when working completely *in vacuo*, with the hydrometer immersed in distilled water, the adjustment would be such as to cause it to float with the stem immersed to a scale reading of C = 17·7 mm.

Weight of glass and shot = W = 136·69884 grams.

On replacing the distilled water by the experimental solution, the added weight necessary to cause the instrument to float at 17·7 mm. *in vacuo* would be w = 23·25860 grams (true).

Weight of shot + glass + added weight = W + w = 136·69884 + 23·25860 = 159·95744 grams. The specific gravity would therefore be :—

$$\frac{W+w}{W}=\frac{159\cdot95744}{136\cdot69884}=1\cdot170144.$$

On admitting air into the hydrometer (Experiment No. 3), the result would be to cause the instrument to be immersed to C′ = 32·2 mm. in distilled water.

This is arrived at in the following manner:—The internal volume occupied by the air is 112·493 c.c., and the density of the air was 0·001208 gram per cubic centimetre. The weight would therefore be a = 0·13592 gram.

Since 0·1 gram added weight produces an immersion of 10·69 mm. of stem, and as the weight of air admitted is distinctly an added weight, the immersion produced by 0·13592 gram would be

$$\frac{10\cdot69\times0\cdot13592}{0\cdot1}=14\cdot5 \text{ mm.}$$

Whence 17·7 + 14·5 = 32·2 mm.

Total weight after admission of air to hydrometer is :—

Weight of shot + glass + air = W + a = 136·69884 + 0·13592 = 136·83476 grams.

On immersing the hydrometer filled with air into the experimental liquid with a similar adjustment as in the first experiment (W + w), the hydrometer would be immersed to a point short of C′, since the air represents the same added weight in this case as when the hydrometer is immersed in distilled water, and the same added weight would not produce so great an immersion of the stem in the experimental liquid as in the distilled water of lower density. Hence an addition must be made to the original added weight (w) to cause the hydrometer to float at C′, the 32·2 millimetres division, when immersed in the experimental liquid, and the value of this addition is the difference between the weights of the same volume of experimental liquid and distilled water represented by the volume of the portion of stem immersed when air was admitted into the hydrometer while experimenting in distilled water.

Then we have seen that the weight required to increase the displacement in distilled water from C to C′ is the weight of the air filling the hydrometer, namely, 0·13592 gram. When the distilled water is displaced by the experimental solution, of specific gravity 1·170144, then the air admitted depresses the hydrometer in the solution from C to a point lower than C′. The total weight required to increase the immersion of the hydrometer in the solution from C to C′ is 0·13592 × 1·170144 = 0·15905 gram. Therefore, in addition to the weight of the air, we require a supplementary weight = 0·15905 − 0·13592 = 0·02313 gram. The total weight of the hydrometer is :—

Glass + shot	= W =	136·69884	grams.
Air	= a =	0·13592	,,
Total added weight	= w' =	23·28173	,,
		160·11649	,, = $W + a + w'$.

In this case 23·25860 + 0·02313 = 23·28173 grams = the total weight added to the top of the stem.

The specific gravity under these conditions is :—

$$\frac{W + a + w'}{W + a} = \frac{160 \cdot 11649}{136 \cdot 83476} = 1 \cdot 170144.$$

When air is admitted generally (Experiment No. 5), the line of flotation will be at another point, C″, for distilled water, and, as explained above, may be represented as a deduction from the added weight, the amount being equal to the weight of air displaced by the non-immersed portion of the stem; let s denote the value of the weight of air displaced.

The volume of the non-immersed portion of stem is 0·9 c.c., and the weight of 1 c.c. air = 0·001208 gram. Therefore the weight of air displaced is s = 0·00109 gram. Now an added weight of 0·1 gram produced an immersion of 10·69 millimetres, when the hydrometer was immersed in distilled water, so an alteration of immersion of the stem will occur, the amount being

$$\frac{10 \cdot 69 \times 0 \cdot 00109}{0 \cdot 1} = 0 \cdot 11 \text{ mm.}$$

Hence the final position of the hydrometer when immersed in distilled water will be C″ = 32·09 mm.; and the total weight :—

Glass + shot + air = $W + a$ = 136·69884 + 0·13592	= 136·83476	grams.
Less correction for non-immersed portion of stem = s =	− 0·00109	,,
$W + a - s$ =	136·83367	,,

In the case of the experimental liquid, the final position is nearly that of C″, the actual correction for the non-immersed portion of the stem being arrived at in the same manner as above, since 0·1 gram added weight produced an immersion of the stem of 9·17 millimetres, and the volume of air displaced being the same as above, as also is the weight, the alteration of immersion will be

$$\frac{9 \cdot 17 \times 0 \cdot 00109}{0 \cdot 1} = 0 \cdot 10 \text{ mm.}$$

The scale reading would therefore be 32·10 millimetres, and as the two readings are such that the difference on the millimetre scale is imperceptible, they are taken as identical.

The final weight is therefore :—

Shot + glass + air + added weight

$$= W + a + w' = 136·69884 + 0·13592 + 23·28170 = 160·11649 \text{ grams.}$$

Correction for non-immersed portion of stem $= s \qquad = -0·00109$,,

$$W + a + w' - s = 160·11540 \text{ ,,}$$

$$\text{Specific gravity} = \frac{160·11540}{136·83367} = 1·170144.$$

C″ is then the final position at which the surfaces of the water and of the experimental liquid cut the stem when the added weight is nothing for water and w' for the experimental liquid, and the experiment is made in air. The effective downward vertical pressures are represented by the true weights $W + a - s$ and $W + a + w' - s$ respectively.

It should be noted that in the example here given the value of the weight added to the stem of the hydrometer to cause it to float at C″ = 32·09 mm. is so very nearly the same as that which caused it to float at C′ = 32·2 mm., that no alteration has been made in the value of this weight, and we have, therefore, used the symbol w' in this first experiment instead of w'', as given in Experiment No. 5.

We have imagined that the open hydrometer was actually weighed in a vacuum, when it contained no air. In practice the hydrometer is weighed full of air and in air. When to this weight we apply the vacuum correction, that is, the weight of air displaced by the whole hydrometer and closed with its weightless cover, we obtain the value of $W + a$ which is the working weight *in vacuo* of the hydrometer. In this expression, for any particular load, W is constant, it is the sum of the weights of glass and shot alone. The weight of air, a, contained in it will vary with the density of the atmosphere at the time.

§ 83. The open hydrometer consists of G_o grams (true) of glass and L_o grams (true) of shot and A_o grams (true) of air, as when weighed *in vacuo*. In order to obtain these constants, we first weigh the glass instrument empty as it comes from the glass-blower, and find that it weighs G grams in air of given density. Taking the specific gravity of the glass to be 2·5, we obtain $\frac{G}{2·5}$ as the volume (in cubic centimetres) of the glass. The weight of $\frac{G}{2·5}$ cubic centimetres of air of the given density is a_g grams, and when added to G gives the weight *in vacuo* of the glass of the instrument :

$$G_o = G + a_g.$$

Similarly, the weight of the shot added as load is found to be L grams in air, and if we take its specific gravity to be 11·35, the volume of air which it displaces is $\frac{L}{11\cdot35}$ c.c., the weight of which at the observed density is a_l grams, whence the weight *in vacuo* of the lead is

$$L_o = L + a_l.$$

If the load L has been so adjusted that at the temperature, T, fixed for the experiments the hydrometer floats in distilled water at the top of the stem, then the weight of distilled water displaced by the hydrometer when so floating is *in vacuo* $G_o + L_o + A_o$, whence the external volume of the whole instrument is obtained; let this volume be V, then we have

$$V = \frac{G_o}{2\cdot5} + \frac{L_o}{11\cdot35} + \frac{A_o}{\phi},$$

where ϕ is the density of the air expressed in grams per cubic centimetre if A_o is expressed in grams.

In this equation V, G_o, and L_o are known, therefore

$$\frac{A_o}{\phi} = V - \left(\frac{G_o}{2\cdot5} + \frac{L_o}{11\cdot35}\right) = v_a,$$

whence

$$A_o = v_a \phi.$$

In any locality ϕ varies with the weather, but it can always be ascertained by the observation of the meteorological elements. The relative humidity of the air in the special room required for this work seldom differs much from 50 per cent.; therefore the variations in the density of the air are due almost wholly to variations of the barometric pressure. In London, the extreme range of barometric pressure may be taken to be between 730 and 770 millimetres, having therefore an amplitude of 40 millimetres. If we suppose that the barometric pressure was 750 millimetres when the instrument was weighed, and that the air then weighed 1·2 milligram per cubic centimetre, the extreme variations of density to be expected will be $\pm\frac{2}{75} \times 1\cdot2 = 0\cdot032$ milligram per cubic centimetre. In the case of hydrometer No. 1, when loaded for work in distilled water the volume of air in the instrument was 112·5 c.c., which at 1·2 milligram per c.c. would weigh 135 milligrams, and the extreme variations of this weight would be ±4·32 milligrams.

It is evident, therefore, that the actual weight of the air in the hydrometer at the time of making an experiment is an essential factor in computing the weight of liquid which it displaces. There is therefore an advantage in making such observations when the weather is settled; the variations of the barometric pressure in the course of a day are then of such an order as to be almost negligible. If, however, a cyclonic depression is passing over the locality, the change of barometric pressure from hour to hour may have to be taken into account.

The dimensions of this instrument, fig. 4, are:—from lower extremity A to contraction B between the ballast bulb and the body of the instrument, 5 centimetres; the body from B to C, 13 centimetres; the stem, from C to F, 14 centimetres, making the total length over all 32 centimetres. On the stem a length of 10 centimetres, DE, is divided in millimetres, and numbered at each centimetre, 0, 1, 2, . . . 10, from below upwards; the lowest division, 0, is 1 centimetre from the junction of the stem with the body of the instrument at C, and the highest division, 10, is 3 centimetres from the upper extremity of the instrument at F. The external diameter of the body of the instrument is 37 millimetres, and that of the ballast bulb 32 millimetres. The external diameter of the stem is 3·5 millimetres, and the internal diameter 2·5 millimetres. As the ballast used is lead shot, and the load of this shot has to be frequently altered, the internal diameter of the contraction at B as well as that of the stem must be such that shot can be added to or removed from the instrument without trouble. G is the button on the cane of white enamel glass which is suspended in the axis of the stem, and, by affording a white background, enables the scale which is etched on the glass to be seen with facility.

The glass shell of the hydrometer, as it came from the glass-blower, was first weighed approximately, and the weight so found was 40·5 grams. It was then loaded with No. 10 lead shot so that it floated in distilled water of 19·5° C. with the zero division, which is the lowest on the scale etched on the stem, exposed above water. 95·6 grams of shot were required for this purpose.

The hydrometer so loaded was found to weigh exactly 136·1022 grams in air.

The atmospheric conditions were as follows:—

Barometer, 760·3 mm.
Temperature of air, 19·0° C.
Relative humidity, 70 per cent.

Whence the weight of 1 c.c. air = 0·001208 gram.

Taking the specific gravity of glass to be 2·5, that of lead to be 11·35, we have:—

For the volume of 40·5 grams glass	16·2 c.c.
For that of 95·6 grams lead	8·4 ,,
And for the total volume of lead and glass . . .	24·6 ,,
The volume of 136·1 grams brass weights is . . .	17·0 ,,
Whence the balance of volume for correction is . .	7·6 ,,

The weight of 7·6 c.c. air under the above conditions is 0·00912 gram.

Therefore the true weight *in vacuo* of the glass and shot is 136·1022 + 0·00912 = 136·11132 grams. To this has to be added the weight of the air which the hydrometer contains. This is arrived at by floating the instrument in distilled water and by adding suitable weights at the top of the stem, so as to immerse it up to the 50-mm. division on the scale. The weight of distilled water so displaced is equal to the sum of the weights of the glass, the shot, the small external weights added, and that of the air enclosed. As mean of the experiments made at 19·5° C., the added external weight required was 0·75471 gram; so that the total solid weight of the hydrometer was 136·86603 grams, which may be taken as the weight of distilled water at the temperature 19·5° displaced by the hydrometer. Dividing this weight by 0·99834, the density of distilled water at 19·5°, we obtain 137·093 c.c. as the volume of the water, which is equal to the external volume of the instrument which displaces it. If from the volume so found we deduct the volume of the glass and lead, we find the volume of the air contained in the instrument to be 112·493 c.c. Under the atmospheric conditions prevailing at the date of the experiment, 1 c.c. of air weighed nearly 1·2 milligram, whence we obtain 0·13592 gram for the weight of the enclosed air at the time. The weight of enclosed air is not constant. It is subject to slight variations, principally those of the barometric pressure, and these have to be taken into account.

§ 84. There remains now only one item to complete the total effective weight of the floating hydrometer, namely, that of the air displaced by the portion of the stem above water when the instrument is in equilibrium with the water. The total effective weight of the hydrometer is diminished by this amount. When it is immersed up to the 50-mm. division, the portion of the instrument not immersed in water is the part of the stem above the 50-mm. division, having a length of 75 millimetres, namely 50 mm. to the upper end of the scale and 25 mm. to the top of the stem.

The experiments made in distilled water at 19·5° C. with this hydrometer showed that the addition of 0·1 gram to the weight at the top of the stem increased the immersion by 10·69 mm., whence we obtain 0·7 c.c. as the volume of the exposed stem; and the weight of this volume of air is found as above to be 0·00084 gram. This has to be deducted from the sum of the weights of glass, lead, and air. We have then :—

Weight *in vacuo* of glass and lead	136·11132	grams.
,, ,, air enclosed	0·13592	,,
,, ,, added external weight	0·75471	,,
Giving as total weight	137·00195	,,
Deducting weight of air displaced by stem	− 0·00084	,,
We have for the final weight of the loaded hydrometer floating at 50 mm. in distilled water at 19·5° C.	137·00111	,,
Deducting from this the added external weight, 0·75471 gram, we obtain the true weight of the hydrometer alone	136·24640	,,

Section XII.—Numerical Details illustrating the Use of the Open Hydrometer.

§ 85. For this purpose we will consider in detail the items of the experimental determination of the weight of liquid displaced by the instrument which is designated "Hydrometer A"; it has for some time been in constant use.

A scheme for recording the items of observation is given in § 86.

In the first vertical column the line corresponding to each item is designated by a letter—a, b, etc. In the second column are the symbols used for the principal items, and in the body of the scheme the items are described or explained.

At the top of the table the standard temperature, T, selected for the experiment is given, along with the designation of the liquid experimented on.

The descriptions in the scheme explain all the items, but we may refer more particularly to one or two of them.

In lines d and k we have the times of the beginning and end of the experiment. These are important, not only with a view to ascertaining the duration of the experiment, but also as a matter of routine in all laboratory work. It is often of great importance in the discussion of the results of experiments to know if errors which appear to be possible, or indeed probable, could in fact have occurred in the time or in the order in which the experiments were made.

Lines e and j, the initial and final temperatures of the liquid. As above indicated, these should be identical, and the condition $T_i = T_f = T$ should hold. The thermometer chiefly used in these experiments was one graduated on the stem into tenths of a Centigrade degree, the length of the whole degree being 12 millimetres. This is a very suitable type of thermometer for the work.

Lines f (f_1, f_2, etc.). Each of these lines contains two entries in each series, namely, w, the "added weight" in grams, and R, the corresponding division, in millimetres, on the stem, at which the hydrometer, when so loaded, floats in the experimental liquid. The addition of external weights proceeds usually by increments of 0·1 gram, and when distilled water was the liquid, and hydrometer A was being used, each such increment of weight produced an average increment of immersion equal to 10·69 millimetres, so that nine observations could be made in each series. In concentrated solutions as many as eleven single observations could be made in one series. The initial added weight was regulated so that the fifth or middle reading should approximate closely to the 50-mm. division, which was the arbitrarily selected division on the stem for the average immersion of the hydrometer in every series.

By dividing the difference between the first and last of n readings by $n-1$ we obtain the mean immersion produced by 0·1 gram, which is given in the line g, and we are thus able to determine the displacement of each millimetre of the stem.

Lines n and o. Line n contains the correction, dw_r, to be applied to the mean added weight $\bar{w}$ in order to make $\bar{R} = 50$ mm.; and line o gives $\bar{w} + dw_r$, the total added weight when the hydrometer floats at 50 mm., the temperature being $\bar{T}$.

Lines p and q. These lines contain the correction, dw_t, to be applied to the mean added weight $\bar{w}$, to compensate for the difference dt of $\bar{T}$ from the standard temperature T.

Line r. This line contains the sum $\bar{w} + dw_r + dw_t$, which is the total added weight which would immerse the hydrometer to 50 mm. when floating in the liquid having the temperature T exactly.

In order to arrive at the value of the correction for temperature difference in terms of weight, two series of observations are made in the experimental solution at two temperatures one degree higher and lower respectively than the standard temperature selected for the specific gravity determination. The same added weights are used in both cases.

Under these circumstances the scale readings for the same added weight in the first case must be higher than those in the second; a series of differences of scale readings for the same added weight is obtained, which in the case of hydrometer A amounted to 5·0 mm. for each pair of observations; hence, for a difference of 0·1° C. in the mean temperature indicated in line *l*, we arrive at a value of 0·25 mm. in scale reading, which represents the effect of the alteration of the temperature of the liquid by 0·1° C., and, as has been indicated above, this scale reading can be interpreted in terms of added weight dw_t, to be added to or subtracted from the total added weight according as the mean temperature, $\bar{T}$, is higher or lower than the selected standard temperature, T.

We thus obtain the total weight of experimental liquid displaced by the hydrometer when floating in it at the 50-mm. division.

When the experimental liquid is distilled water, the entries in lines *t* and *v* are identical, and the corresponding entry in line *w* is unity. Before proceeding with the determination of the specific gravity of solutions, a number of series of observations are made with the instrument in distilled water at the selected standard temperature, T, by which we arrive at the total weight of the instrument when floating and immersed in this liquid up to 50 mm. on the stem. When we are using the closed hydrometer this number is a constant. When the experiments in distilled water are being made with the open hydrometer, the weight of air actually present in it at each experiment is ascertained and taken into account in the computation of the whole displacing weight of the hydrometer in the given conditions. But whether the hydrometer is open or closed, the mass of air in it only contributes so much to the total weight of the instrument at the moment. It makes no difference whether the air enclosed in it forms a greater or less proportion of it.

§ 86. *Scheme for Logging the Observations made with the Hydrometer in the Experimental Liquid at the Selected Standard Temperature, T.*

Line.	Symbol.	Explanation.	
a		Designation of the hydrometer used.	
b	W	Weight *in vacuo* of the closed hydrometer; or of the glass and shot in the case of the open hydrometer.	
c		Date when the experiment was made.	
d		Time of commencement of experiment.	
e	*t*	Initial temperature of liquid.	
f_0		Added weight *w* in grams :—Scale reading R in millimetres.	
f_1		w_1	R_1
f_2		w_2	R_2
f_3		w_3	R_3
f_4		w_4	R_4
f_5		w_5	R_5
f_6		w_6	R_6
f_7		w_7	R_7
f_8		w_8	R_8
f_9		w_9	R_9

Scheme for Logging Observations—continued.

Line.	Symbol.	Explanation.
f_{10} f_{11}		Added weight w_{10} in grams :—Scale reading R_{10} in millimetres. w_{11} R_{11}
g	$\frac{dr}{dw}$	Mean increment of immersion produced by the addition of 0·1 gram to the external load.
h	$\overline{w}$	Mean added weight.
i	$\overline{R}$	Mean scale reading.
j	t'	Final temperature of liquid.
k		Time when experiment was finished.
l	T	Mean of initial and final temperatures.
m	*dr*	Difference of mean reading $\overline{R}$ from 50 mm. $(50-\overline{R})$.
n	*dw*	Weight which immerses *dr* millimetres of stem in the liquid.
o		$\overline{w}+dw_r$: Added weight which produces immersion up to 50 mm. at T° C.
p	*dt*	$=\overline{T}-T$: Departure of mean temperature from the standard temperature.
q	*dw*	Weight to be added to compensate the displacing value of *dt*.
r	$_r\overline{w}$	$=\overline{w}+dw_r+dw_t$: Total mean added weight required to immerse the closed hydrometer in the liquid up to 50 mm. at the standard temperature, T.
s	$a-s$	Weight of air contained in the open hydrometer, less that of the air displaced by the exposed part of the stem.
t	C	$=W+{}^r\overline{w}^t$:—Total weight of liquid displaced by closed hydrometer when immersed in it up to 50 mm. at standard temperature, T.
u	O	$=W+{}_r\overline{w}_t+a$: Total weight of liquid displaced by open hydrometer when immersed in it up to 50 mm. at standard temperature, T.
v	W_{H_2O}	Total weight of distilled water displaced by hydrometer when immersed in it up to 50 mm. at standard temperature, T.
w	S	$=\frac{W+r\overline{w}_t}{W_{H_2O}}$ for closed hydrometer. $=\frac{W+{}_r\overline{w}_t+a}{W_{H_2O}}$ for open hydrometer. Specific gravity of the liquid at the standard temperature, T, referred to that of distilled water at the same temperature as unity.

Numerical Examples in the Case of
(*a*) *Distilled Water at* 19·5° *C.*
(*b*) 7·0 *gram-molecule Solution of* ***Rubidium Chloride.***

	(*a*)	(*b*)		(*a*)	(*b*)
a	Hydrometer A.	Hydrometer A.	*g*	10·69 mm.	7·33 mm.
b	136·11132 grams.	184·67847 grams.	*h*	0·77 gram.	14·75 grams.
c	February 10th, 1910.	July 3rd, 1911.	*i*	51·39 mm.	51·65 mm.
d	10.15 a.m.	11.15 a.m.	*j*	19·50° C.	19·50° C.
e	19·50° C.	19·50° C.	*k*	10.26 a.m.	11.32 a.m.
f_1	0·37 grams. 8·5 mm.	14·15 grams. 7·5 mm.	*l*	19·50° C.	19·50° C.
f_2	0·47 ,, 19·0 ,,	·25 ,, 14·2 ,,	*m*	1·39 mm.	1·65 mm.
f_3	0·57 ,, 30·0 ,,	·35 ,, 21 2 ,,	*n*	0·0130 gram.	0·02251 gram.
f_4	0·67 ,, 40·5 ,	·45 ,, 29·0 ,,	*o*	0·7570 ,,	14·72749 grams.
f_5	0·77 ,, 51·5 ,,	·55 ,, 36·7 ,,	*p*	none	none
f_6	0·87 ,, 62·5 ,,	·65 ,, 44·1 ,,	*q*	none	none
f_7	0·97 ,, 73·0 ,,	·75 ,, 52·2 ,,	*r*	0·7570 gram.	14·72749 grams.
f_8	1·07 ,, 83·5 ,,	·85 ,, 59·3 ,,	*s*	0·13508 ,,	0·13028 ,,
f_9	1·17 ,, 94·0 ,,	·95 ,, 66·8 ,,	*t*	...	...
f_{10}	...	15·05 ,, 74·4 ,,	*u*	...	199·53624 grams.
f_{11}	...	·15 ,, 81·5 ,,	*v*	137·00340 grams.	137·00340 ,,
f_{12}	..	·25 ,, 89·1 ,,	*w*	...	1·456433
f_{13}	...	·35 ,, 95·5 ,,			

The weight 137·0034 grams entered in line *v* includes that of the air contained in the hydrometer. Its value is obtained in the following manner :—

As the result of many determinations, of which the example in § 84 is an instance, the *mean* added weight necessary to immerse the hydrometer to 50 mm. at 19·50° C. was found to be 0·75471 gram.

Hence weight of hydrometer and added weight = 136·86603 grams.

Density of distilled water at 19·50° C. = 0·99834.

Therefore volume of distilled water displaced $= \frac{136·86603}{0·99834} = 137·093$ c.c.

By subtracting from this the volume of glass and shot (= 24·6 c.c. ; see § 83), the resultant volume, 112·493 c.c., is that of the enclosed air.

(The internal volume of the stem above the 50-mm. division is here disregarded.)

The weight of this volume of air is obtained as follows :—

Weight of 1 c.c. air under the atmospheric conditions during the experiments = 0·001208 gram.

Weight of 112·493 c.c. air = 0·13592 gram.

These numbers give the amount of the air contained in the hydrometer when it carries an internal load of 95·6 grams of lead shot. If this load is altered, the residual volume of air experiences a corresponding alteration.

§ 87. *Correction for the non-immersed Portion of Stem.*—When the hydrometer is floating at 50 mm. in distilled water, there is a length of stem of 75 mm. in air—namely, 50 mm. to the end of the scale, and 25 mm. to the open end of the stem.

By line *g* of the table in § 86, we see that 10·69 mm. of scale are immersed by 0·1 gram, and 75 mm. are immersed by 0·7 gram, whence the volume of the non-immersed portion of the stem may be taken as 0·7 c.c.

By the Archimedean principle the non-immersed portion of the stem displacing this volume of air loses weight equal to that of the air so displaced ; so that, were the air removed from the surface of the liquid, the hydrometer would sink into the liquid and the scale reading would be higher. The value of this difference of scale reading is the weight of air displaced by the non-immersed portion of the stem.

The weight of 0·7 c.c. of air, under the atmospheric conditions quoted above, is 0·00084 gram.

We have then, for the total weight which immerses the hydrometer to the 50-mm. division :—

Weight of loaded hydrometer *in vacuo*	= 136·11132	grams.
Weight of enclosed volume of air	= 0·13592	,,
Added weight to immerse stem to 50 mm.	= 0·75471	,,
	137·00195	,,
Correction for exposed portion of stem	= − 0·00084	,,
Sum	= 137·00111	,,

This number represents the weight of distilled water displaced by the hydrometer up to the 50-mm. division when immersed in it at 19·5° C.

The determination of the weight of any experimental solution displaced by the hydrometer up to the 50-mm. division is determined in a precisely similar manner. If any adjustment of the internal load of the hydrometer is made, its volume must be taken into account in estimating the volume of enclosed air.

§ 88. The degree of accuracy attainable by the use of the hydrometer is best illustrated by quoting the results of five series of observations, each series consisting of eleven independent observations made in a solution of calcium chloride containing 6·3 gram-molecules of $CaCl_2$ in 1000 grams of water. The table includes, by the method of least squares, the estimation of the probable error r of a single observation, and r_0 that of the arithmetical mean of each series.

It will be seen from the example given in § 86 that we obtain a series of differences between the consecutive readings corresponding to added weights of 0·1 gram, and, to take the first case, 0·1 gram immerses 10·50 mm. of stem, an added weight of 0·0095 would be required to immerse 1·0 mm. of stem.

When a series of readings has been made in the experimental solution with the hydrometer whose constants are known, the weight of solution displaced to a given scale division, which is one of the actual readings, is known, and the total weight of the hydrometer when floating at the same division in distilled water is obtained. With these data the calculation of the specific gravity of the solution is made.

An example will illustrate this method :—

Taking the first reading in series No. 1, using hydrometer A :

1*st Reading*—

4·0 grams added weight immersed 13·9 mm.

Corrected weight of hydrometer	=	188·53748	grams.
Volume of enclosed air = 108·56 c.c.			
Weight of enclosed volume of air	=	0·12830	,,
Added weight	=	4·00000	,,
		192·66578	,,
Correction for exposed stem (weight of 0·9 c.c. air)	=	− 0·00106	,,
Weight of solution displaced to 13·9 mm.	=	192·66472	,,

Weight of distilled water displaced to 13·9 mm.

Corrected weight of hydrometer	=	136·11132	grams.
Weight of enclosed volume of air	=	0·13592	,,
		136·24724	,,
Added weight to immerse 13·0 mm.	=	0·41042	,,
·9 mm.	=	0·00837	,,
		136·66603	,,
Stem correction (weight of 0·9 c.c. air)	=	− 0·00110	,,
Weight of distilled water displaced to 13·9 mm.	=	136·66493	,,

$$\text{Specific gravity} = \frac{192·66472}{136·66493} = 1·409760.$$

The table in § 90 includes three series obtained with hydrometer A, and two obtained with hydrometer B. The values of the mean specific gravity ($\bar{S}$) furnished by each series, and its probable error ($\pm r_0$), expressed in units of the sixth decimal place, are collected in the following table:—

Hydrometer.	$\bar{S}$.	$\pm r_0$.
A	1·409752	3·1
	1·409746	3·5
	1·409746	3·6
B	1·409727	4·5
	1·409753	6·4

It will be seen that the uncertainty of the means of each series lies entirely in the sixth decimal place. The mean of the five means tabulated is 1·409744, and its probable error is ±3·16 in the sixth decimal place.

§ 89. In order to reap the full benefit of the precision of which the hydrometric method is capable, the operations must be carried out with attention to every precaution, and the experimental data must be recorded according to strict method.

Scrupulous cleanliness is of the first importance, and the operations must be carried out with attention to all the precautions usually observed in laboratories from which exact work is expected to proceed.

It is important that the room in which the observations are made should have a north light and be entirely under the control of the experimenter, who is its only occupant. This is essential, because the management of the temperature of the room, which must be that which the experimenter has found by his own experience to be the one which maintains the experimental liquid constantly at the selected standard temperature while the observations are being made, is the most important element of success and the most difficult of achievement. The conditions are similar to those which have to be observed in the room in which gas analysis is made by Bunsen's original method, only they are rather more stringent. For myself, when I begin hydrometric observations I always lock the door, a practice which I adopted on board the *Challenger* and have adhered to ever since.

The conclusion arrived at from the discussion on temperature conditions which is given in Section IV. on the closed hydrometer applies with equal force in the use of the open hydrometer. An interesting difference occurs in experiments on strong solutions, since they have a lower specific heat, which may fall as low as 0·5, as in the case of most concentrated solutions of $CaCl_2$, so that the thermal mobility of these solutions is greater, and this condition may be met by allowing a somewhat increased margin of difference between air and solution temperature when the compensating luminous flame is used.

§ 90. *Table of Specific Gravities calculated from Single Observations made with Hydrometers A and B when floating in a Solution of Calcium Chloride containing* 6·3 *gram-molecules in* 1000 *grams of Water.*

Added Weight in Grams for Hydrometer A.	Hydrometer A. Series 1. Scale Reading in Millimetres.	Hydrometer A. Series 1. Specific Gravity calculated from Single Observations.	Hydrometer A. Series 2. Scale Reading in Millimetres.	Hydrometer A. Series 2. Specific Gravity calculated from Single Observations.	Hydrometer A. Series 3. Scale Reading in Millimetres.	Hydrometer A. Series 3. Specific Gravity calculated from Single Observations.	Added Weight in Grams for Hydrometer B.	Hydrometer B. Series 1. Scale Reading in Millimetres.	Hydrometer B. Series 1. Specific Gravity calculated from Single Observations.	Hydrometer B. Series 2. Scale Reading in Millimetres.	Hydrometer B. Series 2. Specific Gravity calculated from Single Observations
4·0	13·9	1·409760	14·1	1·409741	14·0	1·409750	2·05	7·1	1·409740	7·0	1·409750
4·1	21·8	735	21·7	745	21·5	764	2·15	15·9	732	15·8	742
4·2	29·6	731	29·7	722	29·5	741	2·25	24·5	744	24·7	724
4·3	37·2	730	37·2	730	37·4	711	2·35	33·5	725	33·8	696
4·4	44·6	744	44·9	722	44·8	731	2·45	42·8	683	42·0	760
4·5	52·2	749	52·1	765	52·1	765	2·55	50·8	751	50·9	742
4·6	59·8	762	59·8	768	60·1	741	2·65	60·0	723	59·0	818
4·7	67·1	776	67·2	772	67·5	744	2·75	67·9	725	68·1	790
4·8	74·7	764	74·9	750	74·8	760	2·85	77·8	694	76·9	782
4·9	82·2	762	82·4	748	82·6	730	2·95	86·0	740	86·0	740
5·0	89·7	761	89·9	747	89·7	767	3·05	94·8	739	94·8	739
Mean Specific Gravity.	S	1·409752		1·409746		1·409746			1·409727		1·409753
Probable error, expressed in units of the 6th decimal place	of a single observation $\pm r$.	10·3		11·6		11·9			14·8		20·1
	of the arithmetical mean $\pm r_0$.	3·1		3·5		3·6			4·5		6·4

The hydrometer A, constructed on the above specification, has proved itself, in use, to be an excellent model. Its volume, about 137 c.c., is very suitable, being sufficiently great to secure precision without rendering it necessary to use extravagant quantities of very soluble salts, which, in the case of costly preparations, might be prohibitive. It is very steady, and this is principally due to the fact that the ballast is all contained in the bulb at the lower extremity. The position of the centre of gravity of the instrument is thus kept very low, and in the spherical bulb the ballast cannot shift.

Section XIII.—On the Specific Gravity and Displacement of Solutions of Salts of the Ennead MR which have nearly the same Molecular Weight and may be looked on as "Isomeric."

§ 91. There are three such groups of salts in the ennead, namely, KBr and RbCl; KI, RbBr, and CsCl; and RbI and CsBr. Experiments have been made on strong solutions of the first group, KBr and RbCl. To these "natural isomers" we have added an (artificial) isomer consisting of a mixture which contains KCl and KI in equal molecular proportions, so that it may be represented by the formula $K\frac{Cl+I}{2}$. This mixture contains 31·00 per cent. of KCl and 69·00 per cent. KI.

It was found that at 19·5° C., the temperature used in these experiments, 702·25 grams of this mixture saturated 1000 grams of water. No more could be dissolved without leaving a residue. This amount was made up of 217·65 grams of KCl and 484·60 grams KI, representing 2·917 gram-molecules of each salt. From the tables of solubility of these well-known salts we find that 217·65 grams of KCl saturate 632·26 grams of water at 19·5° C.; and if we imagine that this quantity of water is wholly taken possession of by 217·65 grams of KCl, there remains 367·74 grams of water to accommodate the 484·60 grams KI. But, at 19·5° C., 367·74 grams of water require 530·28 grams KI to produce saturation. Therefore, though saturated with the mixture, the 1000 grams of water is not saturated with both the individual salts.

§ 92. *Table giving Results of Specific Gravity Determinations made upon Solutions of Rubidium Chloride, Potassium Salt of mixed Halides, and Potassium Bromide, of different Concentrations.*

m. 1.	W. 2.	S. 3.	$\frac{dS}{dm}$. 4.	log Δ. 5.	$\frac{d \log \Delta}{dm}$. 6.	Δ. 7.	$\frac{d\Delta}{dm}$. 8.	$\frac{v}{m}$. 9.	$\frac{\log \Delta_m - 3}{\log \Delta_1 - 3} = x$. 10.
				RbCl = 121·0. T = 19·50° C.					
7	1847·000	1·456464		3·1031672		1268·140		38·306	7·035
6	1726·000	1·406075	0·050389	3·0890324	0·0141348	1227·531	40·609	37·922	6·072
5	1605·000	1·351760	0·054315	3·0745755	0·0144569	1187·341	40·190	37·468	5·086
4	1484·000	1·292983	0·058777	3·0598410	0·0147345	1147·733	39·608	36·933	4·081
3	1363·000	1·229284	0·063699	3·0448789	0·0149621	1108·865	38 868	36·288	3·060
2	1242·000	1·159851	0·069433	3·0297194	0·0151595	1070·827	38·038	35·413	2·027
1	1121·000	1·083782	0·076069	3·0146637	0·0150557	1034·341	36·486	34·341	1·000
				$K\frac{Cl+I}{2}$ = 120·35. T = 19·50° C.					
5	1601·75	1·336904		3·0784945		1198·104		39·620	4·950
4	1481·40	1·280510	0·056394	3·0632893	0·0152052	1156·880	41·224	39·220	3·991
3	1361·05	1·219453	0·061057	3·0477089	0·0155804	1116·115	40·765	38·705	3·009
2	1240·70	1·152989	0·066464	3·0318418	0·0158671	1076·073	40·042	38·036	2·008
1	1120·35	1·080182	0·072807	3·0158567	0·0159851	1037·186	38·887	37·186	1·000
				KBr = 119·1. T = 19·50° C.					
5	1595·500	1·343255		3·0747383		1187·786		37·557	4·996
4	1476·400	1·285584	0·057671	3·0601036	0·0146347	1148·428	39·358	37·107	4·018
3	1357·300	1·223113	0·062471	3·0452092	0·0148944	1109·709	38·719	36·569	3·022
2	1238·200	1·155257	0·067856	3·0301122	0·0150970	1071·796	37·913	35·898	2·013
1	1119·100	1·081211	0·074046	3·0149585	0·0151537	1035·043	36·753	35·043	1·000

§ 93. *Solubility.*—The molecular solubility of each of these salts, that is, its solubility expressed in gram-molecules salt per thousand grams of water at 19·5° C., is :—

Salt.	RbCl.	$K\frac{Cl+I}{2}$.	KBr.
Molecular weight	121	120·35	119·1
Gram-molecules in 1000 grams water .	7·77	5·83	5·7

The discussion will, however, be confined to the relation of solutions of these salts which contain, per thousand grams of water, 5 or a smaller number of gram-molecules of salt.

Considering the change of specific gravity over a range of concentration varying from 5 to 1 gram-molecules per thousand grams of water, that of rubidium chloride varies from 1·351760 to 1·083782, that of the potassium salt of the mixed halides from 1·336904 to 1·080182, and that of potassium bromide from 1·343255 to 1·081211. Although potassium bromide has the lowest molecular weight, there is a closer agreement in the specific gravity of the solutions of this salt with those of rubidium chloride than with those of the potassium salt of the mixed halides. At the same time the nature of the change of values with change in concentration of solution, as indicated by the numbers representing the differences of consecutive specific gravities given in the column dS/dm of the tables, shows that in this respect the potassium salts exhibit a closer relationship among themselves than either of them does with rubidium chloride. Also the nature of the decrease in this value with increasing concentration seems to indicate the fact that the increase in specific gravity becomes more nearly proportional to the increase in concentration in the strongest solutions.

It will be observed that the actual weight of each salt per thousand grams of water in each solution is slightly different, and if the specific nature of the salts were the same in each of these solutions, and the masses of them present in the solution were equal, as their molecular weights would in that case be, the specific gravities of these solutions and the constants derived from them would be different from those in table § 92.

§ 94. For comparison in this sense the specific gravities have been adjusted to the value which they would have if their gram-molecules had the uniform weight 121, which is the actual molecular weight of the heaviest of the three, namely, rubidium chloride.

The following table gives the specific gravities adjusted in this sense :—

Multiples of 121 grams of Salt per 1000 grams Water . .	5.	4.	3.	2.	1.
Observed specific gravities for solutions of RbCl .	1·351760	1·292983	1·229284	1·159851	1·083782
Calculated ,, ,, ,, $K\frac{Cl+I}{2}$	1·338724	1·282025	1·220131	1·153815	1·080575
,, ,, ,, ,, KBr .	1·348731	1·290140	1·226672	1·157734	1·082507

It will be observed that the adjustment of the molecular weight does not materially affect the relations of the solutions as regards their specific gravities.

Returning to the consideration of the data obtained from the original experiments, the comparison of the displacements for the same concentrations in the case of solutions of each of the three salts shows a close agreement to exist between the values for rubidium chloride (which are lower in each case) and potassium bromide, while the values for the potassium salts of the mixed halides stand quite apart and are much higher than the corresponding values for the other two salts.

If we compare the differences of displacements of equivalent solutions of RbCl and KBr for $m = 5$, then $\Delta_{RbCl} - \Delta_{KBr} = 0{\cdot}445$, and for $m = 1$ it is $0{\cdot}702$. The differences of displacements between corresponding solutions of $K\frac{Cl+I}{2}$ and KBr are, for $m = 5$, $\Delta_{K\frac{Cl+I}{2}} - \Delta_{KBr} = 10{\cdot}318$, and for $m = 1$, $\Delta_{K\frac{Cl+I}{2}} - \Delta_{KBr} = 2{\cdot}143$.

The molecular displacement of each of these salts in crystal, as given in § 127, is

$$RbCl = 44{\cdot}710, \quad K\frac{Cl+I}{2} = 46{\cdot}406, \quad KBr = 44{\cdot}460,$$

and if the sum of the displacements of the constituent materials forming the solution, *i.e.* 1 gram-molecule and 1000 grams of water, are compared with the displacement of the solution obtained from the constituents, the following results are arrived at :—

	RbCl.	$K\frac{Cl+I}{2}$.	KBr.
Sum of displacement of constituents	1044·710	1046·406	1044·460
Displacement of solution . .	1034·341	1037·186	1035·043
Difference . .	10·369	9·220	9·417

Here, with regard to the change in displacement when solution is effected, the potassium salts are quite comparable, while the rubidium chloride shows a much greater change, although in the case of values for the sum of the displacements of the constituents, rubidium chloride and potassium bromide are the more comparable.

§ 95. *Difference of Displacement*, $d\Delta$.—$d\Delta$ gives the increment of displacement of a mass of 1000 grams of water produced by successive additions of 1 gram-molecule of salt to that already in solution.

The values of v/m represent the mean increment of displacement of 1000 grams of water per gram-molecule of salt when m gram-molecules have been dissolved in it.

The values of $d\Delta$ from 1 to 4 gram-molecules for each salt show that, with the exception of that for the 1 gram-molecule, they are lowest in the case of potassium bromide, and the values for corresponding concentrations of rubidium chloride very closely approximate to them, while those for the potassium salt of the mixed halides show a considerable divergence.

Thus the value of $d\Delta$ for a 4 gram-molecule solution of potassium bromide, which is 39·358, diminishes to 36·753 for the 1 gram-molecule solution, while that for rubidium chloride diminishes from 39·608 to 36·486 for the same range of concentration, and for the potassium salt of the mixed halides the two values are 41·224 and 38·887.

If we express these pairs of values as ratios, the following values are obtained :—

	RbCl.	$K\frac{Cl+I}{2}$.	KBr.
Actual values =	39·358 : 36·753	41·224 : 38·887	39·608 : 36·486
Ratio =	1 : 0·9212	1 : 0·9433	1 : 0·9338

While, therefore, the values of $d\Delta$ closely approximate in the case of rubidium chloride and potassium bromide, yet the ratios given above show that the rate of decrease in the value of $d\Delta$ for potassium bromide lies between those for the other salts, but closer to that for the potassium salt of the mixed halides.

An inspection of the values of v/m shows that all the values for RbCl are lower than the corresponding values for the other two salts, although the values for potassium bromide approach very close to them, while those for the potassium salt of the mixed halides are much higher.

The values of v/m for the 5 and 1 gram-molecule concentrations for each of the salts are 37·468 and 34·341 for rubidium chloride, 37·557 and 35·043 for potassium bromide, and 39·620 and 37·186 for the potassium salt of the mixed halides; and expressed as ratios, as in the case of the values for $d\Delta$, we have for

	RbCl.	$K\frac{Cl+I}{2}$.	KBr.
Actual values =	37·468 : 34·341	37·557 : 35·043	39·620 : 37·186
Ratio =	1 : 0·9165	1 : 0·9386	1 : 0·9331

Here, the similarity of the ratios for the potassium salt of the mixed halides and potassium bromide shows a similar rate of decrease of this value v/m for these two salts, while that for rubidium chloride shows a considerable departure from either of them.

The agreements which exist, when the values $\frac{d\Delta}{dm}$ and $\frac{v}{m}$ for the three salts are compared, seem to indicate that the molecules of rubidium chloride and potassium bromide exert almost equal effects in the displacement of solution, but the nature of change of displacement with change of concentration shows that the potassium salts are more allied in this respect.

§ 96. *The Displacement of Solutions of the Potassium Salt of the Mixed Halides when considered in reference to the Displacement of Solutions of the Constituent Salts.*—The following table gives full data relating to the displacement, difference of displacement, and mean increment of displacement of solutions of different concentrations of potassium chloride, potassium salts of the mixed halides, and potassium iodide.

The experiments were made with the open hydrometers A and B (see § 82) at the constant temperature 19·50° C.

Concentration in gram-molecules per 1000 grams of Water.	KCl.			$K\frac{Cl+I}{2}$.			KI.		
	Displacement. Δ.	Difference of Displacement. $\frac{d\Delta}{dm}$.	Mean Increment of Displacement. $\frac{v}{m}$.	Displacement. Δ.	Difference of Displacement. $\frac{d\Delta}{dm}$.	Mean Increment of Displacement. $\frac{v}{m}$.	Displacement. Δ.	Difference of Displacement. $\frac{d\Delta}{dm}$.	Mean Increment of Displacement. $\frac{v}{m}$.
5				1198·104		39·620	1240·334		48·067
4	1123·005		30·751	1156·880	41·224	39·220	1190·788	49·546	47·697
3	1090·533	32·472	30·178	1116·115	40·765	38·705	1141·787	49·001	47·262
2	1059·617	30·916	29·808	1076·073	40·042	38·036	1093·384	48·403	46·692
1	1028·904	30·713	28·904	1037·186	38·887	37·186	1046·189	47·195	46·189
1/2	1014·001	29·806	28·002	1018·343	36·308	36·686	1022·778	46·822	45·556

The composite salt was prepared by mixing the component salts KCl and KI in the proportion of their molecular weights, 74·6 : 166·1, so that 1 gram-molecule of the salt, which weighs 120·35 grams, would contain $\frac{1}{2}$ gram-molecule of each of the salts, namely 37·3 grams of KCl and 83·05 grams of KI. When 1 gram-molecule of this salt is dissolved in 1000 grams of water, the displacement of the resultant solution may be compared with the mean of the displacements of the solutions KCl + 1000 grams of water and KI + 1000 grams of water.

The following table gives the results of such a comparison :—

§ 97. *Table of Values of Displacements of Solutions of Potassium Salts of the Mixed Halides which have been obtained—*

A, by experiment.

B, calculated by the method detailed above.

B − A, the difference of calculated and observed results.

Concentration in gram-molecules per 1000 grams Water. *m*.	A.	B.	B − A.
5	1198·104		
4	1156·880	1156·896	0·016
3	1116·115	1116·160	0·045
2	1076·073	1076·500	0·427
1	1037·186	1037·546	0·360
1/2	1018·343	1018·389	0·046
1/4	1009·071	1009·090	0·019
1/8	1004·474	1004·495	0·021
1/16	1002·213	1002·228	0·015
1/32	1001·122	1001·118	− 0·004
1/64	1000·535	1000·559	0·024
1/128	1000·262	1000·282	0·020
1/256	1000·117	1000·133	0·016
1/512	1000·044	1000·076	0·032

This table shows that, with the possible exception of the 1/32 gram-molecule solution, the mixture of *m*KCl + 1000 grams of water and *m*KI + 1000 grams of water is accompanied by *contraction*.

Section XIV.—The Specific Gravity and the Displacement of Solutions of the Chlorides of Beryllium, Magnesium, and Calcium.

§ 98. For the purpose of determining these constants, hydrometric observations were made on solutions of the three salts the concentrations of which ranged from 1/2 to 1/1024 gram-molecule per thousand grams of water, the experiments being made with the closed hydrometers Nos. 3 and 17. Experiments were also made on strong solutions of calcium chloride and magnesium chloride, using the open hydrometers A and B; the concentrations of these solutions varied from 1 gram-molecule per thousand grams of water to the highest attainable degree of supersaturation. It was when the experiments on a supersaturated solution of calcium chloride were in progress that the observations were made which revealed the remarkable state of unrest in that solution which preceded its partition into crystals and mother-liquor with liberation of heat. The details of this experiment are given in Section XV., and from them it will be seen that the range of supersaturation which can be explored hydrometrically when the salt in solution is chloride of calcium is considerable. The solution of magnesium chloride which is saturated at 19·5° C. contains 5·918 gram-molecules (564·123 grams) of $MgCl_2$ in 1000 grams of water. A supersaturated solution, containing 5·982 gram-molecules of salt per thousand grams of water was cooled to 16·5° C., at which temperature the saturated solution contains 5·853 $MgCl_2$ per thousand grams of water; yet, with this small degree of supersaturation, the slightest disturbance, such as lifting the beaker, induced crystallisation in the solution. This shows that the limits of supersaturation are restricted, and that it would certainly be discharged by an attempt to make hydrometric observations in the solution.

This difference in the behaviour of the supersaturated solutions of these two salts is interesting. On the one hand we have the calcium chloride, which produces a high degree of supersaturation with great absorption of heat, and offers great resistance to crystallisation; while magnesium chloride can produce solutions attaining only to a moderate degree of supersaturation with very moderate absorption of heat, and the salt crystallises from such solutions on the slightest provocation. With a view to a comparison with the thermal behaviour of chloride of calcium, some observations were made on the heat of solution of magnesium chloride in water, while experiments were being made to determine the concentration of solutions saturated with the salt at different temperatures. It was found that when the quantities of the crystallised salt $MgCl_2,6H_2O$ and water used were such as to produce a solution the concentration of which was about 2·0 gram-molecules of $MgCl_2$ per thousand grams of water, the dissolution of the salt was accompanied by an appreciable liberation of heat. When the conditions of the experiment were such that a saturated solution was formed, some of the crystals remaining undissolved, the dissolution of the salt was accompanied by absorption of heat. When the saturated solution was produced by fractions, it was observed that during the dissolution of the first fraction the thermometer indicated a

§ 99. *Table giving Specific Gravity Values obtained from Experiments made upon Solutions of the Chlorides of Beryllium, Magnesium, and Calcium.*

m.	W.	S.	log Δ.	$\frac{d \log \Delta}{dm}$.	Δ.	$d\Delta$.	$\frac{v}{m}$.	$\frac{\log \Delta_m - 3}{\log \Delta_1 - 3} = x$.	$x - m$.
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
BERYLLIUM CHLORIDE = $BeCl_2$ = 80·00. T = 19·50° C.									
								$\frac{1}{x}$	$\frac{1}{m} - \frac{1}{x}$
1/2	1040·0000	1·025620	3·0060468		1014·0208		28·04	2·000	0·000
1/4	1020·0000	1·013055	3·0029671	0·0123188	1006·8555	7·1653	27·42	4·076	− 0·076
1/8	1010·0000	1·006599	3·0014648	0·0120181	1003·3785	3·4770	27·03	8·256	− 0·256
1/16	1005·0000	1·003347	3·0007149	0·0119995	1001·6745	1·7310	26·37	16·916	− 0·916
1/32	1002·5000	1·001587	3·0003957	0·0102144	1000·9115	0·7360	29·17	30·562	+ 1·438
1/64	1001·2500	1·000742	3·0002204	0·0112192	1000·5076	0·4039	32·49	54·870	+ 9·130
1/128	1000·6250	1·000325	3·0001302	0·0115430	1000·2999	0·2077	38·39	92·872	+ 35·128
1/256	1000·3125	1·000083	3·0000996	0·0078259	1000·2295	0·0704	58·75	121·362	+ 134·638
1/512	1000·1562	0·999957	3·0000865	0·0067225	1000·1992	0·0303	101·99	139·778	+ 372·222
1/1024	1000·0781	0·999906	3·0000747	0·0120832	1000·1720	0·0272	176·13	166·854	+ 862·146
MAGNESIUM CHLORIDE = $MgCl_2$ = 95·32. T = 19·50° C.									
								x	$x - m$
5·9820	1570·2050	1·338895	3·0692097		1172·7345		28·87	7·538	1·556
5·9182	1564·1230	1·336101	3·0684316	0·0121959	1170·6623	2·0722	28·84	7·453	1·535
5·5	1524·2600	1·317763	3·0632219	0·0124574	1156·7032	13·9591	28·49	6·886	1·386
5·0	1476·6000	1·295011	3·0569894	0·0124650	1140·2220	16·4812	28·04	6·207	1·207
4·0	1381·2800	1·246666	3·0445317	0·0124577	1107·9793	32·2427	26·99	4·850	0·850
3·0	1285·9600	1·193986	3·0322282	0·0123035	1077·0310	30·9483	25·68	3·510	0·510
2·0	1190·6400	1·136425	3·0202398	0·0119884	1047·7070	29·3240	23·85	2·204	0·204
1·0	1095·3200	1·072407	3·0091814	0·0110584	1021·3660	26 3410	21·37	1·000	0·000
								$\frac{1}{x}$	$\frac{1}{m} - \frac{1}{x}$
1/2	1047·6600	1·037385	3·0042803	0·0098020	1009·9047	11·4613	19·81	2·145	− 0·145
1/4	1023·8300	1·019096	3·0020127	0·0090705	1004·6453	5·2594	18·58	4·562	− 0·562
1/8	1011·9150	1·009675	3·0009624	0·0084024	1002·2185	2·4268	17·75	11·982	− 3·982
1/16	1005·9575	1·004893	3·0004598	0·0080422	1001·0593	1·1592	16·95	19·968	− 3·968
1/32	1002·9787	1·002461	3·0002242	0·0075385	1000·5164	0·5429	16·52	40·948	− 8·948
1/64	1001·4893	1·001244	3·0001063	0·0075411	1000·2450	0·2714	15·68	86·299	− 22·299
1/128	1000·7446	1·000639	3·0000458	0·0077516	1000·1055	0·1395	13·50	200·336	− 72·336
1/256	1000·3723	1·000299	3·0000318	0·0035865	1000·0732	0·0323	18·74	288·542	− 36·542
1/512	1000·1861	1·000172	3·0000065	0·0129587	1000·0150	0·0582	7·68	1410·353	− 898·353
1/1024	1000·0930	1·000082	3·0000047	0·0017715	1000·0110	0·0040	11·26	1920·795	− 896·795
CALCIUM CHLORIDE = $CaCl_2$ = 111·00. T = 19·50° C.									
								x	$x - m$
6·627	1735·620	1·424183	3·0858888		1218·6775		33·00	8·279	1·652
6·613	1734·043	1·423500	3·0857023	0·0133214	1218·1540	0·5233	32·99	8·261	1·648
6·6	1732·600	1·422871	3·0855327	0·0130461	1217·6787	0·4753	32·98	8·245	1·645
6·5	1721·500	1·418572	3·0840556	0·0147710	1213·5441	4·1346	32·85	8·102	1·602
6·4	1710·400	1·414247	3·0825725	0·0148310	1209·4069	4·1372	32·72	7·959	1 559
6·3	1699·300	1·409741	3·0811308	0·0144170	1205·3988	4·0081	32·60	7·820	1·520
6·2	1688 200	1·405270	3·0796641	0·0146670	1201·3349	4·0639	32·47	7·697	1·497
6·1	1677·100	1·400460	3·0782883	0·0137580	1197·5352	3·7997	32·38	7·546	1·446
6·0	1666·000	1·395919	3·0768148	0·0147350	1193·4791	4·0561	32·24	7·421	1·421
5·0	1555·000	1·342951	3·0636673	0·0131475	1157·8899	35·5892	31·58	6·165	1·165
4·0	1444·000	1·284536	3·0508210	0·0128463	1124·1413	33·7486	31·03	4·899	0·899
3·0	1333·000	1·227685	3·0357431	0·0150779	1085·7832	38·3581	28·59	3·445	0·445
2·0	1222·000	1·160139	3·0225612	0·0131819	1053·3221	32·4611	26·66	2·175	0·175
1·0	1111·000	1·084776	3·0103741	0·0121871	1024·1745	29·1476	24·17	1·000	0·000
								$\frac{1}{x}$	$\frac{1}{m} - \frac{1}{x}$
1/2	1055·5000	1·043739	3·0048663	0·0110155	1011·2681	12·9064	22·54	2·132	− 0·132
1/4	1027·7500	1·022253	3·0023290	0·0101489	1005·3773	5·8908	21·50	4·454	− 0·454
1/8	1013·8750	1·011231	3·0011340	0·0095604	1002·6146	2·7627	20·92	9·148	− 1·148
1/16	1006·9375	1·005660	3·0005513	0·0093233	1001·2703	1·3443	20·32	18·817	− 2·817
1/32	1003·4687	1·002763	3·0003055	0·0078656	1000·7037	0·5666	22·52	33·954	− 1·954
1/64	1001·7343	1·001423	3·0001349	0·0109145	1000·3109	0·3928	19·90	76·851	− 12·851
1/128	1000·8672	1·000729	3·0000599	0·0096038	1000·1380	0·1729	17·66	173·017	− 45·017
1/256	1000·4336	1·000378	3·0000241	0·0091699	1000·0556	0·0824	14·23	429·747	− 163·747
1/512	1000·2168	1·000179	3·0000163	0·0039680	1000·0378	0·0178	19·35	632·953	− 120·953
1/1024	1000·1084	1·000093	3·0000066	0·0099430	1000·0154	0·0224	15·77	1553·009	− 529·009

rise of temperature. On adding further fractions of salt this ceased, the thermal effect was reversed, and when saturation had been effected the temperature of the solution had fallen below the initial temperature of the water used.

No experiments have been made on solutions of beryllium chloride of greater concentration than 1/2 gram-molecule per thousand grams of water.

The preceding table contains all the experimental results and the deductions therefrom. The form and the symbols used have been already explained.

§ 100. Before discussing the data of the tables, attention must be directed to the distinctive characters of the three salts. While the bases BeO, MgO, and CaO give an alkaline reaction with litmus paper, the chlorides of magnesium and of calcium are neutral, while that of beryllium is acid. In order to obtain, if possible, a neutral solution of $BeCl_2$, the method adopted was to proceed by way of the sulphate and double decomposition with chloride of barium. A solution of pure crystallised sulphate of beryllium, containing exactly 1 gram-molecule of $BeSO_4$ in 1000 grams of water, was made, and with it was mixed a quantity of a solution of barium chloride containing exactly 1 gram-molecule of $BaCl_2$ in 1000 grams of water. The barium sulphate was precipitated completely, and the supernatant liquid contained exactly 1 gram-molecule of $BeCl_2$ in 2000 grams of water, or, at the rate of 1/2 $BeCl_2$ in 1000 grams of water. This solution, which still had an acid reaction, was used for the preparation of the less concentrated ones by exact dilution. It is impossible to produce solutions in this way for which $m>1/2$, on account of the bulk of the barium sulphate produced. Solutions of the highest attainable degree of concentration were prepared in the case of magnesium chloride and calcium chloride, the concentration being determined by the usual chemical methods, and the solutions of a lesser degree of concentration were prepared from these.

In all cases the solutions were prepared by diluting the more concentrated solution immediately preceding it, a method capable of a high degree of precision, which is shown by the fact that, after the experiments on strong solutions of $MgCl_2$ were completed, a single determination of the concentration of the 1 gram-molecule solution of $MgCl_2$ gave a result of 0·9991 gram-molecule of salt in 1000 grams of water. The result was obtained by a determination of the chlorine content.

Chlorine found	6·62	per cent.
,, calculated	6·63	,,
Diff.	0·01	,,

The specific gravity experiments were carried out by the use of the open hydrometers A and B in the case of strong solutions of the salts, magnesium chloride and calcium chloride; and by the use of the closed hydrometers Nos. 3 and 17 in the case of the solutions of each of the three salts where the concentrations were less than 1·0 gram-molecule of salt in 1000 grams of water.

The constant experimental temperature was 19·50° C.

Concentration of the Solutions (*m*).—The highest concentrations were those of slightly supersaturated solutions in the case of magnesium and calcium chlorides. The solution of magnesium chloride which is saturated at 19·50° C. contains 5·9182 gram-molecules of salt in 1000 grams of water, and is the second of the series of strong solutions of this salt. The solution containing 5·5 gram-molecules of salt was prepared from this solution, and then solutions were experimented on having a common difference of 1 gram-molecule, and ranging from 5·0 to 1·0 gram-molecule.

The 6·627 gram-molecule solution of calcium chloride is supersaturated, and the 6·613 gram-molecule forms a solution which is saturated with $CaCl_2$ at 19·50° C. This solution is of interest as being the mother-liquor obtained after crystallisation from the solution which showed the condition of unrest described in Section XV. The experiments on this solution were made at a much earlier date than the others included in this table, but the results are included here in order to give a complete list of the experiments made.

As will be seen, the solutions for which $m = 6·6$ to $6·0$ decrease regularly in concentration by 0·1 gram-molecule. They were made in order to trace the changes of displacement due to small changes of concentration in nearly saturated solutions. For $m = 6·0$ to $1·0$ gram-molecules the common difference of concentration of consecutive solutions is 1·0 gram-molecule.

Discussion of Results. Specific Gravity.—With regard to the agreement of the individual results among themselves for a particular series, the analysis of the specific gravity results in the case of the calcium chloride solution containing 6·3 gram-molecules of salt in 1000 grams of water (see § 90) affords a fair criterion, and the usual number of series of observations made for each concentration was six, three with each hydrometer. In all the experiments the results of which are included in these tables, the temperature of the solution remained constant at 19·50° C. Comparing the specific gravities of solutions of the three salts, having the same molecular concentration, and m being less than 1·0, the values in all cases increase with increasing molecular weight. Thus, for $m = 1/2$ the specific gravities of the solutions rise from 1·025620 for $BeCl_2$ to 1·037385 for $MgCl_2$ and to 1·043739 for $CaCl_2$, and the same feature is observed in comparing the values for all concentrations down to $m = 1/1024$.

It is interesting to compare the increments of specific gravity (S − 1), (which for this purpose are conveniently multiplied by 1000), of the solutions with the molecular weights of the salts dissolved in them. We have them in the following table, for solutions for which $m = 1/2$:—

MR	=	$BeCl_2$.	$MgCl_2$.	$CaCl_2$.
1/2 MR	=	40	47·66	55·5
1000 (S − 1)	=	25·620	37·385	43·739
$\frac{1000\ (S-1)}{1/2\ MR}$	=	0·640	0·784	0·784

When $m = 1/16$ and $1/128$ we have the following values :—

MR	=	$BeCl_2$.	$MgCl_2$.	$CaCl_2$.
$\frac{1000\ (S-1)}{1/16\ MR}$	=	0·669	0·821	0·815
$\frac{1000\ (S-1)}{1/128\ MR}$	=	0·520	0·858	0·841

From this table we see that the increment of specific gravity produced by dissolving 1/2 MR in 1000 grams of water is exactly proportional to the molecular weight of the salts in the case of $MgCl_2$ and $CaCl_2$, and that this proportionality is maintained for values of $m=1/16$ and 1/128. In the case of $BeCl_2$, however, the proportionality fails.

It will be remarked that the specific gravities of the solutions of beryllium chloride for which $m=1/512$ and 1/1024 fall below unity, and the values are quite authentic. It follows that the displacements of these two solutions must be greater than the sum of the displacements of the salt and the water which they respectively contain. A similar feature is observed in the saturated solutions of cæsium salts, § 127, Table III.

Comparing the values of dS for concentrations greater than 1/2 gram-molecule, they diminish from 0·064018 for $MgCl_2$ at 1·0 gram-molecule concentration to 0·048345 at 4·0 gram-molecules, while in the case of $CaCl_2$ the values are 0·075363 at 1·0 gram-molecule, and 0·058415 at 4·0 gram-molecules concentration.

The variation in the values of dS for solutions of $CaCl_2$ between $m=6·0$ and $m=6·6$ does not exhibit itself in a regular decrease but an oscillatory one, for the value at $m=6·0$ is 0·004541, rising to 0·004810 at $m=6·1$, with a fall to 0·004471 at $m=6·2$, rising slightly again at $m=6·3$ to 0·004506, then decreasing to 0·004325 at $m=6·4$ and to 0·004299 at $m=6·5$, the general tendency being to decrease in value with increasing concentration.

§ 101. *Values of* $\frac{d\Delta}{dm}$ *and* $\frac{v}{m}$.—The features of the displacement of the solutions are best exhibited by discussing the values of $\frac{d\Delta}{dm}$ and $\frac{v}{m}$. The values of $\frac{d\Delta}{dm}$ are obtained from columns 7 and 1.

The solutions of the salts with concentrations less than 1 gram-molecule give values for $\frac{d\Delta}{dm}$ which are highest in the case of beryllium chloride, while those of magnesium chloride are lowest, those of calcium chloride being intermediate.

The value of $\frac{d\Delta}{dm}$ for beryllium chloride solution when $m=1/2$ is 28·66, and this decreases to 23·55 when $m=1/32$, rising to 26·58 at $m=1/128$. There are two low values, namely, 18·02 and 15·51 at $m=$ 1/256 and 1/512 respectively, with a value of 27·85 at $m=1/1024$.

In the case of magnesium chloride the value of $\frac{d\Delta}{dm}$ at $m=1/2$ is 22·92, and the value decreases with succeeding concentrations to the value 17·37 at $m=1/32$, which is also the value at $m = 1/64$. There is a slight rise to the value 17·86 at $m = 1/128$, and a sudden fall to 8·27 at $m = 1/256$, with an equally sudden rise to 29·80 at $m = 1/512$. The value at $m = 1/1024$ is 4·10.

In the case of calcium chloride there is a steady decrease from the value of 25·81 at $m = 1/2$ for $\frac{d\Delta}{dm}$ to 8·13 where $m = 1/32$, with a rise to 25·14 at $m = 1/64$. The value decreases to 21·09 at $m = 1/256$, where there is a sudden fall to 9·11 at $m = 1/512$. The value at $m = 1/1024$ is 22·94.

The values for v/m for concentrations below $m = 1$ are on arithmetical grounds more regular in all three cases than the corresponding values for $\frac{d\Delta}{dm}$. The value for beryllium chloride at $m = 1/2$ is 28·04, and this value decreases regularly to 26·37 at $m = 1/16$, where there is a rise to 38·39 at $m = 1/128$, after which the rate of increase is greatly augmented, and reaches a value of 176·13 at $m = 1/1024$.

The value of v/m for magnesium chloride at $m = 1/2$ is 19·81, decreasing to a value of 13·50 at $m = 1/128$, and rises to 18·74 at $m = 1/256$, with a sudden fall to 7·68 at $m = 1/512$, rising to 11·26 at $m = 1/1024$.

Calcium chloride shows the least tendency to sudden variations in the values of v/m, since the maximum amplitude is between 22·54 and 14·23 at $m = 1/2$ and $m = 1/256$ respectively. With the exception of the value 22·52 at $m = 1/32$, there is a fairly regular decrease between the two values quoted above, the rate of decrease taking place in two phases, the rate being greater between $m = 1/32$ and 1/256 than between $m = 1/2$ and 1/32.

There is a rise to a value of 19·35 at $m = 1/512$, and this decreases to 15·77 at $m = 1/1024$.

It is seen that the value of v/m at $m = 1/2$ is highest in the case of beryllium chloride and lowest in the case of magnesium chloride, while that of calcium chloride more nearly approaches that of magnesium chloride. The rise in the value of v/m for calcium chloride, where $m = 1/2$, over that of magnesium chloride is almost exactly proportional to the rise in molecular weight.

§ 102. The values of $(d\Delta - v)$ have been treated in the way fully set forth in Section VIII. for the solutions of the salts of the enneads MR and MRO_3. It is therefore sufficient to give here a table of the values of $(d\Delta - v)$ in the case of solutions of $m BeCl_2$, $m MgCl_2$, and $m CaCl_2$ in 1000 grams of water, for which $m < 1$.

m	1/2.	1/4.	1/8.	1/16.	1/32.	1/64.	1/128.	1/256.	1/512.	1/1024.
MR	Values of $(d\Delta - v)$.									
$BeCl_2$		0·310	0·098	0·056	−0·175	−0·104	−0·092	−0·159	−0·169	−0·145
$MgCl_2$	1·557	0·614	0·208	0·100	0·026	0·026	0·034	−0·041	0·043	−0·007
$CaCl_2$	1·638	0·413	0·148	0·074	−0·137	0·082	0·035	0·027	−0·020	0·007

A very remarkable feature exhibited by this table is the pronounced expansion which accompanies the dilution of solutions of beryllium chloride for which $m<1/16$.

§ 103. In columns 5, 9, and 10 of the tables, § 99, we have the data for judging the degree of conformity of the displacements of the solutions of these salts with the laws of the two hypotheses, Section VII. They do not agree at all with the arithmetic law of the first hypothesis, and their departure from the logarithmic law of the second hypothesis is considerable, as may be seen in the columns headed $(x-m)$ and $(1/m-1/x)$. Thus, in the strong solutions of $MgCl_2$ and $CaCl_2$, taking the exponent for $m=1$ as unity, we see that the exponents for the solutions of higher concentrations increase more rapidly than the corresponding concentration factors m, but for both strong and weak solutions $x>m$. Considering the solutions of $BeCl_2$, we see that the values of $(1/m-1/x)$ change sign for a value of m lying between 1/16 and 1/32. The behaviour of the solutions of $BeCl_2$ is, in this respect, quite remarkable. If column 6 be referred to, it will be seen that for

m	=	1/128.	1/256.	1/512.	1/1024.
$\Delta - 1000$	=	0·2999	0·2295	0·1992	0·1720

so that for these very different concentrations the displacements are very nearly the same. This depends on the remarkably low specific gravities of these solutions, which was commented on in § 100.

In the following table, which is constructed on the same scheme as Table VIII., § 45, the solutions of each salt are taken in successive pairs. The numbers in it represent the values $\frac{\log \Delta_m}{\log \Delta_{\frac{m}{2}}}=x$, or the exponent of the displacement Δ_m when the exponent of $\Delta_{\frac{m}{2}}$ is taken as unity. If the solutions conformed to the logarithmic law, the value of x should be 2.

Table giving the Values of $\frac{\log \Delta_m}{\log \Delta_{\frac{m}{2}}}$ for the three Salts, Beryllium Chloride, Magnesium Chloride, and Calcium Chloride.

m.	6·0.	4·0.	2·0.	1·0.	1/2.	1/4.	1/8.	1/16.	1/32.	1/64.	1/128.	1/256.	1/512.
$BeCl_2$					2·038	2·025	2·049	1·807	1·795	1·692	1·307	1·152	1·158
$MgCl_2$	2·154	2·200	2·204	2·145	2·127	2·091	2·093	2 051	2·107	2·321	1·440	4·888	1·362
$CaCl_2$	2·149	2·253	2·175	2·132	2·089	2·054	2·057	1·804	2·263	2·251	2·484	1·473	2·454

Here again the radical changes which take place in the properties of the solutions when the values of m fall below 1/16 are apparent, and particularly so in the case of the solutions of $BeCl_2$.

Section XV.—On a Remarkable State of Unrest in a Supersaturated Solution of Calcium Chloride before Crystallising.

§ 104. The primary purpose for which the open hydrometer was designed was to investigate the specific gravity and the displacement of solutions having concentrations in the neighbourhood of that of saturation. In § 90 we have seen the satisfactory result of experiments made for this purpose with solutions of chloride of calcium containing 6·3 grm.-mols. $CaCl_2$ per 1000 grams of water. Experiments in the same direction were made with solutions of chloride of calcium of still higher concentration. A parent solution was made, which, on the basis of published data relative to the solubility of the salt, should be supersaturated at 19·5° C. With it, it was intended to produce the solution saturated at this temperature, and to study its specific gravity and that of solutions formed by diluting it with small quantities of water.

As the solution showed no inclination to crystallise, although every opportunity was offered to it to do so, it seemed to me to be an example of a supersaturated solution peculiarly adapted to closer study.

Its composition was determined, and it was found to contain 7·225 grm.-mols. of chloride of calcium dissolved in 1000 grams of water. Its specific gravity was determined with the hydrometer exactly as if it had been a non-saturated solution. Two hydrometers were used for this purpose. They are designated A and B respectively. That designated A is the hydrometer whose constants have been set out in detail in Section XI. The hydrometers were made at different dates and on different specifications, though possessing the same general characteristics.

In Table I. are given the constants of both these instruments when loaded so as to float with small added weight, (a) in distilled water, and (b) in a supersaturated solution of calcium chloride, respectively. The entries opposite "weight of glass" and "weight of lead shot" in these tables are the approximate weights in air of these substances, which are required for the estimation of the corresponding volumes. The entries opposite "weight of the loaded hydrometer" are the exact weights, as *in vacuo*, of the glass + lead forming part of each instrument. To each of these weights has to be added that of the air contained in the instrument. The external volume of the hydrometer is independent of the internal load which it carries. It is entered only in (a), line 6. The "internal space occupied by air" is arrived at by subtracting the sum of the volumes of glass and of lead from the external volume of each hydrometer respectively. The mass of the air which fills this space depends not only on the volume of that space, but also on the density of the air which forms the atmosphere of the laboratory at the time of the experiment.

TABLE I.

Constants of Hydrometers A and B when loaded so as to float with small added Weight,

(a) In Distilled Water, and

(b) In a Supersaturated Solution of Calcium Chloride.

(a) DISTILLED WATER.		Hydrometer A.	Hydrometer B.
Weight of glass	grams	40·5	38·11
Volume ,,	c.c.	16·2	15·24
Weight of lead shot used for internal load	grams	95·6	78·00
Volume ,, ,, ,,	c.c.	8·4	6·87
Weight of the loaded hydrometer as *in vacuo*	grams	136·1113	115·1026
External volume of hydrometer	c.c.	137·09	118·09
Internal space occupied by air	c.c.	112·49	95·98
Volume of stem from zero scale division to the top of the stem	c.c.	1·2	1·0
(b) SUPERSATURATED CALCIUM CHLORIDE SOLUTION.			
Weight of glass	grams	40·5	38·11
Volume ,,	c.c.	16·2	15·24
Weight of lead shot used for internal load	grams	148·3	125·4
Volume ,, ,, ,,	c.c.	13·1	11·05
Weight of the loaded hydrometer as *in vacuo*	grams	188·8312	163·5755
Internal space occupied by air	c.c.	107·79	91·80

The use of these constants in arriving at the volume of air enclosed in the hydrometer, and in the reduction of the weight of the hydrometer in air to its value *in vacuo*, has been described in Section XI.

A little consideration and experience enables the experimenter to adjust the internal load so that the greatest possible range of specific gravity may be covered without altering it. The inferior limit of this range corresponds to a solution of such density that the hydrometer floats in it, immersed up to the highest division in the scale, without the addition of any external weight. The superior limit of the range corresponds to a solution of such density that the external weight to be added in order to immerse it to the lowest division on the scale begins to endanger the stability of the hydrometer as a floating body.

§ 105. *Experiments and Observations with Hydrometers A and B.*—The first set of determinations of the specific gravity of the supersaturated solution (7·225 $CaCl_2 + 1000$ grams of water) was made on 11th May 1910 in one of the smaller rooms of the Davy-Faraday Laboratory. The room has a northerly exposure, which is essential, and in other respects it is well suited for this class of investigation (§ 24).

A series of observations had been made with each hydrometer, and further observations were proceeding, when it was noticed that discrepancies between successive readings and corresponding ones in the earlier experiments made with the same added weights were occurring, and that these were far greater than any which could be attributed to errors of observation.

They persisted while four series of observations were made—two sets with each hydrometer—and were so great that in the fifth series of observations it was necessary to reduce the initial added weight in order that the complete series of observations might be made.

Throughout each series of experiments the temperature of the solution remained absolutely constant at 19·50° C. After the removal of hydrometer A from the experimental solution, on the completion of the fifth series of observations, the solution was stirred carefully with the standard thermometer, and its temperature was found to be 19·50° C., that of the air being 19·30° C. It was not until after these observations had been made that a cloudiness indicating the commencement of crystallisation appeared in the solution. It increased rapidly, and the temperature rose smartly to 23·16° C. and remained constant from 1.10 p.m. to 2.35 p.m.—a period of 85 minutes—when the temperature began to fall.

A careful record of the thermal and other observations throughout the whole experiment was kept, and the following is a *résumé* of these data :—

Weight of solution + cylinder	= 1270·190 grams.
Weight of cylinder	= 463·580 ,,
Weight of solution taken for observations with the hydrometers	= 806·610 ,,

This solution was 7·225 $CaCl_2$ + 1000 grams of water, and contained 44·48 per cent. $CaCl_2$.

§ 106. *Thermal Data.*—When the hydrometer A had been removed from the solution after the fifth series of observations, the time was 1.5 p.m. The solution was stirred with the thermometer, gently, and the temperature noted at 1.8 p.m. It was at this time that the crystals appeared in the solution, and its temperature rose in less than one minute to 23·16° C. *and then remained stationary until* 2.35 *p.m.*, while that of the air in the room varied only between 19·2° and 19·4° C. When the temperature of the crystals and the solution had fallen somewhat the cylinder was placed in water of 19·3° C. and cooled to 19·5° C., when the mother-liquor was found to have the specific gravity 1·423500 and to contain 42·33 per cent. $CaCl_2$.

§ 107. *Rate of Cooling of Original Solution.*—The crystals, together with the mother-liquor in the cylinder, were then heated to a temperature of 30° C. by placing the cylinder in a water-bath of that temperature and keeping it there until the crystals were re-dissolved. The system was then allowed to cool in the air, the temperature of which remained constant at 19·3° C., and the temperature of the cooling liquid was taken at intervals of 30 seconds.

The series of observations extended over 41 minutes, during which the temperature fell from 23·82° C. to 21·99° C. and the solution remained liquid to the end. The cooling had proceeded for 13 minutes before the temperature fell to 23·16° C., and the loss of heat was taking place quite regularly. The following are the temperatures observed at each half-minute for two minutes before and two minutes after the temperature 23·16° C. was passed :—

Time in minutes :	−2·0	−1·5	−1·0	−0·5	0·0	+0·5	+1·0	+1·5	+2·0
Temperature :	23·23°	23·21°	23·19°	23·17°	23·16°	23·14°	23·12°	23·09°	23·07°

During the four minutes the temperature fell 0·16° C., whence 0·04° C. per minute represents the mean rate of fall of temperature when the system has the temperature 23·16° C. and is cooling in air of constant temperature 19·30° C.

§ 108. *Calculation of Heat liberated during Crystallisation.*—The first thermal effect observed was when crystallisation began. The temperature of the system rose in less than a minute from 19·5° to 23·16°. During this phase the glass cylinder, as well as its contents, was warmed 3·66°. The heat liberated in this act depends on the weight of the solution, on its specific heat, and on the rise of temperature. In determining the thermal exchange which has taken place, we have to take account of the capacity for heat, which is generally represented by the "water-value," of the cylinder. The numerical data required in this calculation are the following :—

Weight of $CaCl_2$ solution	806·61 grams.
Its specific heat (Regnault)	0·636
Whence, water-value of solution	513·00 grams.
Weight of cylinder	463·58 ,,
Specific heat of glass	0·2
Whence, water-value of cylinder	92·72 grams.
Rise of temperature	3·66° C.

Whence, heat liberated in first act (513·00 + 92·72)3·66 = 2217 gr.° C.

After the first minute, when the temperature had become constant at 23·16°, the rate of liberation of heat was exactly equal to its rate of dissipation, which we have found to be represented by a fall of temperature of 0·04° per minute. This state was maintained for 85 minutes, which requires a liberation of heat, in the second act, of

$$85 \times 605{\cdot}72 \times 0{\cdot}04 = 2059 \text{ gr.}^\circ \text{ C.}$$

Adding the 2217 gram-degrees liberated in the first act, we find the total heat evolved during the interval of 85 minutes to be

$$2059 + 2217 = 4276 \text{ gr.}^\circ \text{ C.}$$

§ 109. In order to verify the state of unrest above described, the experiment was repeated with a 7·196 $CaCl_2$ solution (§ 113), and after crystallisation was completed, the crystals were removed and freed as far as possible from adherent mother-liquor, and their composition ascertained by estimation of the chlorine contained in a weighed quantity. The results of duplicate determinations gave the composition of the crystals so obtained as $CaCl_2 6 \cdot 3H_2O$ and $CaCl_2 6 \cdot 4H_2O$ respectively. It is obvious, therefore, that the crystals which were deposited had the composition $CaCl_2 6H_2O$, and that the excess of water indicated by the analyses was due to some adherent mother-liquor, from which it is almost impossible to free the crystals.

That the crystals deposited in the first experiment had the composition $CaCl_2 6H_2O$ is confirmed by the thermal data already set forth.

The weight of original calcium chloride solution which crystallised in the cylinder was 806·61 grams. It contained 44·48 per cent. $CaCl_2$.

The mean of the first series of specific gravities—1·446019—is taken as the specific gravity of this solution when at rest.

The concentration of the solution is therefore :—

	Per Cent.	By Weight.
$CaCl_2$ . .	44·48	359·0 grams.
Water . .	55·52	447·6 ,,
Total . .	100·00	806·6 ,,

After the crystallisation was ended, and with the solution at a temperature of 19·5° C., one determination of the specific gravity of the mother-liquor gave the result 1·423500.

The concentration of the mother-liquor determined by analysis was 42·33 per cent., equivalent to 734·0 grams, or 6·613 gram-molecules $CaCl_2$ per 1000 grams water.

The crystals ($CaCl_2 6H_2O$) contain 50·685 per cent. $CaCl_2$.

The cooling observations showed that the heat evolved in the act of crystallising was 4276 gr.° C.

According to THOMSEN, the heat of solution of $CaCl_2 6H_2O$ is $-4340 \cdot 0$ gr.° C. ; therefore on thermal evidence alone 215·5 grams, or 0·984 $CaCl_2 6H_2O$, has separated out. But, on the basis of the analytical estimations made on the supersaturated solution and the mother-liquor, we find that 210·3 grams of crystals separated out of 806·61 grams of solution, or 0·96 gram-molecule $CaCl_2 6H_2O$. The agreement of these two computed values is excellent.

We accept then as the quantities of crystals and mother-liquor 210·3 grams and 596·3 grams respectively.

§ 110. The nature of the experiments having been indicated, and the general character of the thermal change and the alterations in specific gravity mentioned, the following table, IIA., gives a complete account of the individual observations of specific gravity made, together with the corresponding displacements calculated from them.

Table IIB. gives a similarly complete account of the individual observations of specific gravity in five series made in the solution 6·3 $CaCl_2 + 1000$ grams of water at 19·5° C., with the corresponding displacements calculated therefrom. The solution of calcium chloride saturated at 19·5° C. is 6·613 $CaCl_2 + 1000$ grams of water ; therefore the 6·3 $CaCl_2$ solution, though of high concentration, is sufficiently removed from saturation to exhibit the tranquillity of a dilute solution.

Table IIa.

Table of Observations made with Hydrometers A and B when floating in the Supersaturated Solution 7·225 $CaCl_2$ + 1000 grams of Water at 19·5° C. before Crystallisation.

Hydrometer A.				Hydrometer B.			
Series 1 (10.45 a.m.–11.5 a.m.).				Series 2 (11.30 a.m.–11.45 a.m.).			
Added Weight in Grams.	Scale Reading in Millimetres.	Specific Gravity calculated from Single Observations.	Displacement.	Added Weight in Grams.	Scale Reading in Millimetres.	Specific Gravity calculated from Single Observations.	Displacement.
8·65	13·7	1·445961	1245·71	6·1	7·8	1·445905	1245·76
8·75	20·9	5972	·70	6·2	16·9	5826	·83
8·85	28·1	6034	·64	6·3	25·3	5837	·82
8·95	35·6	6044	·63	6·4	34·0	5828	·83
9·05	42·8	6033	·64	6·5	42·1	5880	·78
9·15	50·4	6015	·66	6·6	50·7	5895	·77
9·25	57·8	6039	·64	6·7	58·3	5962	·72
9·35	65·0	6049	·63	6·8	67·1	5948	·71
9·45	72·4	6037	·64	6·9	75·8	5936	·64
9·55	79·9	6001	·68	7·0	83·6	5998	·67
9·65	87·1	6028	·65	7·1	91·3	6076	·61

Hydrometer A.				Hydrometer B.				Hydrometer A.			
Series 3 (12.10 p.m.–12.25 p.m.).				Series 4 (12.35 p.m.–12.49 p.m.).				Series 5 (12.55 p.m.–1.5 p.m.).			
Added Weight in Grams.	Scale Reading in Millimetres.	Specific Gravity calculated from Single Observations.	Displacement.	Added Weight in Grams.	Scale Reading in Millimetres.	Specific Gravity calculated from Single Observations.	Displacement.	Added Weight in Grams.	Scale Reading in Millimetres.	Specific Gravity calculated from Single Observations.	Displacement.
8·65	16·0	1·445741	1245·90	6·1	7·8	1·445905	1245·76	8·6	12·5	1·445719	1245·92
8·75	23·9	5780	·87	6·2	16·9	5826	·83	8·7	20·1	5703	·93
8·85	31·6	5686	·95	6·3	25·3	5837	·82	8·8	27·9	5681	·95
8·95	39·1	5663	·97	6·4	34·3	5796	·86	8·9	36·9	5525	1246·09
9·05	46·5	5711	·93	6·5	42·7	5820	·84	9·0	45·1	5448	·16
9·15	53·6	5831	·82	6·6	51·8	5794	·88	9·1	53·9	5313	·27
9·25	61·0	5717	·92	6·7	60·0	5852	·81	9·2	62·9	5165	·40
9·35	68·1	5732	·91	6·8	68·1	5855	·80	9·3	70·5	5133	·43
9·45	75·3	5768	·88	6·9	77·1	5805	·79	9·4	78·2	5096	·46
9·55	82·6	5796	·85	7·0	85·2	5838	·82	9·5	86·8	4959	·58
9·65	89·9	5824	·83	7·1	94·5	5759	·89	9·6	94·7	4886	·64

TABLE IIB.

Table of Observations made with Hydrometers A and B when floating in the Solution 6·3 $CaCl_2$ + 1000 *grams of Water at* 19·5° *C., for Comparison with the Results given in Table IIA.*

Hydrometer A. Series 1. Added Weight in Grams.	Scale Reading in Millimetres.	Specific Gravity calculated from Single Observations.	Displacement.	Hydrometer B. Series 2. Added Weight in Grams.	Scale Reading in Millimetres.	Specific Gravity calculated from Single Observations.	Displacement.
4·0	13·9	1·409760	1204·94	2·05	7·1	1·409740	1204·96
4·1	21·8	735	·97	2·15	15·9	732	·96
4·2	29·6	731	·96	2·25	24·5	744	·95
4·3	37·2	730	·96	2·35	33·5	725	·97
4·4	44·6	744	·95	2·45	42·8	683	1205·00
4·5	52·2	749	·95	2·55	50·8	751	1204·95
4·6	59·8	762	·94	2·65	60·0	723	·97
4·7	67·1	776	·92	2·75	67·9	725	·97
4·8	74·7	764	·93	2·85	77·8	694	·99
4·9	82·2	762	·93	2·95	86·0	740	·96
5·0	89·7	761	·94	3·05	94·8	739	·96
Mean . .		1·409752		Mean . .		1·409727	
Probable error of mean expressed in units of the sixth decimal place		$\pm r_0 = 3·11$				4·46	

Hydrometer A. Series 3. Added Weight in Grams.	Scale Reading in Milli-metres.	Specific Gravity calculated from Single Observa-tions.	Displace-ment.	Hydrometer B. Series 4. Added Weight in Grams.	Scale Reading in Milli-metres	Specific Gravity calculated from Single Observa-tions.	Displace-ment.	Hydrometer A. Series 5. Added Weight in Grams.	Scale Reading in Milli-metres.	Specific Gravity calculated from Single Observa-tions.	Displace-ment.
4·0	14·1	1·409741	1204·95	2·05	7·0	1·409750	1204·95	4·0	14 0	1·409750	1204·95
4·1	21·7	745	·95	2·15	15·8	742	·95	4·1	21·5	764	·94
4·2	29·7	722	·97	2·25	24·7	724	·97	4·2	29·5	741	·95
4·3	37·5	730	·96	2·35	33·8	696	·99	4·3	37·4	711	·98
4·4	44·9	722	·97	2·45	42·0	760	·94	4·4	44·8	731	·96
4·5	52·1	765	·94	2·55	50·9	742	·95	4·5	52·1	765	·94
4·6	59·8	768	·93	2·65	59·0	818	·89	4·6	61·0	741	·95
4·7	67·2	772	·93	2·75	68·1	790	·92	4·7	67·5	744	·95
4·8	74·9	750	·95	2·85	76·9	782	·92	4·8	74·8	760	·94
4·9	82·4	748	·95	2·95	86·0	740	·95	4·9	82·6	730	·96
5·0	89·9	749	·95	3·05	94·8	739	·96	5·0	89·7	767	·93
Mean . .		1·409746		Mean . .		1·409753		Mean . .		1·409746	
Probable error of mean expressed in units of the sixth decimal place		$\pm r_0 = 3·49$				6·35				3·6	

Table IIc.

Table of Observations made with Hydrometers A and B in the Supersaturated Solutions, 7·196 $CaCl_2$ + 1000 grams of Water, at 19·50° C.

Index of Series of Observations.	Hydrometer used.	Time Interval in Minutes of Successive Observations from first Observation.	Added Weight in Grams.	Reading in Millimetres.	Corresponding Specific Gravity.	Corresponding Displacement.
		1·4	8·65	13·4	1·443843	1245·81
		2·7	8·75	20·5	876	·78
		4·1	8·85	28·0	884	·78
		5·5	8·95	35·6	868	·79
		6·8	9·05	43·1	859	·80
1st	A	8·2	9·15	50·9	824	·83
		9·6	9·25	58·2	851	·80
		10·9	9·35	65·8	825	·83
		12·3	9·45	73·0	831	·82
		13·6	9·55	80·3	832	·82
		15·0	9·65	87·5	843	·81
Time interval of 8 minutes between experiments.						
		24·5	6·1	10·0	1·443941	1245·73
		25·9	6·2	18·8	911	·75
		27·4	6·3	27·3	912	·75
		28·8	6·4	36·7	839	·82
		30·3	6·5	45·0	867	·79
2nd	B	31·7	6·6	54·2	806	·84
		33·2	6·7	62·6	830	·82
		34·6	6·8	71·1	830	·82
		36·1	6·9	79·9	807	·84
		37·5	7·0	88·0	844	·81
		39·0	7·1	96·6	838	·82
Time interval of 8 minutes between experiments.						
		48·5	8·65	13·9	1·443793	1245·85
		50·1	8·75	21·3	799	·85
		51·6	8·85	28·2	865	·79
		53·2	8·95	36·6	769	·88
		54·7	9·05	43·8	800	·85
3rd	A	56·3	9·15	51·5	767	·88
		57·8	9·25	58·3	841	·81
		59·4	9·35	66·2	784	·86
		60·9	9·45	73·8	752	·89
		62·5	9·55	81·0	762	·88
		64·0	9·65	88·7	724	·92

TABLE III.

Giving Details of the Order of Succession of the Experiments, of their Duration, and of the Temperatures of the Solutions and the Air respectively during the Experiments made on the Solution, 7·225 $CaCl_2$ + 1000 *grams of Water.*

Number of Experiment. (1)	Time of Commencement of Experiment. (2)	Time of Completion of Experiment. (3)	Duration of Experiment. (4)	Time Interval between Successive Experiments. (5)	Initial and Final Solution Temperatures. (6)	Initial and Final Air Temperatures. (7)	Hydrometer. (8)
1	10.45 a.m.	11.5 a.m.	20 minutes	25 minutes	19·5° C. 19·5° C.	19·3° C. 19·1° C.	A
2	11.30 a.m.	11.45 a.m.	15 minutes	25 minutes	19·5° C. 19·5° C.	19·3° C. 19·3° C.	B
3	12.10 p.m.	12.25 p.m.	15 minutes	10 minutes	19·5° C. 19·5° C.	19·3° C. 19·4° C.	A
4	12.35 p.m.	12.49 p.m.	14 minutes	6 minutes	19·5° C. 19·5° C.	19·4° C. 19·3° C.	B
5	12.55 p.m.	1.5 p.m.	10 minutes		19·5° C. 19·5° C.	19·3° C. 19·35° C.	A

§ 111. *Comparison of Results obtained with Hydrometers A and B when floating in the Supersaturated Solution of Calcium Chloride with those obtained when the Hydrometers are floating in a Solution containing* 6·3 *gram-molecules of Calcium Chloride in* 1000 *grams of Water.*—We will first draw attention to Table III., which gives the duration of each experiment, the initial and final solution temperatures, and the air temperatures before and after each experiment, in connection with the observations and results recorded in Table IIA. Table III. also affords a fair criterion of the usual duration of these experiments, and it gives suitable relief to the constancy of the temperature, both of the experimental liquid and of the atmosphere of the laboratory, which can, and must be secured, if the full precision of which the hydrometric method is susceptible is to be achieved. It will be observed that the temperature of the air was generally 0·2° C. lower than that of the experimental liquid, and that, in these conditions, the temperature of the experimental liquid remained perfectly constant during the time of the experiment. The absolute degree of constancy covered by this statement depends on the specification of the thermometer used. It was a standard instrument, divided, on the stem, into tenths of a Centigrade degree, and the length of a whole degree was 12 millimetres. On such an instrument variations of one-hundredth of a degree are easily appreciated by the practised eye. It must, however, never be forgotten that *what is directly observed is, at the best, the most probable value of the temperature of the bulb of the thermometer. The legitimacy of the conclusion that this is the temperature of the medium in which the thermometer is immersed depends on the expertness and experience of the experimenter.* The use of an instrument of the degree of delicacy above specified is justified only when all the precautions have been taken which are required in order to justify the experimenter in concluding that he has the temperature of the system under such control that its uncertainty is not greater than $\pm 0{\cdot}005^\circ$ C. Nothing short of first-rate work secures this.

In the work which Mr S. M. BOSWORTH has been doing under my direction, upwards of 3000 hydrometer observations have been made during the last twelve months, and the temperature conditions have been controlled with such skill that only three of the series showed a sensible variation of the temperature of the solution from the standard temperature during the time the experiment lasted. Their results were rejected, not because they were not very good, but because in this respect the others were perfect.

We will now proceed to a comparison of the figures in Tables IIA. and IIB.

§ 112. It would be useless to take the means of each series of specific gravity results in Table IIA. and compare them with the mean results given in Table IIB. But these numbers show the progressive character of the alteration of the specific gravity in each consecutive series, as well as the considerable differences of these numbers *inter se*, when contrasted with the agreement which holds among the mean results set out in the other table, IIB.

The mean results are given in the following table:—

TABLE IV.

Giving the Mean Specific Gravities calculated from the Series in Table IIA.

Series.	Hydrometer.	Mean Specific Gravity.
1	A	1·446019
2	B	1·445917
3	A	1·445750
4	B	1·445826
5	A	1·445330

Taking the five mean results given above, we find that the maximum amplitude of variation is 689 units in the sixth decimal place. If we turn to Table IIB., we find the maximum amplitude of variation to be only 26 such units.

Although the consideration of the mean specific gravities shows clearly that there is an unstable condition in the supersaturated solution as contrasted with the stable condition of the 6·3 $CaCl_2$ solution, this is made more evident if we compare, as in the following table, the greatest amplitude of the variation in each series given in Table IIA. with those occurring in series made with the same hydrometer, as given in the other table, IIB.

Series		I	2	3	4	5
Hydrometer		A	B	A	B	A
Amplitudes of Variation	Table IIA.	88	250	168	146	833
	Table IIB.	46	68	50	66	56

In series 4 of Table IIB. there are two extreme values of the specific gravity, namely, 1·409696 and 1·409818, giving an amplitude of 122, but they are quite exceptional, and if they are omitted the greatest amplitude in the series is 66, so that the mean maximum amplitude in the series made with hydrometer B is 67, while that in the series made with hydrometer A is 50.

On examining the figures for the supersaturated solution, their irregularity when compared with those of the 6·3 $CaCl_2$ solution is at once apparent; moreover, the irregularity increases very rapidly in each consecutive series, so that the amplitude of variation is in the first series 88, rising to 250 in the second, then falling to 146, and reaching 833 in the last series, immediately after which crystallisation commenced.

The rate of increase is not regular, for in the fourth series the variation amounts to 146, which seems to show that the solution was for the time being in a state of comparative calm.

This table, therefore, serves the useful purpose of indicating the fluctuating character of the alteration of the specific gravity, while Table IV. shows that in spite of these fluctuations there is a definite decrease in specific gravity from the first to the fifth series.

§ 113. *Displacement.*—The displacement of the 6·3 gram-molecule solution throughout the series is for all practical purposes constant, as might be expected, since the solution is quite stable and no disturbing influences, such as the imminence of crystallisation, are present.

But the displacement in the case of the supersaturated solution is subject to variations corresponding to those of the specific gravity. *These afford evidence of considerable and, to some extent, spasmodic acts of expansion and contraction, unaccompanied by any change of temperature of the solution or of the external pressure to which it was subjected. These spasmodic changes of volume exhibit a veritable species of labour, going on in the solution in its efforts to become a mother-liquor. In this it is finally successful, but not before it has succeeded in forcing the door which confined its store of heat. The birth of the crystal was synchronous with, and dependent on, the liberation of heat.*

If we consider the mean specific gravities of the five series given in Table IIA., we find a progressive decrease in them from the first to the fifth, with an interruption in the fourth, where a slight increase is observable over that of No. 3. This resultant decrease of the mean specific gravity of the solution is accompanied by a series of fluctuations in the results of the single observations of each series which indicates a condition of unrest in the solution, which is most accentuated in the fifth series of observations, during which the specific gravity fell from 1·445719 to 1·444886. The most remarkable feature of these changes is that they occurred without being accompanied by any change in the temperature of the solution. Confirmation of the occurrence of this remarkable condition of unrest was furnished by a repetition of the experiment, made with a solution containing 7·196 gram-molecules $CaCl_2$ per 1000 grams H_2O, recorded in Table IIc., and in it similar fluctuations of density, although not quite so pronounced, occurred as the forerunner of crystallisation. When the temperature of the solution had been observed after completion of the third series, the side of the cylinder was accidentally rubbed by the thermometer and crystallisation took place; but the behaviour of this solution and that of the 7·225 $CaCl_2$ solution, in the case of the first three series, exhibit similar features of unrest.

§ 114. It is interesting to inquire into the nature of the changes of displacement which have occurred in the transition of the solution from a condition of supersaturation to that of a mixture of saturated solution and crystals at the same temperature, 19·5° C. They are clearly shown in the accompanying table.

We commence with 806·61 grams of a solution having a specific gravity 1·444886, the displacement of this weight of solution being therefore 558·25 grams, and this resolves itself into 596·3 grams of mother-liquor with a specific gravity of 1·423500, giving a displacement of 418·89 grams, and 210·3 grams of $CaCl_26H_2O$ crystals with a specific gravity of 1·654 (LANDOLT's Tables), giving a displacement of 127·15 grams, from which we see that the displacement of the mixture of crystals and mother-liquor is 12·21 grams, or 2·2 per cent less than that of the original volume of supersaturated solution.

The following table shows this clearly :—

	Weight in Grams.	Displacement in Grams of Water.	Volume in c.c. at 19·5° C.	Percentage Displacement.
Original solution . .	806·6	558·25	558·75	100·00
Mother-liquor . . .	596·3	418·89	419·48	75·03
$CaCl_26H_2O$ . . .	210·3	127·15	127·36	22·83
Total . .	806·6	546·04	546·84	97·86
Difference . . .	...	12·21	11·91	2·14

§ 115. When we compare the behaviour of the chlorides of magnesium and calcium in supersaturated solution, it seems strange that the salt which has the greater amount of heat to lose by crystallising should be the more difficult to bring to crystallisation. The difficulty, however, lies only in the starting of crystallisation; there is none in its continuation. To start crystallisation in any solution, no matter how supersaturated it may be, is an operation which has some of the elements of an act of creation—*something appears where there was nothing* OF THE KIND *before.* The actions and reactions which take place before the first element of crystal appears are withdrawn from our sight, but their existence has been revealed by the hydrometer, which faithfully reports to him who can use it the dilatations and contractions which precede the crystal's birth.

They are illustrated in the accompanying diagram. In it the ordinates represent displacements of the solution and the abscissæ intervals of time, dating from the first observation of the first series. The lowermost curve, No. 1, represents graphically the data of displacement relating to the supersaturated solution 7·225 $CaCl_2$ + 1000 grams of water given in Table IIA. The uppermost curve, No. 2, represents those relating to the non-saturated solution 6·3 $CaCl_2$ + 1000 grams of water given in Table IIB.; and the intermediate curve, No. 3, represents the data relating to the supersaturated solution 7·196 $CaCl_2$ + 1000 grams of water given in Table IIC.

The displacements are plotted in the order in which they were observed, and the series follow each other in chronological order. The letters A, B designate the hydrometer which was used for the series represented under each respectively. The time intervals between successive series of observations are included in the diagram and are traced by dotted lines. The interval of time which separated two consecutive series of observations was on an average seventeen minutes, and the duration of each series of experiments was about fifteen minutes. The diagram shows at a glance the contrast between the tranquillity of the 6·3 $CaCl_2$ solution and the unrest indicated by the curves for the two supersaturated solutions.

The curve No. 2 for the 6·3 $CaCl_2$ solution pursues an even course, the displacements oscillating between the extremes 1205·00 and 1204·89. Otherwise the curve differs little from a straight line, and there is perfect agreement between the last result in one series and the first in the succeeding series. This is shown by the horizontality of the four dotted lines connecting the successive serial curves.

Curve No. 1 for the 7·225 $CaCl_2$ supersaturated solution is in striking contrast with No. 2. There is little agreement between the displacements in any of the corresponding series, and the oscillations of the serial curves are very marked, culminating in the continuous expansion shown in series 5, after which crystallisation took place. This is also well shown by the difference between the last displacement of one series and the first displacement of the following series. It is evident that the state of unrest continued when the solution was left to itself in the cylinder. The slight contraction shown in passing from the third to the fourth series indicates an effort on the part of the solution to regain a more stable condition. It is, however, clear that the state of unrest continued during the whole of the 140 minutes represented by the line of abscissæ in the diagram, and it suggests the possibility, and indeed the probability, that the supersaturated solution, even when confined in a closed vessel, may never be at rest.

§ 116. If we consider attentively what took place before the supersaturated solution of calcium chloride was brought to shed its salt as crystals, it is seen that it differs very

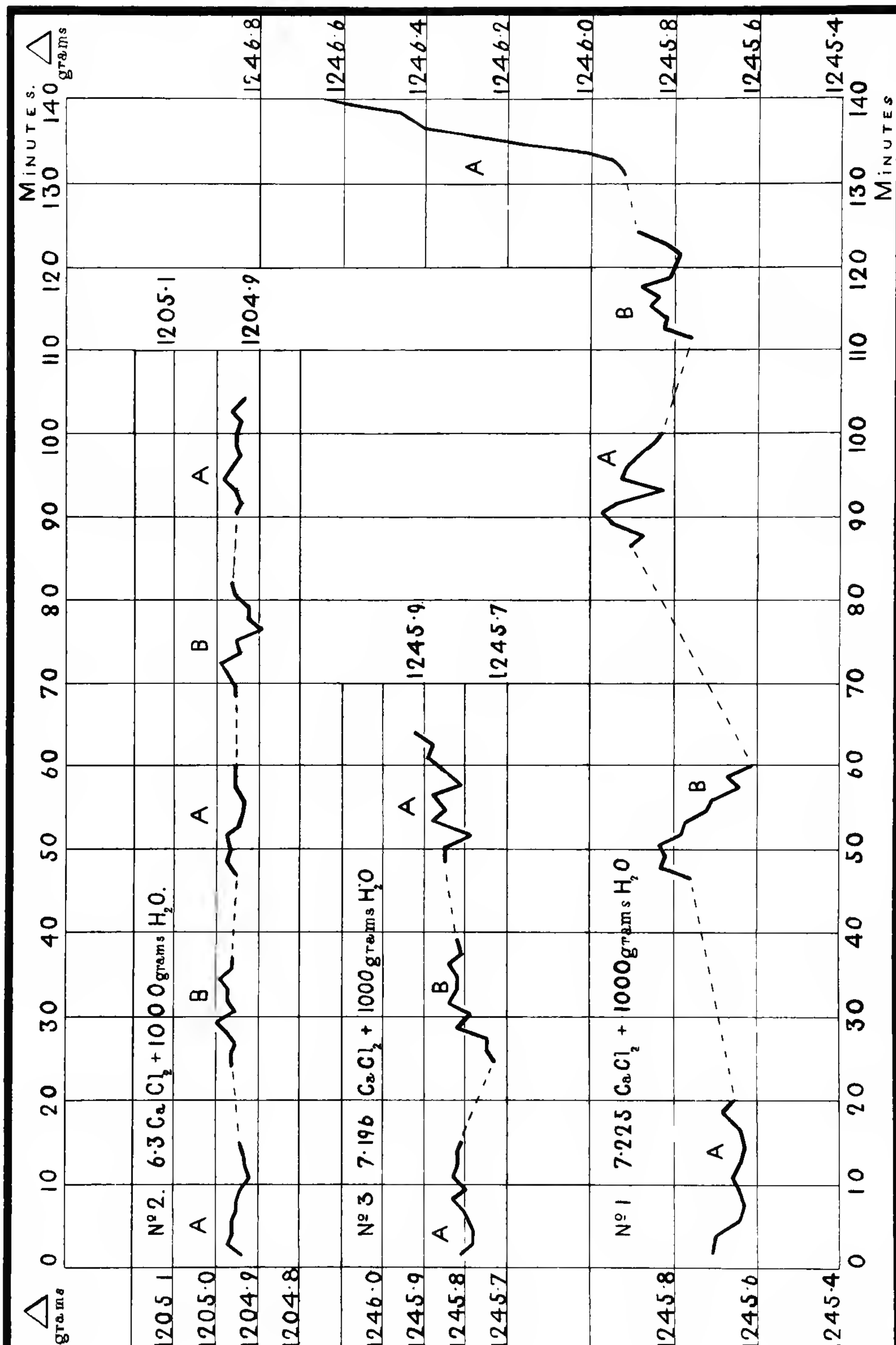


Diagram representing the variations in the displacement (Δ) of supersaturated solutions of $CaCl_2$ before crystallisation begins (curves Nos. 1 and 3), compared with the constancy of displacement of a non-saturated solution of $CaCl_2$ (curve No. 2).

little from what takes place when its non-saturated solution is brought to shed part of its water as crystals of ice. In each case considerable depression of the temperature of the solution below that of crystallisation is required before crystallisation sets in. When it does set in, an immediate rise of temperature takes place in the solution, and it stops only when the ordinary temperature of crystallisation has been reached. The same sequence of events is observed when water, containing no salt, is brought to crystallisation ; it also rises to its ordinary temperature of crystallisation. Unless these experiments are made in conditions which are unusual in chemical laboratories, none of these liquids begins to crystallise immediately when its temperature has been lowered exactly to the crystallising point.

The extent to which any particular solution can be cooled below its crystallising temperature, and the amount of mechanical disturbance which it can withstand when in this condition, vary with the nature of the solution. We have seen that the resistance so offered by a supersaturated solution of calcium chloride is considerable. But even that offered by pure water to the starting of crystallisation is greater than is generally believed to be the case.

When the mass of water used is small, and the capacity for heat of the vessel which contains it is large, it is possible, with care, to reduce the temperature of the system so far that, when crystallisation takes place, the whole of the water is transformed into ice and its temperature does not rise so high as 0° C. An instance of this is given in the following passage quoted from a lecture on "Ice and its Natural History" which I delivered before the Royal Institution on 8th May 1908 :—*

"Evidence of the uncertainty which exists regarding the temperature at which ice begins to form in water, when it is cooled in contact only with a solid other than ice, is furnished by the wet-bulb thermometer when it is being prepared for use at temperatures below 0° C., by freezing on it the quantity of water which is supported, against gravity, by the perfectly clean bulb. When this is rotated in air of $-10°$ to $-20°$ C., ice never begins to form until the temperature of the bulb of the thermometer has fallen to $-2°$ or $-3°$ C., and rarely before it has fallen to $-4°$ C. *In many cases I have observed it fall to temperatures as low as* $-8°$ *or* $-9°$ *C.; and in such cases, when freezing begins, the whole of the water is frozen without its being able, by the liberation of latent heat alone, to raise the temperature of the bulb of the thermometer to 0° C.*"

Whether, when in this unstable state, it would stand the mechanical disturbance which is resisted by a supersaturated solution of calcium chloride, can only be determined by experiment. This I have not as yet attempted. All that is required is to set about determining the specific gravity of pure water hydrometrically in a laboratory having the constant temperature $-4°$ or $-5°$ C., and using at least equal precautions with those observed in the case of supersaturated solutions at ordinary room temperatures.

If water, in these conditions, is sufficiently unsensitive to mechanical disturbance, it will undoubtedly do its part in manifesting its unrest ; it will then be the part of the

* *Proceedings of the Royal Institution of Great Britain*, 1909, xix., Part I. p. 248.

experimenter to receive the message, and if he succeeds he will have done a very fine piece of work.

§ 117. In considering the dilatation of the 7·225 $CaCl_2$ solution before crystallisation, we may pass over the first four series, although the oscillations of displacement which they exhibit would be remarkable enough if they stood alone, and confine our attention to the expansion in the fifth series which is continuous from the first to the eleventh observation. The displacement at the first observation was 1245·92, and at the eleventh 1246·64, corresponding to an increase of 0·72 gram in ten minutes. But the absolute minimum displacement observed was 1245·61 in the second series, so that the extreme amplitude of expansion was 1·03 gram. While these changes of displacement were going on, the liquid was perfectly homogeneous and its temperature was absolutely constant. Therefore the dilatations were not of thermal but of mechanical origin. We can apply no mechanical power which would produce such a stretching effect on a liquid, but we can easily arrive at the mechanical power which could effectually counteract it.

DRECKER * gives 0·0000217 as the coefficient of compressibility of a 40·9 per cent. solution of $CaCl_2$, and we may take this as the compressibility of our 7·225 solution, although it would be rather less. On this basis we obtain 38 atmospheres as the pressure required to reduce the volume of the solution 7·225 $CaCl_2$ + 1000 grams of water from 1246·64 to 1245·61 cubic centimetres, and we conclude that, *if we could place the solution in conditions such that its internal pressure should be increased by* 38 *atmospheres, the extreme dilatation observed would be mechanically provided for.* These isothermal oscillations cease immediately when the first element of crystal appears in the solution and affords an outlet for its latent heat, after which crystallisation proceeds in perfect tranquillity at a rate proportional to that at which heat is removed from the solution. It is stopped if heat is supplied at this rate to the solution from without.

§ 118. There is a remarkable resemblance between the state of unrest preceding the crystallisation of a supersaturated solution and that preceding the liquefaction of a gas, under a pressure not inferior to its critical pressure, when its temperature is reduced slightly below its critical temperature.

ANDREWS, in reporting his discovery of the critical state of liquids and gases, incidentally describes this state of unrest as follows :—" On practically liquefying carbonic acid by pressure alone, and gradually raising at the same time the temperature to 88° Fahr. (31·1° C.), the surface of demarcation between the liquid and gas became fainter, lost its curvature, and at last disappeared. The space was then occupied by a homogeneous fluid which exhibited, when the pressure was suddenly diminished or the temperature slightly lowered, a peculiar appearance of moving or flickering striæ throughout its entire mass. At temperatures above 88° Fahr. no apparent liquefaction of CO_2 or separation into two distinct forms of matter could be effected, even when a pressure of 300 or 400 atmospheres was applied" (*Phil. Trans.* (1869) vol. clix. p. 575).

* *Wied. Ann.*, 1888, vol. xxxiv. p. 955.

When a gas such as carbonic acid, under a pressure which is not inferior to its critical pressure, is confined in a tube as in ANDREWS' experiment and has a temperature ever so little higher than its critical temperature, it fills the tube as a homogeneous fluid which no pressure, however high, can liquefy. If, by removal of heat or by sudden relief of pressure, its temperature is reduced ever so little below its critical temperature, the homogeneous fluid begins to exhibit the peculiar appearance of moving or flickering striæ throughout its entire mass, as described by ANDREWS. These moving or flickering striæ indicate oscillations of density accompanying the effort on the part of the homogeneous fluid to shed a portion of its mass in the liquid state before there is a liquid nucleus for it to condense on and to afford the first outlet to the latent heat, the escape of which is an essential condition of liquefaction.

§ 119. We do not know the temperature at which the dry gas can condense on the dry walls of the envelope, but there can be little doubt that it is lower than that at which it condenses on its own liquid.

It is only in the conditions of ANDREWS' experiment that we can witness a substance persisting in the gaseous state under a greater than the critical pressure and having a temperature lower than the critical temperature, because *it is only when the gas and the envelope which contains it have been maintained at a temperature higher than the critical temperature of the gas that the inner walls of the envelope have a chance of being perfectly dry.* By "perfectly dry" I mean free from every and any trace whatever of the liquid substance. If the cooling process as specified above be then carried out, the temperature may be reduced slightly below the critical temperature, and yet the substance may persist in the gaseous state because there is none of itself in the liquid state for it to begin to condense on.

I have defined * *the boiling point of a substance, under a particular pressure, to be the temperature at which it as a liquid evaporates into itself as a gas, and as a gas or vapour condenses on itself as a liquid.* When the gas condenses on any other substance, or in a space filled only by itself, the temperature at which liquefaction commences is uncertain. In the moment, however, that the first, even the minutest trace of liquid appears, whether in the gas or on the walls, the temperature of condensation is defined, because the gas is then condensing on itself as a liquid. How, in such an experiment, the first element of liquid appears, we do not know. We say, it is by accident; but we may with equal right say, it is by an act of creation—because *in the process something appears where there was nothing* OF THE KIND *before.*

We thus see that there is a close analogy, in all important particulars, between the state of unrest which exists in a supersaturated saline solution before crystallisation commences and that indicated by the flickering striæ in a supersaturated gas before liquefaction takes place.

* "Chemical and Physical Notes," by J. Y. BUCHANAN, F.R.S., *The Antarctic Manual*, 1901, p. 97.

SECTION XVI.—THE DETERMINATION OF THE SPECIFIC GRAVITY OF THE CRYSTALS OF A SOLUBLE SALT BY DISPLACEMENT IN ITS OWN MOTHER-LIQUOR, AND THE VOLUMETRIC RELATIONS BETWEEN THE CRYSTALS AND THE MOTHER-LIQUOR WHICH ARE ESTABLISHED BY THE EXPERIMENT.*

§ 120. The work on the specific gravity of dilute solutions at 19·5° C. reported in the early part of this memoir was interrupted by the arrival of the great anticyclone or heat-wave of the summer of 1904, during which observations at a temperature of 19·5° were quite impossible. Indeed, the temperature of the laboratory, whether by night or day, hardly ever fell below 23° C. or rose above 25° C. It persisted over Northern Europe for nearly six weeks, and produced tropical conditions, which were evidenced alike by the high temperature of the air and by its insignificant diurnal variation.

In these circumstances I decided to make use of the time by putting into practice a method of determining the specific gravity of soluble salts which I had long intended to try. I took it up at first merely as a *tour de force* in experimentation with which to occupy myself during the hot weather, but it turned out to be a valuable method of research, and the duration of the spell of hot weather enabled me to prove and to use it.

The specific gravity of an insoluble substance is determined by the amount of distilled water which a known weight of it displaces. In the case of soluble salts it has been the custom to replace the water by a hydrocarbon or mineral oil. The objections to the use of this liquid are numerous, especially when the salt, the specific gravity of which it is desired to determine, is rare or costly. Moreover, to judge by the want of agreement among the values of the specific gravity of the same salt found by different chemists, there is greater uncertainty about the numerical results than there should be. One reason for this may be that the salts are not insoluble, but only sparingly soluble in the oil, and that sufficient attention has not been given to this point.

There is one liquid in which every soluble salt is quite insoluble, and that is its own mother-liquor at the temperature at which the one parted from the other. By immersing the salt in its own mother-liquor at the temperature of what we may call its birth, and by making the maintenance of this temperature a *conditio sine quâ non* of every manipulation during which the two are brought together again, errors due to uncertain solubility are eliminated, and contamination of valuable preparations is avoided. It is therefore by the immersion of each salt in its own mother-liquor that I determined its displacement; and this, combined with the weight of the salt and the specific gravity of the mother-liquor, gave the specific gravity of the salt.

* This formed the subject of a paper which was read at the meeting of the Chemical Society of London on 6th April 1905, but it was not published by the Society. I owe it to the courtesy of Professor E. S. DANA that the hospitality of the pages of the *American Journal of Science* was extended to it. It appeared in the January number of 1906, vol. xxi. p. 25, under the title :—"On a Method of Determining the Specific Gravity of Soluble Salts by Displacement in their own Mother-Liquor; and its Application in the case of the Alkaline Halides. By J. Y. Buchanan."

It is obvious that the method is applicable only to salts which *have* a mother-liquor, such as KCl; RbBr; $CaCl_26H_2O$; $BaCl_22H_2O$; it is inapplicable to salts such as $CaCl_2$; $BaCl_2$; and the like, which have no legitimate mother-liquor.

§ 121. It is an essential condition of success that the work be carried on in a room, for the time being, especially devoted to the purpose, and occupied by one investigator. He must have in it everything that he requires, including his balance. The window of the room must face the north, and the precautions generally to be observed are similar to those prescribed by BUNSEN for the practice of his original gasometric method.

The salts used in this research were the chlorides, bromides, and iodides of potassium, rubidium, and cæsium. The rubidium and cæsium preparations were from the works of SCHUCHARDT in Goerlitz, and on examination proved to be of the highest degree of purity. The potassium salts were also unexceptionable as regards quality, and were supplied by MERCK. All of these salts dissolve easily, and most of them abundantly, in water. They also crystallise with great readiness.

§ 122. The first operation is to prepare a hot solution of the salt such that, after standing over night, or for such length of time as may be deemed sufficient, it shall furnish about 60 c.c. of mother-liquor and about 15 c.c. of crystals. In the case of the potassium salts there was no difficulty, as their solubility at all temperatures is well known. The solubility of the rubidium and cæsium salts had to be determined, at least approximately, in each case, in order to economise the costly material. The following simple method furnished the required information easily and expeditiously. A suitable vessel, beaker or flask, is weighed empty, and then with 25 grams of distilled water, of the temperature of the air. The salt is then gradually added and the mixture stirred with the thermometer. In the case of every one of these salts the temperature falls rapidly while dissolving, and by as much as from 15° to 20°. The salt is added as rapidly as it is taken up by the water. When the fall of temperature slackens, a minimum is soon reached, while some salt still remains undissolved at the bottom of the vessel. It is then continually stirred; the temperature rises slowly while the salt gradually passes into solution, until, at a certain temperature, the amount of salt remaining undissolved is such that a further rise of one degree of temperature will evidently cause it to disappear. The vessel is now weighed, and, as result, we have the weight of salt dissolved in 25 grams of water at about the last observed temperature. With a little care it is easy to arrange that this temperature shall be in the neighbourhood of that of the air. The vessel with its contents is now heated, and salt added by degrees, while the temperature rises and finally reaches the boiling point or whatever other temperature may have been determined on. Salt is added until the liquid is saturated at this temperature. The vessel is again weighed and the salt dissolved at the higher temperature is ascertained. These simple experiments, which are completed in very few minutes, furnish all the information that is required for the economical employment of the material. In the absence of more detailed information, the following results obtained in the above way are worth quoting :—

100 Grams of Water Dissolve

Grams of at °C.	98 RbBr 12	164 RbI 20	264 RbI boiling	225 CsCl 25	51 CsI 12	157 CsI 107	93 CsBr 7·5	121 CsBr 24·5	156 CsBr 50	222 CsBr 93·5

With this information there is no difficulty in preparing the solution which shall, after allowing for unavoidable loss in preparation, give the required amounts of mother-liquor and of crystals. The water is warmed and the pure salt is added while the temperature is raised to that of ebullition, or to any lower temperature that may have been selected. When the salt has all passed into solution, the liquid is poured into a flat crystallising dish and crystallisation begins immediately. The area of the dish should be such that the layer of solution shall not be more than half a centimetre thick. The mother-liquor is then everywhere in close touch with the crystals. The dish is then put away in a cupboard for the night.

In the morning, the temperature of the contents of the crystallising dish and that of the air are taken very carefully. The mother-liquor is then poured off clear into a stoppered bottle, while the crystals are collected, allowed to drain, and dried in the ordinary way. The temperature which the mixture had when separated is noted as that at which the crystals and the mother-liquor are in equilibrium; and it is exactly at this temperature that they have to be brought together again in order to determine the specific gravity of the salt. It is at this temperature also that the specific gravity bottle is weighed when filled with distilled water and with mother-liquor respectively. In fact the temperature of equilibrium and of separation is the only temperature used.

§ 123. In Table I. the experimental details are given in full in the case of one salt, namely, cæsium chloride. For the other salts the results only are given, and they are collected in Table II.

All the weights as given represent the weight *in vacuo*.

The specific gravity bottle which was used was one of the common and convenient form which has a thermometer for a stopper and a lateral capillary tube for the adjustment of level. Its nominal capacity was 50 cubic centimetres. On three occasions one of 25 c.c. capacity was used for determining the displacement of the mother-liquor.

The molecular concentration (m) of the mother-liquor is determined by titration with tenth-normal silver nitrate solution. This solution was made with the greatest care and contained exactly 17 grams of silver nitrate in one litre, at the ordinary temperature of the laboratory at the time. The burette used was divided into tenths of a cubic centimetre and had a capacity of 50 c.c. The determination of the halogen was not made until the specific gravity had been determined, and, if the concentration was not already known within narrow limits, a preliminary titration was made, after which the volume of mother-liquor was weighed which would certainly require 40 ± 1 c.c. for titration. The capacity of the burette from 0 to 40 c.c. was determined by weight with great care. The concentration is stated in gram-molecules salt per 1000 grams of water.

TABLE I.—*Experimental Details in the case of Cæsium Chloride.*

Formula and molecular weight of salt	MR	CsCl = 168·5
Temperature	T	23·1° C.
MOTHER-LIQUOR.		
Determination of Specific Gravity.		
Weight of specific gravity bottle	w_1	38·8900 gms.
Weight of specific gravity bottle filled with distilled water	w_2	89·2399 ,,
Weight of water which fills it, $w_2 - w_1$	w_3	50·3499 ,,
Weight of sp. gr. bottle + mother-liquor	w_4	135·0620 ,,
Weight of mother-liquor, $w_4 - w_1$	w_5	96·1720 ,,
Specific gravity of mother-liquor, $\frac{w_5}{w_3}$	S	1·9101
Analysis.		
Weight of mother-liquor taken	w_6	1·0334 gms.
Cb. cents. $\frac{1}{10}$ $AgNO_3$ solution used	w_7	41·21 c.c.
Weight of salt equivalent to silver used, $\frac{w_7 \times MR}{10000}$	w_8	0·6944 gms.
Weight of water in W gms. mother-liquor, $w_6 - w_8$	w_9	0·3390 ,,
Concentration of mother-liquor expressed in gm.-molecules salt per thousand gms. water, $0·1\frac{w_7}{w_9}$	m	12·1563
SALT IN CRYSTAL.		
Determination of Specific Gravity.		
A. Weight of first portion of salt	w_{10}	22·1229 gms.
Weight of sp. gr. bottle + salt + mother-liquor	w_{11}	146·5514 ,,
Weight of mother-liquor, $w_{11} - (w_1 + w_{10})$	w_{12}	85·5385 ,,
Weight of water displaced by mother-liquor, $\frac{w_{12}}{S}$	w_{13}	44·7828 ,,
Weight of water displaced by salt, $w_3 - w_{13}$	w_{14}	5·5671 ,,
Specific gravity of salt, $\frac{w_{10}}{w_{14}}$	D_1	3·9739
B. Weight of second portion of salt	w_{15}	26·6220 gms.
Sum of weights of the two portions, $w_{10} + w_{15}$	w_{16}	48·7449 ,,
Weight of sp. gr. bottle + salt + mother-liquor	w_{17}	160·4249 ,,
Weight of mother-liquor, $w_{17} - (w_1 + w_{16})$	w_{18}	72·7900 ,,
Weight of water displaced by the mother-liquor, $\frac{w_{18}}{S}$	w_{19}	38·1085 ,,
Weight of water displaced by the salt, $w_3 - w_{19}$	w_{20}	12·2414 ,,
Specific gravity of salt, $\frac{w_{16}}{w_{20}}$	D_2	3·9820
C. Weight of water displaced by salt, $w_{20} - w_{14}$	w_{21}	6·6743 gms.
Specific gravity of salt, $\frac{w_{15}}{w_{21}}$	D_3	3·9890
Accepted specific gravity of salt	D	3·982

For weighing out the salt and passing it directly into the specific gravity bottle a special and convenient form of weighing tube was used. It was made out of a stoppered specimen tube with an internal diameter of 2 centimetres and a length of 7 or 8 centimetres. The lower end of this tube was opened and a piece of narrower glass tube joined to it before the blowpipe. This tube, which had a length of about 3 centimetres, had an external diameter such that it could just pass freely through the neck of the specific gravity bottle. The wide end was closed with a glass stopper, and the narrow end with a small india-rubber cork.

It was the custom to work so as to have about 15 c.c. of dry salt to be added in two charges to the specific gravity bottle. These charges were intended to be nearly, though not quite, equal. The available supply was distributed between two weighing tubes by approximate weight, after which the exact weight of each portion was determined in the usual way. The two portions of cæsium chloride weighed respectively 22·1229 and 26·6220 grams, so that in the first determination of specific gravity 22·1229 grams and in the second 48·7449 grams were concerned. It is not immaterial whether the first portion is charged into the empty specific gravity bottle and the mother-liquor poured over the dry powder, or is charged into the bottle which is already about half full of mother-liquor. In the former case the elimination of the entangled air is difficult and takes time, during which it is not easy to prevent the temperature getting out of hand. By the latter process very little air is carried past the surface of the liquid and very little stirring with the thermometer, which is required on other grounds, suffices to eliminate it.

§ 124. Owing to the readiness with which these salts crystallise and to the slowness with which all salts dissolve in an almost saturated solution, the temperature of the mixture of salt and mother-liquor, during the adjustment of level in the specific gravity bottle, must on no account be permitted to fall below T by even 0·01°, nor should it be allowed, even momentarily, to rise above it by more than 0·1°. The regulation of temperature was effected entirely with a standard thermometer divided into tenths of a degree, each tenth occupying a length of rather more than one millimetre on the stem. The thermometer which forms part of the specific gravity bottle is used chiefly as a stopper of convenient form. So soon as the level of the liquid has been adjusted in the bottle, it is weighed. The temperature and pressure of the air are kept account of for the reduction of all weights to the vacuum.

When the first weighing has been completed, about 20 or 25 c.c. of the clear mother-liquor are drawn off and the second charge of dry salt is added and mixed, after which the level is adjusted, and the weight determined. In the absence of experience it might be thought that it would be difficult to draw off so much of the liquid without some of the solid salt; but no matter how much they may be stirred up, these crystallised salts settle at once and completely to the bottom when immersed in their saturated solutions, and the operation presents no difficulty. It was at first intended to make a series of three determinations with each salt, but two were found to be sufficient. During all these manipulations the temperature of the air in the laboratory never differed from that of crystallisation ($T = 23·1°$) by more than one or two tenths of a degree, and, when the solubility of the salt is great, it is only in such conditions that operations of this kind can be carried out successfully.

Before bringing the crystals together with the mother-liquor in the specific gravity bottle, the operator must realise that their common temperature when mixed is to be *exactly that of crystallisation or equilibrium (T); and he must take such measures as his experience dictates to arrive at this end. Preliminary experiments on a somewhat extensive scale are absolutely necessary, and the success of an operation depends almost entirely on the operator and the trouble that he is prepared to take.*

§ 125. Table II. gives for each salt, MR, the temperature, T, of equilibrium between crystals and mother-liquor, and, in condensed form, the experimental data of the determination of S, the specific gravity at T of the mother-liquor, that of water at the same temperature being unity, of m, the concentration of the mother-liquor in gram-molecules salt per 1000 grams of water, and of D_1, D_2, D_3, the three observed values, as well as D, the finally accepted value, of the specific gravity of the salt, all at T, and referred to that of water at the same temperature as unity.

TABLE II.—*Experimental Results regarding each Salt in the Ennead.*

Salt: Formula MR	KCl.	KBr.	KI.	RbCl.	RbBr.	RbI.	CsCl.	CsBr.	CsI.
Salt: Mol. weight	74·6	119·1	166·1	121·0	165·5	212·5	168·5	213·0	260·0
Temperature, T	23·4°	23·4°	24·3°	22·9°	23·0°	24·3°	23·1°	21·4°	22·8°
Specific Gravity.	MOTHER-LIQUOR.								
Weight taken, gms. w_5	59·4068	34·3044	85·9636	74·7356	81·3282	46·2696	96·1720	42·3756	78·0087
Displacement, gms. w_3	50·3524	24·9554	49·9140	49·9188	49·9196	24·9478	50·3499	24·9744	50·3658
Specific gravity, $\frac{w_5}{w_3} = S$	1·1798	1·3746	1·7222	1·4971	1·6292	1·8548	1·9101	1·6968	1·5488
Concentration.									
Gm.-mols. p. 1000 gm. H_2O, m	4·7619	5·7250	8·9344	7·7670	6·7229	8·2307	12·1563	5·3057	3·5454
Specific Gravity.	SALT IN CRYSTAL.								
A. Weight of first portion of salt, gms. w_{10}	13·3684	36·7928	27·1751	19·0112	27·0906	26·4777	22·1229	27·8926	26·3890
Displacement, gms. w_{14}	7·3271	13·7498	8·9703	7·0256	8·4700	7·7248	5·5671	6·2453	5·8545
Specific gravity $\frac{w_{10}}{w_{14}} = D_1$	1·8245	2·676	3·0295	2·706	3·198	3·428	3·974	4·466	4·5075
B. Weight of both portions of salt, gms. w_{16}	27·4258	52·5142	52·1768	43·7750	51·5438	50·6025	48·7449	57·5390	53·3916
Displacement, gms. w_{20}	14·5322	19·6005	17·1465	15·9627	16 0568	14·7658	12 2414	12 9466	11·8423
Specific gravity $\frac{w_{16}}{w_{20}} = D_2$	1·887	2·679	3·043	(2·74)	3·210	3·428	3·982	4·455	4·5085
C. Weight of second portion of salt, gms. w_{15}	14·0574	15·7214	25·0017	(24·76)	24·4532	24·1248	26·6220	29·6464	27·0026
Displacement, gms. $w_{20} - w_{14} = w_{21}$	7·2051	5·8501	8·1762	8·9371	7 5868	7·0410	6·6743	6·7013	5·9878
Specific gravity, $\frac{w_{15}}{w_{21}} = D_3$	1·951	2 688	3·058	(2·77)	3·223	3·426	3·989	4·424	4·509
Accepted specific gravity, D	1·951	2·679	3·043	2·706	3·210	3·428	3·982	4·455	4·508

The letters and suffixes have the same significance as in Table I.

The numbers in line T show how uniform the temperature was during the period over which the experiments were spread. All the experiments were made between the 12th and 22nd of July 1904, with the exception of those on cæsium bromide, which were made on 10th August. By that time the anticyclone had begun to break, and the value of T for this salt is 21·4°. For all the other salts, T lies between 22·8° and 24·3°.

During the whole of the period the barometer was very steady, varying between 758 and 761 millimetres, and the relative humidity of the air in the laboratory varied between 40 and 50 per cent.

Of the three values D_1, D_2, D_3 for the specific gravity of the salt, D_1 is obtained directly from the first portion of the salt, D_2 from the sum of the two portions, and D_3 is derived from D_1 and D_2 by subtraction.

D_2 represents very nearly the mean of D_1 and D_3, and is the accepted value for the majority of the salts. It is expressed to three places of decimals, of which units in the second place are exact.

It will be noticed that in the case of rubidium chloride the value of D_1 is accepted. The second determination depends on the approximate weight of the second portion of salt when the tube was being filled, the exact weighing on the balance of precision having been accidentally omitted. The operation was however completed, and the calculation made with the approximate weight was used as a control. The result shows that the value of D_1 may be safely accepted. In the case of potassium chloride the value of D_3 (1·951) is accepted, and the reason for this is as follows: The first portion of salt was in very coarse powder, and in mixing it with the mother-liquor numerous crystalline particles were observed which contained gaseous enclosures, easily perceptible by the naked eye. As was expected, the observed specific gravity proved to be low. The second portion was much more finely powdered and the specific gravity resulting from the two was higher (1·887). But this result is affected to the full extent by the gaseous enclosures in the first portion. We therefore calculate the specific gravity from the second portion alone, which gives 1·951 for the specific gravity.

It is an advantage of the method just described that it furnishes more than the mere determination of the specific gravity of the salt. Thus, by ascertaining almost simultaneously the specific gravity of the mother-liquor and the displacement in it of the crystals, both being at the temperature of equilibrium, data are obtained for the determination of the relation between the displacement of the salt in crystal and the increment which it produces in the displacement of 1000 grams of water when it is dissolved in this mass of water and forms a saturated solution with it at that temperature. It has not hitherto been permissible to make exact comparisons of this kind, owing to the independence of the observations on the salt and on the solution which have been available.

§ 126. In discussing the results of observation it is convenient to arrange them in a more articulate form than that of Table II.

The group of salts which forms the subject of these experiments is one of the most remarkable in nature. The salts are nine in number and include all the possible binary combinations of the members of the electro-positive triad K, Rb, Cs with those of the electro-negative triad Cl, Br, I. The two triads of simple bodies make three triads, or one ennead, of binary compounds. The relations of the different members of the ennead are shown in Table III., in which the different features of the salts are exhibited in separate sub-tables. In these sub-tables the data referring to salts of the same metal (M) are found in the same column under the symbol for the metal (K, Rb, Cs), and those relating to salts of the same metalloids (R) are found in the same line opposite the symbol for the metalloid (Cl, Br, I). The symbol MR is used to represent both the formula and the molecular weight of the salt.

Sub-table (*a*) of this table contains the formula and sub-table (*c*) the molecular weight of each salt. The latter is the fundamental attribute of a substance, on which all its properties depend. The molecular weights of the salts which occur in one column differ by the amount of the difference of the atomic weights of the metalloids which they contain, that is, by 44·5 or 47 Similarly, contiguous salts in one line have molecular weights which differ by 46·4 or 47·5. If we consider the two diagonal triads in the ennead, we see that they are characterised by the fact that both the elements in each unit are different from those in either of the other units. Further, along the diagonal KCl–CsI the molecular weights of the units differ as much as possible from each other, while the atomic weights of the components of each unit are as nearly as possible identical, being close neighbours in the atomic series. On the other diagonal, KI–CsCl, the molecular weights of the units agree with each other as nearly as possible, while the atomic weights of the constituents of the units differ from each other as much as possible.

In sub-table (*b*) we have the values of T, the temperature at which the crystals and mother-liquor of each salt were in equilibrium, and that at which the various displacements were observed.

Under the experimental conditions, which have been minutely described above, it is impossible to fix in advance the exact temperature of equilibrium of the crystallising liquid. This is given by the meteorological conditions, modified by the structural features of the laboratory and of the apartment or enclosure where crystallisation takes place.

§ 127. *The Crystal.*—In compartment (*g*) we have the values of D, or the specific gravity of the salt in crystal at T, referred to that of distilled water of the same temperature as unity. The data in this compartment are in most cases for different, but always neighbouring, temperatures. The differences of the values of T are, however, so small and those of D are so great that we may discuss the specific gravities as if they had been made at one common temperature.

TABLE III.—*Table giving Numerical Relations between the Crystallised Salts of the Ennead MR and their Mother-Liquors.*

	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
	(*a*) Formula of each salt. MR.			(*b*) The common temperature at which the determinations of the specific gravity of the crystals and the mother-liquor respectively were made. T.			(*c*) Molecular weight of each salt. MR.		
Cl.	KCl.	RbCl.	CsCl.	23°·4	22°·9	23°·1	74·6	121·0	168·5
Br.	KBr.	RbBr.	CsBr.	23·4	23·0	21·4	119·1	165·5	213·0
I.	KI.	RbI.	CsI.	24·3	24·3	22·8	166·1	212·5	260·0
	(*d*) Weight of salt per 1000 grams of water in each solution. w.			(*e*) Concentration of the mother-liquor expressed in gram-molecules salt dissolved in 1000 grams of water. $\frac{w}{MR}=m$.			(*f*) Weight of the mass of the mother-liquor which contains 1000 grams of water. $1000+w=W$.		
Cl.	355·24	939·81	2048·34	4·7619	7·7670	12·1563	1355·24	1939·81	3048·34
Br.	681·85	1112·64	1130·11	5·7250	6·7229	5·3057	1681·85	2112·64	2130·11
I.	1484·00	1749·02	921·80	8·9344	8·2307	3·5454	2484·00	2749·02	1921·80
	(*g*) Specific gravity of the crystal at T, referred to that of distilled water at the same temperature as unity. D.			(*h*) Specific gravity of the mother-liquor at T, referred to that of distilled water at the same temperature as unity. S.			(*i*) Displacement of W grams of mother-liquor at T, expressed in grams of water at T. $\frac{W}{S}=\Delta$.		
Cl.	1·951	2·706	3·982	1·1798	1·4971	1·9101	1148·70	1295·71	1595·90
Br.	2·679	3·210	4·455	1·3746	1·6292	1·6968	1223·52	1296·73	1255·37
I.	3·043	3·428	4·508	1·7222	1·8548	1·5488	1442·34	1482·11	1240·83
	(*j*) Displacement of one gram-molecule of the crystal at T, expressed in grams of water at T. $\frac{MR}{D}$.			(*k*) Increment of displacement of 1000 grams of water caused by the dissolution in it of m . MR. $\Delta-1000=v$.			(*l*) Mean increment of displacement of mother-liquor per gram-molecule of salt dissolved in 1000 grams of water at T. $\frac{\Delta-1000}{m}=\frac{v}{m}$.		
Cl.	38·233	44·710	42·310	148·70	295·71	595·90	31·223	38·069	49·021
Br.	44·460	51·553	47·820	223·52	296·73	255·37	39·038	44·131	48·137
I.	54·580	61·986	57·670	442·34	482·11	240·83	49·506	58·575	67·907
	(*m*) Displacement of one gram-molecule of crystal, expressed in gram-molecules of water at T. $\frac{MR}{18D}$.			(*n*) Difference of the molecular displacement of the salt in crystal from the mean molecular increment of displacement of the water in the mother-liquor. $\frac{MR}{D}-\frac{v}{m}$.			(*o*) Ratio. $\frac{MR}{D}\cdot\frac{m}{v}$.		
Cl.	2·124	2·489	2·350	7·010	6·641	− 6·711	1·225	1·175	0·863
Br.	2·470	2·864	2·657	5·422	7·422	− 0·317	1·139	1·168	0·993
I.	3·032	3·444	3·204	5·074	3·411	− 10·237	1·103	1·059	0·849

On examining the values of D, we see that they increase with those of MR, but the increase is not continuous, it is remittent. It takes place *triadwise*; and this holds whether we take the triads in column or in line. Comparing salts in the same line, we see that replacing Rb by Cs causes a rise of specific gravity which is twice as great as that caused by the substitution of Rb for K. Comparing salts in the same column, the replacement of Cl by Br causes more than double the rise caused by the substitution of I for Br. However we regard it, we see that *the specific gravity of the salts is a periodic function of their molecular weight, within the ennead.*

In compartment (j) we have the values of $\frac{MR}{D}$ or the displacement of one gram-molecule (MR) of salt stated in grams of water, and in compartment (m) the same constant is stated in gram-molecules of water $\left(\frac{MR}{18D}\right)$. In dealing with the specific gravities, we saw that, whether we follow the columns or the lines, they increase with increase of molecular weight. In the case of the molecular displacements this holds for the columns but not for the lines. In these the salts of rubidium have the greatest molecular displacement, the potassium salts have the least, and the cæsium salts occupy an intermediate position. As we shall see later, this irregularity is due to a specific peculiarity of the cæsium salts. Meantime it may be noted that the values of $\frac{MR}{18D}$, which may be called the *volumetric equivalent* of one gram-molecule of any of the salts of the ennead, varies from 2·124 H_2O to 3·204 H_2O, the iodides having the highest and the chlorides the lowest equivalents. The average difference between the volumetric equivalents of the iodides and bromides is 0·563 H_2O, and that between those of the bromides and chlorides is 0·343 H_2O.

§ 128. *The Mother-liquor.*—The values of T are the same for the mother-liquor as for the crystals, and are presented in (b). In (e) we have the values of m or the molecular concentration of the mother-liquor. This is expressed in gram-molecules salt per 1000 grams of water, its equivalent w in grams is given in sub-table (d), and the total weight in grams (W) of the solution is given in sub-table (f). The concentration, m, of the mother-liquor represents with great exactness the molecular solubility of the salt at T, and we shall consider it for a moment from this point of view.

The least soluble, molecularly, of the nine salts is cæsium iodide, which has the highest molecular weight, and potassium chloride, which has the lowest molecular weight, comes next to it. Next to cæsium iodide, in molecular weight and in solubility, we have cæsium bromide; and, similarly, next to potassium chloride, in molecular weight and in solubility, we have potassium bromide. In the latter case the solubility increases with the molecular weight, while in the former it decreases with it. But, if sub-table (c) be referred to, it will be observed that, as regards molecular weight, KCl and CsI occupy singular positions in the ennead. On the other hand, KBr (119·1) and RbCl (121) have almost identical molecular weights, as have also CsBr (213) and RbI (212·5), yet the solubilities in each pair respectively are very different. The lowest solubilities are on the diagonal KCl–CsI, and the highest solubilities on the diagonal KI–CsCl. RbBr, which occupies the middle place on both these diagonals, is also in the middle of the middle column and of the middle line, and is the centre of the ennead. Its solubility, besides being nearly the average of the group, has a symmetrical position with respect to those of the other salts. On one diagonal the solubility of its neighbours is lower, on the other higher, than its own. In its column the solubility of its neighbours is higher, in its line it is lower, than its own. Turning from the molecular solubilities in sub-table (*e*) to the ordinary solubilities given in sub-table (*d*), we see that the positions of CsI and KCl are reversed; the least soluble salt of the ennead is KCl, with 355·24 grams, and next to it comes CsI, with 921·80 grams per 1000 grams of water. Other great differences occur which are obvious on inspection and need not be further referred to here, because in the research only the molecular weights of the salts are taken into account.

In compartment (*h*) we have the values of S, the specific gravity of the mother-liquor at T, referred to that of distilled water of the same temperature as unity. These numbers cannot, as they stand, be compared with each other because they refer to solutions of such different concentrations. They enable us, however, to arrive at the increment of the displacement of 1000 grams of water caused by its being saturated with the particular salt at T. Thus, taking again cæsium chloride as an example, we have for the weight of salt dissolved in 1000 grams of water

$$w = m.\mathrm{CsCl} = 2048{\cdot}34 \text{ grams.}$$

Adding 1000 grams to this, we have for the weight of the solution

$$\mathrm{W} = 1000 + w = 3048{\cdot}34 \text{ grams.}$$

The specific gravity (S) being 1·9101, the displacement of the solution is

$$\Delta = \frac{\mathrm{W}}{\mathrm{S}} = 1595{\cdot}90 \text{ grams of water,}$$

whence the increment of displacement of the water by its saturation with the salt is

$$v = \Delta - 1000 = 595{\cdot}90 \text{ grams,}$$

and the mean increment of displacement per molecule is

$$\frac{v}{m} = 49{\cdot}021 \text{ grams.}$$

$\dfrac{m.\mathrm{MR} + 1000}{\mathrm{S}} = \Delta =$ displacement of the mass of mother-liquor containing 1000 grams of water.

$\Delta - 1000 = v =$ increment of displacement due to dissolution of $m.\mathrm{MR}$ in 1000 grams of water.

In compartment (*l*) we have the value of $\dfrac{v}{m}$ for each member of the ennead.

§ 129. Before commenting on the numbers in the table, it is important to form a clear conception of their physical meaning. We shall best arrive at this by returning to our detailed example of chloride of cæsium. As the quantity of saturated solution which contains 1000 grams of water weighs 3048·34 grams and displaces 1595·90 grams of water, we may imagine it to have been prepared in the following way:—1595·90 grams of water are taken, and cæsium chloride is dissolved in it so that each portion, as it is added, forms a saturated solution with the exact quantity of water which it requires for this purpose, and the remainder of the water remains uncontaminated. Parallel with the dissolution of the salt, pure water is removed at such a rate as to keep the displacement or bulk of the liquid always the same. When no more salt will dissolve, we have a saturated solution which contains 1000 grams of water. The weight of cæsium chloride which has entered the solution is 2048·34 grams, and the weight of water which has left it is 595·90 grams, whilst the displacement of the liquid is the same at the end of the operation as it was at the beginning. In thus describing the preparation of the saturated solution, we have described an operation of substitution. *It is therefore permissible to regard saturated solutions as products of substitution.* If we give to the above numbers their molecular interpretation, we see that the mean increment of displacement produced by the presence of one molecule of cæsium chloride in its saturated solution at 23·1° is equal to that of 2·723 gram-molecules of free water, and therefore, that, in these conditions, *CsCl is, in a sense, volumetrically equivalent to 2·723* H_2O.

If we study sub-table (*l*), we see that the average molecular increment of displacement produced by the salts increases with their molecular weight, whether we follow the columns or the lines. The only exception is furnished by cæsium bromide, the increment produced by which is very slightly lower than that of cæsium chloride. The greatest increment is that due to cæsium iodide, which has the highest molecular weight; and the least increment is that due to potassium chloride, which has the lowest molecular weight. The pair, potassium bromide and rubidium chloride, which have almost equal molecular weights, cause also almost equal molecular increments of displacement. The same is true of the pair, potassium iodide and cæsium chloride, but rubidium bromide causes a markedly lower increment of displacement. Finally, the pair, rubidium iodide and cæsium bromide, which have almost identical molecular weights, present no resemblance in the increment of displacement which they produce.

§ 130. *Comparison of the Displacement of the Salt in Crystal and the Increment of Displacement which it produces in the Water of its Mother-Liquor.*—The molecular displacement $\frac{MR}{D}$ of the salts in crystal is given in sub-table (*j*) in terms of grams of water; that of $\frac{v}{m}$, the salts in mother-liquor, is similarly given in sub-table (*l*).

If we compare these two tables, we find the remarkable result that while in the case of the potassium and the rubidium salts the numbers for the displacement in crystal are greater than those for the increment of displacement in mother-liquor, in the case of the cæsium salts the reverse is the case.

In sub-table (*n*) we have the difference $\left(\frac{MR}{D} - \frac{v}{m}\right)$ of the molecular displacement of the salt in crystal from its mean molecular increment of displacement of the water in the mother-liquor. In compartment (*o*) we have the ratio $\left(\frac{\text{MR}}{\text{D}} \cdot \frac{m}{v}\right)$ of these quantities.

Taking the figures in compartment (*n*), we see that in the case of the salts of potassium and rubidium crystallisation is accompanied by considerable expansion, and this is what is usually met with. In the case of the cæsium salts the reverse is the case, and very decidedly so in that of the chloride and of the iodide, but much less so in the case of the bromide, which, in this, as in other particulars, maintains its singular position.

In this connection it should be noted that among the ratios $\left(\frac{\text{MR}}{\text{D}} \cdot \frac{m}{v}\right)$ given in compartment (*o*), the two which are nearest to unity are those for RbI (1·059) and for CsBr (0·993) respectively; and their molecular weights are almost identical. Further, the salts situated *co-diagonally* to them, namely RbBr and CsI, have ratios whose differences from unity are, numerically, almost equal, namely + 0·168 for RbBr and − 0·151 for CsI.

Taking a general view of the numbers in (*o*) which give the ratios of displacement in crystal and in mother-liquor, we see great differences. The most striking examples are, as in the case of solubility, the extreme members of the ennead, KCl and CsI. The former expands by more than 25 per cent., and the latter contracts by 15 per cent. on crystallising.

These figures accentuate the peculiarity of the cæsium salts, that crystallisation is accompanied by contraction. An interesting conclusion can be drawn from the behaviour of the different salts in this respect, namely, that *the crystallisation of the potassium and rubidium salts of the ennead must be hindered by increased pressure, while that of the cæsium salts must be helped by the same agency.*

§ 131. *Extension of the Research to the Salts of the Ennead MRO_3, or the Oxyhalides of Potassium, Rubidium, and Cæsium.*—It appeared to be interesting to extend this work so as to include the salts of the ennead of the *oxyhalides*, having the general formula MRO_3, in which M may be K, Rb, Cs, and RO_3 may be ClO_3, BrO_3, IO_3.

In contrast with the salts of the ennead MR, which are very soluble, the oxyhalides are only sparingly soluble. The determination of the specific gravity of the crystals in their mother-liquors is therefore much easier, and was effected quite successfully by my assistant, Mr H. F. Fermor. The results so obtained are given in Table IV., which is identical in form with Table II., dealing with the salts of the halides, which has already been explained.

§ 132. The results of the discussion of the observations made with the salts of the ennead MRO_3 are given in Table V., which is constructed on the same plan as Table III. It consists of a number of sub-tables, (*a*), (*b*), (*c*), etc., and the nature of each is specified in its title. The molecular weight of each salt, represented by the general formula MRO_3, differs from that of the corresponding salt of the general formula MR by $O_3 = 48$. Therefore the differences between the molecular weights in the same column and between those in the same line in sub-table (*c*) are the same as those between the molecular weights of the corresponding salts of the ennead MR to be found in sub-table (*c*) of Table III., and what was said in this respect about the linear, columnar, and diagonal relations of the molecular weights of the salts of the ennead MR applies equally in the case of the ennead MRO_3. The concentration, *m*, of the mother-liquor, given in sub-table (*e*) is derived from its specific gravity.

§ 133. In sub-table (*g*) we have the values of D, the specific gravity of the salt in crystal at T, referred to that of distilled water at the same temperature as unity. If we examine the values of D, we see that they rise triadwise and parallel to the values of the molecular weight. In order to study their differences the accompanying table has been constructed :—

Table giving the Specific Gravities, D, of the Salts of the Ennead MRO_3, and their Differences.

	K.	Diff.	Rb.	Diff.	Cs.
ClO_3 . .	2·319	0·857	3·176	0·406	3·582
Diff. .	0·900		0·505		0·527
BrO_3 . .	3·219	0·462	3·681	0·428	4·109
Diff. .	0·705		0·655		0·740
IO_3 . .	3·924	0·412	4·336	0·513	4·849

In this table we have the nine entries of the specific gravity of the crystals, and these furnish six entries of independent differences taken column-wise, and an equal number taken line-wise. The differences occurring in the lines correspond to pairs of salts having the same acid and different bases; those occurring in the columns correspond to pairs of salts having the same base but different acids. In the upper left-hand corner we have in the top line 0·857, which is the excess of the specific gravity of $RbClO_3$ over that of $KClO_3$, and 0·406, which is the excess of the specific gravity of $CsClO_3$ over that of $RbClO_3$; so that 0·857 is the increase of the specific gravity of the salt $MClO_3$ when the substitution of Rb for K as the value of M is effected. Similarly, the increase of specific gravity caused when the substitution of Cs for Rb in $MClO_3$ is effected, is 0·406.

Replacing Cl by Br as R in KRO_3 produces a rise of 0·900 in the specific gravity, while the replacement of K by Rb as M in $MClO_3$ produces a rise of 0·857. When Rb is replaced by Cs as M in $MClO_3$ and $MBrO_3$ the effects are similar, namely, a rise of 0·406 and 0·428 respectively. The replacement of Br by I as R in KRO_3 and $CsRO_3$ causes a rise of 0·705 and 0·740 respectively, while the replacement of Rb by Cs as M in $MClO_3$ is very close to that produced by the replacement of K by Rb as M in MIO_3, namely, 0·406 and 0·412 respectively. These examples illustrate the similarity of the substitution effect produced by elements having nearly identical atomic weights but antagonistic chemical and physical properties.

Table IV.

Experimental Results regarding each Salt in the Ennead MRO_3.

Salt: Formula MRO_3	$KClO_3$	$KBrO_3$	KIO_3	$RbClO_3$	$RbBrO_3$	$RbIO_3$	$CsClO_3$	$CsBrO_3$	$CsIO_3$
Salt: Mol. weight	122·6	167·1	214·1	169·0	213·5	260·5	216·5	261·0	308·0
Temperature, T	14·8°	19·2°	18·6°	16·2°	16·0°	15·6°	16·0°	16·0°	15·4°
	Mother-Liquor.								
Specific Gravity.									
Weight taken, gms., w_5	52·2451	52·7937	53·9820	52·1316	51·2751	51·6035	52·4485	51·4449	51·3776
Displacement, gms., w_3	50·4321	50·3973	50·4135	50·4209	50·4223	50·4258	50·4235	50·4219	50·4274
Specific gravity, $\frac{w_5}{w_3} = S$	1·0360	1·0475	1·0708	1·0339	1·0169	1·0233	1·0402	1·0203	1·0188
Concentration.									
Gm.-mols. p. 1000 gms. H_2O, m	0·4764	0·3990	0·4027	0·2938	0·1029	0·1072	0·2596	0·0995	0·0720
Specific Gravity.	**Salt in Crystal.**								
A. Weight of first portion of salt, gms., w_{10}	6·5566	23·5976	38·2490	29·0782	32·6042	39·0514	29·0122	25·4094	38·9291
Displacement, gms., w_{14}	2·8211	7·2897	9·7153	9·1539	8·8402	8·9315	8·0905	6·1773	8·0102
Specific gravity, $\frac{w_{10}}{w_{14}} = D_1$	2·3241	3·2371	3·9370	3·1766	3·6882	4·3723	3·5860	4·1134	4·8600
B. Weight of both portions of salt, gms., w_{16}	12·3282	53·0730	79·6206	58·6266	68·1770	78·6761	64·9130	60·7122	83·3354
Displacement, gms., w_{20}	5·3153	16·4973	20·2904	18·4619	18·5218	18·1450	18·1219	14·7769	17·1872
Specific gravity, $\frac{w_{16}}{w_{20}} = D_2$	2·3192	3·2171	3·9240	3·1755	3·6809	4·3360	3·5820	4·1086	4·8487
C. Weight of second portion of salt, gms., w_{15}	5·7716	29·4754	41·3716	29·5484	35·5728	39·6247	35·9008	35·3028	44·4063
Displacement, gms., $w_{20} - w_{14} = w_{21}$	2·4942	9·2076	10·5751	9·3080	9·6816	9·2135	10·0314	8·5996	9·1770
Specific gravity, $\frac{w_{15}}{w_{21}} = D_3$	2·3140	3·2012	3·9122	3·1745	3·6743	4·3008	3·5789	4·1052	4·8389
Accepted specific gravity, D	2·319	3·219	3·924	3·176	3·681	4·336	3·582	4·109	4·849

Table V.

Table giving Numerical Relations between the Crystallised Salts of the Ennead MRO_3 and their Mother-Liquors.

	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
	(*a*) Formula of each salt. MRO_3.			(*b*) The common temperature at which the determinations of the specific gravity of the crystals and the mother-liquor respectively were made. T.			(*c*) Molecular weight of each salt. MRO_3.		
ClO_3.	$KClO_3$.	$RbClO_3$.	$CsClO_3$.	14°·8	16°·2	16°·0	122·6	169·0	216·5
BrO_3.	$KBrO_3$.	$RbBrO_3$.	$CsBrO_3$.	19·2	16·0	16·0	167·1	213·5	261·0
IO_3.	KIO_3.	$RbIO_3$.	$CsIO_3$.	18·6	15·6	15·4	214·1	260·5	308·0
	(*d*) Weight of salt per 1000 grams of water in each solution. w.			(*e*) Concentration of mother-liquor expressed in gram-molecules salt dissolved in 1000 grams of water. $\frac{w}{MRO_3}=m$.			(*f*) Weight of the mass of the mother-liquor which contains 1000 grams of water. $1000+w=W$.		
ClO_3.	58·41	49·65	56·20	0·4764	0·2938	0·2596	1058·41	1049·65	1056·20
BrO_3.	66·67	21·97	25·97	0·3990	0·1029	0·0995	1066·67	1021·97	1025·97
IO_3.	86·22	27·92	22·18	0·4027	0·1072	0·0720	1086·22	1027·92	1022·18
	(*g*) Specific gravity of the crystal at T referred to that of distilled water at the same temperature as unity. D.			(*h*) Specific gravity of the mother-liquor at T, referred to that of distilled water at the same temperature as unity. S.			(*i*) Displacement of W grams of mother-liquor at T, expressed in grams of water at T. $\frac{W}{S}=\Delta$.		
ClO_3.	2·319	3·176	3·582	1·0360	1·0339	1·0402	1021·63	1015·24	1015·38
BrO_3.	3·219	3·681	4·109	1·0476	1·0169	1·0203	1018·21	1004·98	1005·56
IO_3.	3·924	4·336	4·849	1·0708	1·0234	1·0188	1014·40	1004·42	1003·31
	(*j*) Displacement of one gram-molecule of the crystal at T, expressed in grams of water at T. $\frac{MRO_3}{D}$.			(*k*) Increment of displacement of 1000 grams of water caused by the dissolution in it of $m . MRO_3$. $\Delta-1000=v$.			(*l*) Mean increment of displacement of mother-liquor per gram-molecule of salt dissolved in 1000 grams of water at T. $\frac{\Delta-1000}{m}=\frac{v}{m}$.		
ClO_3.	52·867	53·212	60·441	21·63	15·24	15·38	45·399	51·858	59·264
BrO_3.	51·910	58·001	63·519	18·21	4·98	5·56	45·629	48·444	55·849
IO_3.	54·603	60·078	63·518	14·40	4·42	3·31	35·756	41·251	44·634
	(*m*) Displacement of one gram-molecule of crystal, expressed in gram-molecules of water at T. $\frac{MRO_3}{18D}$.			(*n*) Difference of the molecular displacement of the salt in crystal from the mean molecular increment of displacement of the water in the mother-liquor. $\frac{MRO_3}{D}-\frac{v}{m}$.			(*o*) Ratio. $\frac{MRO_3}{D}\cdot\frac{m}{v}$.		
ClO_3.	2·937	2·956	3·358	7·467	1·354	1·177	1·164	1·026	1·020
BrO_3.	2·884	3·222	3·530	6·281	9·557	7·670	1·137	1·197	1·137
IO_3.	3·033	3·337	3·529	18·847	18·827	18·884	1·527	1·456	1·423

§ 134. In sub-table (j) is given the molecular displacement, MRO_3/D, of the crystal in grams of water, and in sub-table (m) the same constant $MRO_3/18D$ is given in molecules of water.

In the potassium salts the values of this constant is least for $KBrO_3$, and greatest for KIO_3. In the rubidium salts there is a progressive increase from the chlorate to the bromate and the iodate. In the cæsium salts the values for the bromate and iodate are identical, and that for the chlorate is only very little lower.

§ 135. Sub-tables (d), (e), and (f) give the concentration of the mother-liquor for each salt, expressed in three different ways. In (e) it is expressed in gram-molecules, m, of salt per 1000 grams of water, and for none of them is the value of m as high as 0·5. Therefore, although saturated, they cannot be called concentrated or strong solutions. As was pointed out in § 132, these values of the concentration of the mother-liquor are derived from its specific gravity by extrapolation from the ratios of concentration to specific gravity in the most concentrated solutions of the salts, as given in § 26, Tables 16 to 24. This course was adopted owing to the difficulty of determining analytically the concentration of solutions of the salts of the ennead MRO_3 and the uncertainty of the results obtained by desiccation. The dependence of the value of the concentration on that of the specific gravity of the mother-liquor excludes certain lines of discussion which were followed in the case of the solutions of the salts of the ennead MR.

It will be remarked that the specific gravities of the non-saturated solutions were all determined at 19·5° C., and are referred to that of distilled water at the same temperature as unity, while those of the mother-liquors are determined at temperatures inferior to 19·5° C., but the specific gravity of each solution is referred to that of distilled water of the same temperature as unity. This almost completely eliminates any error in the determination of the concentration of the solution which might accrue from the difference of temperature at which the specific gravities were determined. If Table 66 in § 28 be referred to, the value of possible error due to this cause can be ascertained for the two temperatures 19·5° and 23° C. The concentration of the $KClO_3$ solution would be given too low by 2 per cent.; in the case of the other solutions the error would be less than 1 per cent. But the specific gravities of the mother-liquors were determined at temperature lower than 19·5°, and the error would be less and in the opposite sense.

§ 136. In sub-table (m) we have the values of $\frac{MRO_3}{D} - \frac{v}{m}$. They are all positive; therefore in every case crystallisation is accompanied by expansion. This is small in the case of $RbClO_3$ and $CsClO_3$, considerable in that of $KClO_3$ and the bromates, and very high in that of the iodates. It is remarkable that the crystallisation of each of the three iodates is accompanied by identical expansion.

§ 137. Finally, attention must be called to the effect on the molecular displacement in crystal of the salts of the ennead MR by the addition of O_3 so as to form the corresponding salts of the ennead MRO_3.

In the following table we have in the first line the values of MR, in the second and third lines the molecular displacements MRO_3/D and MR/D respectively, in the fourth line their differences, in the fifth line their ratios, and in the sixth line the corresponding ratios MRO_3/MR of their molecular weights.

Salt. MR =	KCl.	KBr.	KI.	RbCl.	RbBr.	RbI.	CsCl.	CsBr.	CsI.
$\frac{MRO_3}{D}$	52·86	51·88	54·60	53·31	58·00	60·07	60·44	63·52	63·52
$\frac{MR}{D}$	38·23	44·46	54·58	44·71	51·55	61·99	42·31	47·82	57·67
$\frac{MRO_3}{D} - \frac{MR}{D}$	14·63	7·42	0·02	8·60	6·45	−1·92	18·13	15·70	5·85
$\frac{MRO_3}{D} \Big/ \frac{MR}{D}$	1·38	1·17	1·00	1·19	1·12	0·97	1·43	1·33	1·10
$\frac{MRO_3}{MR}$	1·64	1·40	1·28	1·39	1·29	1·41	1·28	1·22	1·18

§ 138. *Concluding Remarks.*—These will be very short. The paper has already expanded to an unexpected length, and yet, owing to the enormous amount of experimentally established material, the discussion of it which has been possible is far from adequate, but an end must be made somewhere.

The Table of Contents has been drawn up in a form which constitutes it really a recapitulation of the principal features of the paper, with reference to the paragraph and page where they are to be found, so that the reader has no difficulty in making himself acquainted with the matters dealt with in the paper, or in studying those which more particularly interest him. This being so, I will content myself by indicating here the points about the research which present the greatest interest or novelty.

Two methods of determining specific gravities are used. Neither of them is new in principle, but there are innovations in the details of both. To take the case of the determination of the specific gravity of a soluble salt in its own mother-liquor, the principle is not new, because, if the common practice of determining the specific gravity of a salt in petroleum is adopted, the liquid in which it is weighed is, or ought to be, a saturated solution of the salt from which, as a mother-liquor, crystals of the salt can be obtained; but it is obvious that this is a very different case from determining the specific gravity of chloride of cæsium in its mother-liquor, which contains in solution something like two parts of salt to one part of water. To carry out correctly this operation, the experimenter must be a trained and very experienced chemist; but it is not necessary to be an experienced chemist to perceive the experimental difficulties of the operation; it is therefore unlikely to be attempted by unsuitable hands.

The principal method used, namely, that in which the very ancient instrument, the hydrometer, is used, also requires to be practised by a trained and experienced chemist if it is proposed to obtain results of the exactness recorded in this memoir. But to most people the hydrometer is associated with a rough-and-ready method of ascertaining the specific gravity of liquids in public works, and in other similar places, where its use is commonly entrusted to a workman; and the idea of readiness, if not of roughness, is, it may be said, habitually associated with the instrument and its use. An important part of this paper is devoted to showing how the hydrometer has to be used if the best results of which the instrument is capable are to be obtained from it. It will be seen that experimental skill and perseverance, and constant attention to many minute precautions, are necessary. If this care is taken, the results will be good; if it is not taken, they will be bad.

When the hydrometric method is practised in the manner here specified, it is possible to obtain the specific gravity of liquids with greater accuracy than by any other means. Hence, in the case of saline solutions it is possible with it to carry the exact determination of the specific gravity of the solutions of a salt to much higher dilutions than is possible by other methods. It was to experiment on solutions of such high dilution that their specific gravities have hitherto escaped experimental determination, that this systematic research was originally undertaken. It will be seen that the results obtained fully justify the time and labour expended on them. It has hitherto been the general experience that, when two equal quantities of a salt are dissolved *seriatim* in a quantity of water, the diminution of the total volume of the salt and the water produced by the dissolution of the first quantity is greater than that produced by the further dissolution of the second quantity. It has been proved in this memoir that for the solutions of many salts there is a concentration below which this law is reversed. It is the first time that this has been unequivocally demonstrated. In the case of some salts which, when dissolved so as to furnish solutions of moderate concentration, exhibit considerable contraction, they at high dilutions exhibit an expansion, which may cause the volume of the solution to exceed the sum of the volumes of the salt and the water.

A similar and very remarkable feature of saturated solutions is shown in the case of the salts of the ennead MR. In the saturated solutions of the salts of potassium and rubidium the sum of the volumes of the salt and water is greater than that of the solution produced, while in the case of the solutions of the cæsium salts the reverse is the case. From this it follows that increase of pressure must assist the crystallisation of the solutions of the cæsium salts, and hinder that of the solutions of the potassium and rubidium salts.

The main purpose of this investigation was to determine the specific gravity of solutions of moderate concentration and of high dilution. In order to use the same hydrometer for these different classes of solutions, its weight was altered by the use of accessory weights attached to the top of the stem. It occurred to me during the course of the investigation that, by carrying this principle further, the use of the hydrometric method, in all its delicacy, might be extended to solutions of any degree of concentration by increasing the additions made to its weight. It was found that for our hydrometers, closed at the top, solutions having a specific gravity of 1·2 could be experimented on, but the accessory weight required was so great as almost to disturb the equilibrium of the instrument. In order to meet this difficulty, the stem of the hydrometer was left open, so that the internal weight or ballast could be varied at will. With the *open hydrometer* so constructed, saturated and even supersaturated solutions of very soluble salts have been experimented on, and results of the highest interest have been obtained.

The most noteworthy case is that of calcium chloride in supersaturated solution. In it a very remarkable state of unrest was observed before crystallisation took place. When the crystallisation of this solution is finished, the sum of the volumes of the crystals and the mother-liquor is less than that of the original supersaturated solution. The state of unrest which precedes the actual appearance of the first crystal consists in a rhythmic series of *isothermal expansions and contractions*, which cease the moment the first crystal appears and heat is liberated. The supersaturated solution exhibits veritable symptoms of *labour* before giving birth to the crystals and becoming itself a mother-liquor. The details of this remarkable phenomenon are to be found in Section XV.

It now only remains for me to discharge the pleasant duty of acknowledging my obligations to the gentlemen who have acted as my assistants in the experimental work and in the preparation of this memoir. The work has been hard and continuous, having extended to nearly ten years, and it is impossible for me adequately to express my thanks to these gentlemen for the intelligence, skill, and perseverance with which they have all devoted themselves to it.

The secretarial work connected with it has been very heavy, and it has been managed with great ability and success by Mr W. G. Royal-Dawson, to whom my best thanks are due. The pages of Tables in the memoir will suggest to anyone who is familiar with such work the amount of labour which has been expended in their preparation and verification.

The experimental work has for nearly three years been in the hands of Mr S. M. Bosworth, B.Sc., who has carried it out in a room in the Davy-Faraday Laboratory, which was admirably suited to the purpose. My thanks are especially due to Sir James Dewar and the Managers of that Institution for their generosity in putting it at my disposal. Mr Bosworth's name appears several times in the text in connection with some of the more remarkable features chronicled, more particularly in connection with the state of unrest occurring in the supersaturated solution of calcium chloride before crystallisation. It was owing to his confidence in the exactness of the readings of the hydrometer which he observed in this solution, and in the reality of the discrepancies which he observed, that the state of unrest was not only noticed but measured. Mr Bosworth was preceded as my assistant by Mr H. F. Fermor, now of the Metropolitan Water Board, to whom a large part of the experimental work recorded in the Tables is due. His work was of the highest order, and justified his selection for the responsible office which he now holds. Before him, my laboratory assistant was Mr H. Royal-Dawson, brother of Mr W. G. Royal-Dawson, and he, like all the gentlemen whom

I have been fortunate enough to have as assistants, attained the same high degree of exactness in his experimental work, so soon as he perceived that, when he took the necessary trouble with the work, it was rewarded by increased accuracy of results. This has been my invariable experience. Comparisons of work done on solutions of the same concentration of the same salt by Mr ROYAL-DAWSON, and afterwards by Mr BOSWORTH, are quoted in the memoir, and they furnish evidence of the excellence of the work put out by both these chemists.

Nearly the whole of the experimental work of this memoir has been done by the gentlemen just mentioned. It would, however, be unjust if I did not refer to the great and valuable work with the hydrometer done for me at an earlier date by my old and valued friend and former assistant, Mr ANDREW KING of the Heriot-Watt College, Edinburgh. The exactness of his work is of the highest order, and his intimate knowledge of, and sympathy with, my work for many years has been of the utmost value to me, and I wish to take this occasion to make public acknowledgment of the debt of gratitude which I owe to him.

APPENDIX A.

DENSITIES OF THE SOLUTIONS AT T.

In the following tables the specific gravities of the solutions have been reduced to their value when referred to that of distilled water at 4° C. as unity. The factors used for this purpose are:—

for T =	15·0°	19·5°	23·0°	26·0°
factor =	0·999173	0·998372	0·997614	0·996879

For example, $_{15^\circ}S_{15^\circ}$ of ½ NaCl is 1·020564. Therefore its density $_{4^\circ}S_{15^\circ} = 0{\cdot}999173\ _{15^\circ}S_{15^\circ} = 1{\cdot}019720$.

CHLORIDES. MCl.

M =	Na.	K.	K.	Rb.	Cs.	K.	Rb.	Cs.
T =	15·0° C.		19·5° C.			23·0° C.		
m.								
1/2	1·019720	1·022321	1·021312	1·041446	1·060842			
1/4	1·009597	1·011063	1·010023	1·020204	1·030059			
1/8	1·004427	1·005080	1·004251	1·009377	1·014340			
1/16	1·001821	1·002123	1·001340	1·003903	1·006395	1·000531	1·003086	1·005549
1/32	1·000494	1·000659	0·999859	1·001139	1·002400			
1/64	0·999827	0·999888	0·999112	0·999770	1·000396			
1/128	0·999495	0·999538	0·998736	0·999078	0·999395			
1/256			0·998565	0·998721	0·998885			
1/512			0·998454	0·998535	0·998620			

BROMIDES. MBr.

M=	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	19·5° C.			23·0° C.		
m.						
1/2	1·039583	1·059519	1·079175			
1/4	1·019241	1·029411	1·039316			
1/8	1·008883	1·013935	1·019040		1·013316	1·018237
1/16	1·003642	1·006227	1·008764	1·002907	1·005490	1·007975
1/32	1·001006	1·002310	1·003545		1·001597	1·002847
1/64	0·999676	1·000326	1·000999		0·999577	1·000242
1/128	0·999023	0·999354	0·999640		0·998598	0·998943
1/256	0·998696	0·998828	0·998978			
1/512	0·998530	0·998605	0·998679			
1/1024		0·998450	0·998517			

IODIDES. MI.

M=	K.	Rb.	Cs.	K.	Rb.	Cs.	Cs.
T=	19·5° C.			23·0° C.			26·0° C.
m.							
1/2	1·057205	1·076665	1·096000	1·056113			
1/4	1·028229	1·038085	1·048131	1·027260			
1/8	1·013451	1·018349	1·023304	1·012568	1·017641	1 022635	
1/16	1·005948	1·008402	1·010880	1·005140	1·007682	1·010221	1·009463
1/32	1·002156	1·003404	1·004661	1·001366	1·002645	1·003940	
1/64	1·000268	1·000873	1·001487	0·999528	1·000163	1·000769	
1/128	0·999220	0·999607	0·999915	0·998562	0·998878	0·999206	
1/256	0·998851	0·998923	0·999108	0·998100	0·998265	0·998526	
1/512	0·998607	0·998644	0·998644				
1/1024	0·998494	0·998518	0·998472				

NITRATES. $M'NO_3$ and $M''(NO_3)_2$.

M′ or M″ =	Na.	K.	Sr″.	Ba″.	Li.	Na.	Ba″.	Pb″.	Rb.	Cs.
T=	15·0° C.				19·5° C.				23·0° C.	
m.										
1/2		1·030021			1·018047	1·026137				
1/4		1·014860			1·008389	1·012472			1·023143	
1/8		1·007140			1·003395	1·005479			1·010648	1·015514
1/16	1·002623	1·003136			1·000916	1·001954	1·011652	1·016131	1·004182	1·006672
1/32	1·000887	1·001145	1·004513	1·005886	0·999660	1·000171	1·005058	1·007304	1·000960	1·002139
1/64	1·000036	1·000157	1·001844	1·002547	0·999025	0·999271	1·001734	1·002869	0·999341	0·999897
1/128	0·999604	0·999663	1·000523	1·000865	0·998707		1·000079	1·000618	0·998567	0·998797
1/256			0·999838	1·000008			0·999227	0·999498	0·998017	0·998193
1/512			0·999502	0·999595			0·998804	0·998948		
1/1024				0·999391			0·998577	0·998672		

TABLES giving a Summary of the Densities of the Solutions of different Salts at different Temperatures.

TRIADS OF NITRATES, CHLORATES, BROMATES, AND IODATES. MRO_3.

RO_3=	NO_3.			ClO_3.			BrO_3.			IO_3.		
M=	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.
T=	19·5° C.			19·5° C. and *23·0°* C.			19·5° C. and *23·0°* C.			19·5° C. and *23·0°* C.		
m.												
1/2	1·028886	1·048924										
1/4	1·013880	1·024028	1·043933	1·017422	1·027478	1·037351	1·030146			1·042602		
1/8	1·006232	1·011314	1·016304	1·007994	1·013027	1·018026	1·013570			1·020662		
1/16	1·002334	1·004958	1·007397	1·003227	1·005716	1·008181	1·006022	1·008609	1·011107	1·009523	1·012027	1·014644
1/32	1·000382	1·001721	1·002950	1·000758	1·002057	1·003317	1·002212	1·003487	1·004739	1·003952	1·005215	1·006501
1/64	0·999374	1·000119	1·000615	0·999623	1·000232	1·000777	1·000290	1·000934	1·001578	1·001128	1·001771	1·002388
1/128	0·998880	0·999290	0·999516	0·999004	0·999289	0·999586	0·999328	0·999630	0·999986	0·999773	1·000059	1·000317
1/256		0·998829	0·998975	0·998691	0·998830	0·998923	0·998847	0·999013	0·999245	0·999080	0·999198	0·999300
1/512				0·998554	0·998590	0·998581	0·998609	0·998691	0·998746	0·998732	0·998807	0·998820
1/16				*1·002362*	*1·004927*	*1·007476*	*1·005164*	*1·007752*	*1·010343*	*1·008734*	*1·011206*	*1·013801*

STRONG SOLUTIONS (*Pyknometer*).

TABLES giving a Summary of the Densities of the Solutions of different Salts at different Temperatures.

R or RO_3=	Cl.			Br.			I.			NO_3.		
M=	K.	Rb.	Cs.	K.	Rb.	Cs.	K.	Rb.	Cs.	Rb.	Li.	Na.
T=	19·5° C.			19·5° C.		21·4° C.	19·5° C.		23·1° C.	19·5° C.		
m.												
1/2		1·0409	1·0599		1·0596			1·0753		1·0488		
1	1·0432	1·0814	1·1060	1·0790	1·1175	1·1467	1·1128	1·1469	1·1718	1·0964	1·0372	1·0525
2	1·0835	1·1573	1·2250	1·1526	1·2261	1·3015	1·2157	1·2814	1·3431	1·1842	1·0730	1·1012
3	1·1204	1·2263	1·3223	1·2200	1·3250	1·4297	1·3097	1·4003	1·4887	1·2654	1·1063	1·1459
4	1·1543	1·2915	1·4090	1·2832	1·4155	1·5516	1·3959	1·5077			1·1373	1·1871
5		1·3519	1·4931	1·3403	1·4985	1·6590	1·4766	1·6055			1·1665	1·2247
6		1·4061	1·5644		1·5746		1·5458	1·6944			1·1940	1·2592
7		1·4562	1·6447				1·6115	1·7676			1·2194	1·2918
8			1·6997				1·6722				1·2437	1·3231
9			1·7561								1·2663	1·3517
10											1·2885	

APPENDIX B.

TABLE giving the Number of Series as well as the Number of Single Observations made with the various Hydrometers, from which the results recorded in this Memoir were obtained.

Hydrometer.		Number of Series obtained.	Number of Single Observations made.
Type.	Designation.		
Closed	No. 3	394	3,546
,,	,, 17	1287	11,583
,,	,, 21	511	4,599
Open	A	207	2,183
,,	B	164	1,616
,,	1910, No. 3	22	198
Total . .		2585	23,725

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